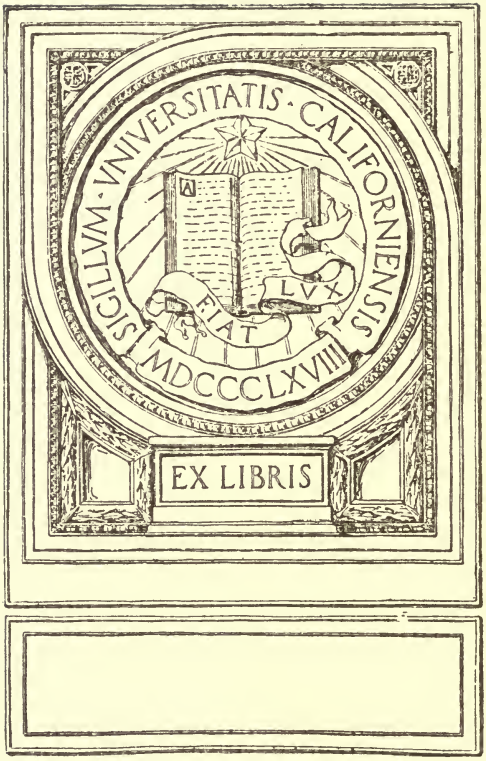
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THERMODYNAMICS,

HEAT MOTORS,

AND

REFRIGERATING MACHINES.

BY

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SEVENTH EDITION.

REVISED AND ENLARGED.

NEW YORK:

JOHN WILEY & SONS,

53 EAST 10TH STREET.

1895.

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PREFACE.

The following work has been prepared to meet a want experienced by myself in my course of instruction in Thermodynamics.

After reading several works upon the subject, including those of the founders of the science—Rankine, Clausius, Thomson—I was most favorably impressed with the spirit of Rankine's mode of discussing the subject. It is in keeping with the modern method of treating Analytical Mechanics, in which the analysis is founded upon ideal conditions established by definitions, and the resulting formulas modified to represent the infinite variety of conditions in nature.

But Rankine's giant-like processes are not adapted to the wants of the average student. Article 241 of his Steam Engine and other Prime Movers reaches the height of sublimity in regard to terseness, comprehensiveness, and obscurity. Without a proper preliminary, he crowds into a few words a principle which has cost other writers protracted labor and heroic efforts to establish.

My aim has not been to bring down the subject to the comprehension of the reader, but to lead him up, by a more easy and uniformly graded path, to the same height, and at the same time familiarize him with the way by a free use of illustrations, exercises, historic references, and numerical examples.

The body of the work contains a development of the essential principles of the subject, to which I have added an Addenda, for the purpose of enlarging upon the matter contained in some of the articles, more especially those pertaining to vapors. This enabled me to follow the thread of the subject more closely without turning aside to consider applications to a variety of substances, and to enlarge more freely upon those secondary matters when separated from the body of the text.

Special attention is called to the graphical representation of internal work, as in Figs. 36 and 37, supposed to be new, as well as many of the exercises and the discussion of *Entropy*, or the *Thermodynamic Function*.

De Volson Wood.

Hoboken, Sept., 1888

THIRD EDITION.

The treatment of the theoretical part of Thermodynamics, including its application to the steam engine, as far as page 180, is the same in this edition as in the first and second editions. Since the first edition there have been added the following subjects: Vapor Engine; Sterling's Engine; Ericsson's Hot-Air Engine; Gas Engine; Naphtha Engine; Ammonia Engine; Steam Injector; Pulsometer; Compressed-Air Engine; The Compressor; Steam Turbine; Refrigerating Machines; Miscellaneous matter in an Addenda; Combustion of Fuel; Steam, Ammonia and other Tables. The Ammonia Tables have been computed from the formulas of the author and are new.

The Author.

August, 1889.

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ERRATA.

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21. Second line, art. 33*a*, cancel "closed."
25. In table, last column, second number up, read "(774.3)."
29. Line 10, read "The unit-mass as here used is."
48. Line 7, for "mechanical" read "dynamical."
49. After Eq. (27), for "$p\ d\ \tau$" read "$R\ d\ \tau$."
62. For the last two equations of Ex. 1 read

$$\text{“}W\ (65 - 50) = 2 \times 0.1689\ (\tau_2 - 525) \times 0.0807 \times \frac{873}{492};$$

$$\therefore\ W = 1.122 + \text{”}$$

106. Line 8, cancel "If p be the pressure," etc., to end of sentence.
113. For Eq. (99) read "If the vapor be steam generated from water at T_0 degrees Fah. evaporated at T_1 degrees, and superheated at constant pressure to T_2 degrees,

$$H = 778\,[(T_1 - T_0) + 1121.7 - 0.695\ T_1 + 0.48\,(T_2 - T_1)]$$
$$= 778\,[1121.7 - 0.175\ T_1 - T_0 + 0.48\ T_1]. \qquad (99)\text{”}$$

146. Sixth line, at end of, insert "(135*a*)."
146. Fourth line up, to read "$x\left(\frac{d\ H_e}{d\ \tau}\right)$."
146. Third line up, transfer "(" to just before "in which."
147. Eq. (136), for "$\frac{H_e}{d\ \tau}$" read "$\frac{H_e}{\tau}$."
147. Above Eq. (138), for "(35*a*)" read "(135*a*)."
147. Eq. (139), for "x" read "S."
147. Eq. (136) is equivalent to $C\ d\ \tau + \tau\ d\left(\frac{x\ H_e}{\tau}\right)$.
153. Eq. (149*a*), for "C" read "c."
153. Above Eq. (149*a*), for "$c\ J\ \frac{h_e}{\tau}$" read "$J\ \frac{h_e}{\tau}$."
162. For "H_3" read "H'_3."
162. Two lines up, for "D" read "J."
174. Line 5, for "τ" read "T."
215. Sixteenth line up, for "47.83" read "37.83."
217. Line 7, for "18.5" read "15.4."
247. Change "backward" to "forward," and the converse.
252. Line 15, for "Dugold" read "Dugald."
254. Opposite Fig. 66, for "C_3" read "$C\ O$."
257. Line 15, for "128" read "129."
265. Line 12, for "5½" read "8."
303. Eq. (320), for "τ_0" read "τ_1."
303. One line below Eq. (320), for "(309)" read "(315)."
319. Line 8 up, for "$M\ N$" read "$m\ N$."
320. Line 15 up, for "D" read "d."
325. Line 1, for "$S\ O_2$" read "$S_2\ O$."
349. Line 8, for "370.65" read "360.65."
351. Line 6, for "Eq. (358).... 565.6" read "Eq. (360).... 556.7."
351. Line 10, for "565.6" read "556.7;" for "509.92" read "501.02."
360. Line 8 up, for "Bisulphuret" read "Bisulphide."
361. Line 12 up, for "sulphate" read "Bisulphide."
435. Line 6, for "saturation" read "boiling points."

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THERMODYNAMICS.

CHAPTER I.

FUNDAMENTAL PRINCIPLES AND GENERAL EQUATIONS.

1. Heat is energy.—Energy is a capacity for doing work, and it has been shown in many ways that heat, by its action upon substances, can do work. Thus, it may cause steam to drive a piston; it causes solids and liquids to expand, and changes the molecular condition of bodies, as when solids are fused or liquids vaporized. Heat is also recognized as a sensation.

2. Heat is not material.—A body has the same weight when hot as when cold. Count Rumford, in 1798, discovered that he could boil a large quantity of water by the heat produced in boring a piece of cannon. Sir Humphry Davy (about 1799) melted ice by rubbing two pieces together without heat being imparted to them.

3. Heat consists of a motion of the particles of a body.—The only known method of directly measuring energy is by a combination of mass and velocity; thus, if m be the mass of a body and v its velocity, then will its kinetic energy be $\frac{1}{2}mv^2$. The mass being constant while the body is heated we infer that its heat energy is produced by the velocity of its mass elements. These motions are invisible, and hence their character can only be inferred; it may be a motion of the ultimate atom, or of an atmosphere

about the atom, a vibratory or periodic motion of some kind, or a combination of simple motions. It is probably not a to-and-fro rectilinear motion of the molecule. A development of the theory of heat, fortunately, does not require a knowledge of this motion, or even a particular hypothesis, beyond the fact that there is a motion of some kind. Rankine constructed an hypothesis called "molecular vortices," from which he deduced many important consequences pertaining to heat. (See *Edinburgh Trans.*, vol. xx.; *Philosophical Mag.*, 1851 and 1855.)

4. Velocity of heat.—The perfect identity of the laws of radiant heat with those of light as to reflection, refraction, interference and absorption, and the identity of their velocities, being 186,300 miles per second, requires essentially the same theory as to their nature and mode of propagation. Electricity is also a form of energy and governed by laws similar to those of heat. As light is transmitted by means of a subtle ether pervading all space, and called the *luminiferous ether*, so it is believed that the same ether transmits heat, electricity and magnetism. (See Appendix.)

5. Heat-energy is measured only by its effects.—The kinetic energy of a mass may be computed if its mass and velocity are known, or it may be determined by the work it does in being brought to rest, but since the velocity of the particles producing heat cannot be measured, heat-energy can be measured only by its effects. Thus, if a ball of hot iron would just melt one pound of ice, and after being heated again would just melt two pounds of ice, then would the ball in the second case have contained twice the heat above the melting-point of ice that it did in the first case. Similarly, it requires about twice as much heat to raise the temperature of a given amount of water two degrees as it does one degree. The same principle applies

to other bodies of one substance having different weights and to bodies composed of different substances, or to heterogeneous substances. For scientific purposes some specific effect must be assumed as a *standard*, and considered as a *unit*.

6. The thermal unit is the heat necessary to raise the temperature of unity of weight of water at its maximum density one degree.

Water is at its maximum density at 39.1° F. (4° C.). According to the experiments of Kopp, its volume is 1.00012 at 32° F., 1.00000 at 39.1° F., 1.00011 at 46° F., and 1.04312 at 212° F.

The British Thermal Unit (B. T. U.) is the heat necessary to raise the temperature of one pound of water from 39° F. to 40° F.

The French Calorie is the heat necessary to raise the temperature of one kilogramme of water from 4° C. to 5° C., and is 3.968 times the B. T. U.

Some writers, in defining the thermal unit, start the measurement with the temperature of melting ice, instead of at 39° F., and although there is but little difference between the two values thus obtained, yet for scientific purposes and for physical reasons, the latter is preferable, and should be generally adopted.

7. Work.—When heat-energy disappears as heat, it must, according to the principles of the conservation of energy, appear or exist in some other form of energy. When the heat in steam drives the piston of an engine, the steam loses heat by the operation, and an exact equivalent of the energy so disappearing reappears as work done or as energy in the moving parts of the engine, no allowance being made in this illustration for losses due to radiation or friction. To aid one in this conception conceive that one end of the cylinder is filled with small, perfectly elastic balls, bounding

and rebounding between the head of the cylinder and the piston; they will, by their continued action, produce a pressure upon the piston. If the piston moves forward the energy of the balls will be diminished, as is shown in mechanics in the discussion of the impact of elastic bodies, and this loss of energy will equal that imparted to the piston, or, if the piston moves uniformly, equal to the work done. In general, when heat-energy disappears it is said that an equivalent amount of work has been done, although the entire work may not be visible energy. Some of it may produce molecular changes in the substance. In the preceding illustration, if the piston be forced inward against the rebounding balls, their velocity will be increased, and hence their energy will be increased by an amount equal to the work imparted to them by the piston.

8. Internal work is some kind of effect produced upon the molecular character of a substance. Thus, if one pound of water at 32° F. be mixed with one lb. at 33° F. it will produce two pounds of water at 32½° F., but if one pound of ice at 32° F. be mixed with 1 pound of water at 33° F. the temperature of the mixture will be 32° F. Indeed, it is found by experiment that it will require about 144 pounds of water at 33° to melt one pound of ice producing 145 pounds of water only a very little above 32° F., so that nearly all the heat between 32° F. and 33° F. in the 144 pounds of water is necessary to change solid water (ice) at 32° to liquid water at the same temperature. Similarly, a large amount of heat is absorbed in changing liquid water at 212° F. to gaseous water (steam) at the same temperature. This disgregation of the molecular structure is called *internal work*, or energy of a potential form.

9. The actual heat-energy of a substance is dependent upon its temperature. The heat absorbed by a substance may do external work, as in driving a piston, and

internal work, as shown in the preceding article, and in addition to both it may increase the temperature of the substance, thus increasing its energy. The last is called *actual energy*. The actual energy is some function of the temperature.

10. Latent heat is heat which produces effects other than that of change of temperature. Strictly speaking, it is not heat, but is a measure of the heat which has been destroyed in producing effects other than that of changing the actual energy of the substance. Thus, heat becomes latent in producing changes in the state of aggregation of the substance, as in fusion, vaporization or sublimation; and as defined in Article 8, constitutes internal work. But it also becomes latent in doing external work by expansion, and if the temperature be maintained constant during expansion, the heat destroyed in doing the work is called *the latent heat of expansion.*

11. General expression.—The total heat in a definite weight of any substance is unknown, although if gases were perfect it might be computed, as will hereafter be shown; but it is possible to find expressions for the heat absorbed in passing from one known state to another, for we have

$$\begin{aligned} \textit{Heat absorbed} &= \textit{change of actual energy} + \textit{change of potential energy} + \textit{external work};\\ &= \textit{total change of internal energy} + \textit{external work};\\ &= \textit{change of actual energy} + \textit{total work}. \end{aligned}$$

In this expression the total internal energy includes all the heat involved both in changing the temperature and the internal structure of the substance.

12. Temperature is a condition of relative heat. Experience shows that when two bodies, one hotter than the

other, are near each other, the hot body becomes cooler and the cooler one hotter. Heat of itself passes from a hotter to a colder body, and this process cannot be reversed except by an expenditure of mechanical energy. The hotter body is said to have a higher temperature than the colder one.

Temperature is not an indication of the quantity of heat absorbed by a body, nor of the amount of heat in a body, but of the intensity of the heat. Thus, if a pound of iron has the same temperature as a pound of water, the latter will contain about eight times as much heat as the former for each degree, as would be found by putting each pound into another quantity of some liquid at a different temperature.

Temperature is a measure of the sensible heat—that is, actual heat—which can affect the senses.

13. Thermometers are instruments for measuring differences of temperature. The more common ones depend for their action upon the expansibility of a liquid—such as mercury or alcohol. The liquid is confined in a tube as nearly cylindrical as possible, within which it expands. When the expansion of metals is employed for determining temperature, the instruments used are called *pyrometers*.

The air thermometer depends for its action upon the pressure produced by heat at constant volume.

All thermometers have *two fixed points:* one the melting point of ice, the other the boiling point of water at atmospheric pressure.

The melting point of ice is a more nearly fixed point than the freezing point of water. In some carefully conducted experiments water has been reduced several degrees below the ordinary freezing point, 32° F., before freezing. To secure such a result, the water must be kept in a condition of as perfect rest as possible. The boiling point of water depends upon the pressure to which it is subjected; and since the pressure of the atmosphere is continually changing, as shown by the barometer, the pressure of one

atmosphere must be *fixed* for scientific purposes. The value determined by Regnault, and now generally adopted, is 2116.2 pounds per square foot, or 14.7 pounds, very nearly, per square inch. In determining the boiling point of water the thermometer should be placed in the vapor near the water.

The Fahrenheit scale has 180 equal divisions between the fixed points, and the zero of the scale is 32 such divisions below the melting point of ice. It is designated by *F.*, or Fahr.

The Centigrade scale has 100 equal divisions between the fixed points, its zero being at the lower or melting point of ice. It is indicated by *C.* It is sometimes called the Celsius scale.

The Réaumur scale has 80 equal divisions between the fixed points, its zero being at the lower.

To reduce the readings of one scale to those of another, the following equations may be used:

$$C = \tfrac{5}{9}(F-32°);\ F = \tfrac{9}{5}C + 32°;\ R = \tfrac{4}{5}C.$$

The construction here implied assumes that liquids expand equally for equal quantities of heat, and that the tubes containing them are uniform; but neither of these conditions are exactly realized, the practical considerations of which belong to *Thermometry.*

14. The Air Thermometer.—In order to gain an idea of an elementary air thermometer, conceive a small, perfectly cylindrical tube closed at the lower end to contain a quantity of air, limited at its upper end by a drop of mercury acting as a piston. Subject this instrument to the temperature of melting ice under the pressure of one atmosphere, 29.922 inches of mercury, and mark the upper end of the air column; then, next subject it to the temperature of boiling water under the same pressure and mark the upper end of the air column at this temperature. The two marks will be the *fixed points* before described. If the length of

the column from the lower end to the lower mark be unity, then will its length to the upper mark be 1.3665 as found by Regnault. The expansion is 0.3665 of its original volume. For the Fahrenheit scale the space between the fixed points would be divided into 180 equal parts, and hence each part would be $\frac{0.3665}{180} = 0.00203611$ of the distance below the lower fixed point. If the length below the lower fixed point be divided into equal parts of the same magnitude, the number of such spaces will be,

$$\frac{1}{0.00203611} = \frac{180}{0.3665} = 491.13.$$

If these parts are numbered according to the natural numbers, 0, 1, 2, 3, etc., beginning with zero at the extreme lower end of the tube—called the absolute zero of the air thermometer—then would the temperature of melting ice be 491.13° F. from the absolute zero of the air thermometer, and that of boiling water 671.13° F. from the same zero. If air were a perfect gas, this would constitute an *absolute* scale, but as it is not, a correction is required in order to establish such a scale. For air thermometers the pressure at constant volume is commonly used, instead of the volume at constant pressure as above described.

15. A perfect gas is defined to be such that, under a constant pressure, its rate of expansion would be exactly equal to its rate of increase of temperature, and, the volume being constant, increments of pressure will be equal for equal increments of heat. In other words, it would be a substance in which no internal work would be done by changes of temperature or pressure. No such substance is known—it is *ideal*, subjected merely to a definition and to laws to be assigned—and yet it is of great service in this science. The idea of a perfect gas was the result of experiments upon existing gases, as air, oxygen, hydrogen, etc., which, at first, were supposed to be represented by the per-

fect law. In mechanics, at the present time, the bodies treated are, at first, the subjects of definition, and considered perfect, as *perfect solids*, *perfect liquids*, *perfectly elastic*, etc., and the results obtained from these hypotheses made practical by the introduction of moduli the values of which are found by experiments. The same method is adopted in this science.

16. An absolute scale is one whose divisions would be indicated by a perfect gas thermometer. On such a scale the divisions would be exactly equal for equal increments of heat down to the zero of the scale. Since a perfect gas is unknown, the *zero of the absolute scale* can be determined only approximately by computation, as will be shown hereafter, where the best result yet obtained fixes it at 492.66° F. below the melting point of ice. The letter F. here affixed implies that there are 180 divisions between the fixed points, as in Fahrenheit's scale. This zero on the centigrade scale is $\frac{5}{9}$ of $492.66° = 273.7°$ C. Temperature on the absolute scale will generally be indicated by the Greek letter τ, and the temperature of melting ice by τ_0. If T° F. indicate the temperature from the zero of the Fahrenheit scale, and T° C. from the zero of the centigrade scale, we will have

$$\tau_0 = 492.66° \; F = 273.7° \; C.$$
$$\tau = 492.66° \; F + T° \; F.$$
$$= 273.7° \quad C + T° \; C.$$

It is found that air is so nearly a perfect gas within the ranges of temperature and pressure for which it has been tested that it may be considered as such for all practical purposes, and will be so considered theoretically except in the determination of the place of the zero of the absolute scale. Further, the ordinary mercurial thermometer agrees sufficiently well with the air thermometer for the more ordinary ranges of temperature met with in engineering prac-

tice to be used in such cases. But for scientific purposes and for extreme cases in practice, the difference is too large to be ignored. Regnault found that when the air thermometer marked 630° F. above the melting point of ice, the mercurial thermometer indicated 651.9° above the same point, a difference of about 22°. Liquids generally expand more rapidly the higher the temperature.

17. The absolute zero of temperature is the zero of the absolute scale, and corresponds to the condition of total deprivation of heat; at which temperature no substance could exercise any expansive power. This temperature has never been reached, and the nearest approach to it has been produced by expansion in liquefying air, oxygen and nitrogen, reaching —373° F. (—225° C.), or more than $\frac{4}{5}$ the distance from the zero of Fahrenheit to the absolute zero. (*Comptes Rendus*, Feb. 9, 1865; *Jour. Frank. Inst.*, Sept. 1886, p. 213). The absolute zero is about 492.66—491.13 =1.5 degrees Fahr. below the zero of the air thermometer, as computed on the hypothesis of the same rate of contraction of air below 32° as from 32° to 212°. This law might change as the temperature was extremely reduced, but it would continue uniform for the ideally perfect gas.

18. The equation of a gas is an equation expressing a continuous relation between its volume, pressure and temperature throughout a finite range of the same. Let p be the pressure on a unit of area of the substance when the volume of one pound is v and absolute temperature is τ, then, generally,

$$p = f(v, \tau),$$

which may be considered as the equation to a surface, called the *thermodynamic surface.*

19. Equation of a perfect gas.—According to the definition in Article 15,

$$(p)_v \propto \tau, \text{ and } (v)_p \propto \tau,$$

where the subscripts represent the quantities which are constant while the others vary, and combining these in one expression, we have

$$p\,v \propto \tau,$$

or,

$$\frac{p\,v}{\tau} = \frac{p_1\,v_1}{\tau_1}, \qquad (1)$$

where p_1, v_1, τ_1 are contemporaneous fixed values. Let p_0 be the pressure of one atmosphere, τ_0 the absolute temperature of melting ice, and v_0 the volume of one pound of the gas at that pressure and temperature, then will equation (1) become

$$\frac{p\,v}{\tau} = \frac{p_0\,v_0}{\tau_0} = R \text{ (say)}, \qquad (2)$$

which is the equation required. The values of p_0 and τ_0 have already been given and are independent of the nature of the gas, but v_0 depends upon the density of the gas. A cubic foot of dry air weighs at sea level at the temperature of melting ice

$$w_0 = 0.080728 \text{ pounds};$$

hence,

$$v_0 = \frac{1}{0.080728} = 12.387;$$

$$\therefore \frac{p_0\,v_0}{\tau_0} = \frac{2116.2 \times 12.387}{492.66} = \frac{26214}{492.66} = 53.21,$$

when τ_0 is measured from the zero of the absolute scale; but if it be from the zero of the air thermometer, we have

$$\frac{p_0\,v_0}{\tau_0} = \frac{26214}{491.13} = 53.37;$$

and equation (2) becomes,

for the absolute scale, $p\,v = 53.21\,\tau$, (3)

for the air thermometer, $p\,v = 53.37\,\tau$. (3′)

For French units, let

p_0' = the pressure of one atmosphere in kilogrammes per square metre,

v_0' = the volume of a kilogramme of the gas in cubic metres,
τ_0' = the absolute temperature of melting ice on the centigrade scale; then

$$\frac{p_0' v_0'}{\tau_0'} = \frac{\frac{(3.28)^2}{2.2} p_0 \times \frac{2.2}{(3.28)^3} v_0}{\frac{5}{9} \tau_0} = \frac{1}{1.8222} \frac{p_0 v_0}{\tau_0},$$

which for air becomes in kilogramme—metre—centigrade units (k. m. c.), omitting the accents,

$$p v = 29.20 \tau;$$

and in decimetre—kilo.—centigrade units,

$$p v = 2.920 \tau.$$

20. Mariotte's law.—If the temperature be constant, equation (2) shows that the volume varies inversely as the pressure; a law discovered by direct experiments upon gases, and known as Mariotte's law, supposed by some to have first been discovered by that investigator, but by others this credit is given to Boyle. For a time after the announcement of the law it was supposed to be *perfect* for the so-called permanent gases, but more refined experiments have shown that the actual law governing them is only a very close approximation to it.

21. Law of Gay Lussac (or of Charles).—According to equation (2), if the pressure be constant, the volume will increase directly as the temperature, or

$$d v = \frac{R}{p} d \tau,$$

a law discovered by Gay Lussac (or, according to some, by Charles) by experiments upon actual gases, and known as the law of Gay Lussac. At first it was supposed to be the perfect law of the so-called permanent gases, but it is now known not to be exact though very nearly so. Originally, it was not stated in terms of the absolute temperature, as that term was not then known, but the law of the increments is the same on any thermometric scale.

22. The so-called imperfect gases include all such as cannot be represented with sufficient accuracy by

equation (2). All known gases are imperfect, strictly speaking, but the permanent gases are so nearly perfect that they may, for engineering purposes, be considered as perfect. No single formula can represent exactly the law of imperfect gases, but the most comprehensive one, and one which may be made to represent actual substances with sufficient accuracy for practical purposes, was deduced by Rankine from his theory of *Molecular Vortices*, and is

$$\frac{p\,v}{\tau} = \frac{p_0\,v_0}{\tau_0} - \frac{a_0}{\tau} - \frac{a_1}{\tau^2} - \frac{a_2}{\tau^3} - \&c., \qquad (4)$$

in which a_0, a_1, a_2, &c., are functions of the density to be determined by experiment; but as the theory here referred to is not a recognized part of science, the formula is accepted only so far as it conforms to the results of experiment. (Rankine's *Sc. Papers*, 32.)

For carbonic acid gas the form of the equation, as confirmed by the experiments of Regnault, becomes

$$p\,v = \frac{p_0\,v_0}{\tau_0}\,\tau - \frac{b}{\tau\,v} = R\,\tau - \frac{b}{\tau\,v}, \qquad (5)$$

in which $p_0 = 2116.2$, $v_0 = 8.1572$, $p_0\,v_0 = 17262$, $\tau_0 = 492.66°$, $b = 481600$;

$$\therefore\ p = 35\,\frac{\tau}{v} - \frac{481600}{\tau\,v^2}. \qquad (6)$$

Sir William Thomson and Dr. Joule used, for imperfect gases, the formula,

$$p\,v = R\,\tau - \left(\alpha - \frac{\beta}{\tau} + \frac{\gamma}{\tau^2}\right)\frac{1}{v}, \qquad (7)$$

in which for air the constants for French units are

$R = 2.8659$, $\alpha = 777.386$, $\beta = 844560$, $\gamma = 214325840$. (*Phil. Trans.* [1854], CXLIV., 360).

23. Thermal lines.—Any line the co-ordinates of which represent the contemporaneous relation between the pressure, volume, and temperature of a body subjected to

thermal conditions, is a thermal line. Ideally, it may be any line on a thermodynamic surface; actually, the *projection* of a thermal line on any one of the co-ordinate planes is called a thermal line, and geometrically it is called *the path of the fluid*, although the latter refers to the projection on the co-ordinate plane, $p\,v$, unless otherwise stated. Thermal lines on the plane $p\,v$ constitute *a diagram of energy*. If the pressure p be constant, the line is called an *isobar*; if the volume v be constant, it is called an *isometric*. Thermal lines were introduced into this science by M. Clapeyron.

24. Isothermal lines represent the relation between the pressure and volume when the temperature is maintained constant. In equation (2) if τ be constant we have

$$p\,v = R\,\tau = m, \tag{8}$$

for the equation of an isothermal of a perfect gas. It is an equilateral hyperbola referred to its asymptotes as shown in Fig. 1, in which $O.a$ is the axis of the hyperbola, the branches of which will be asymptotic respectively to the axes $O\,v$ and $O\,p$; $O\,v$ being the axis of volumes and $O\,p$ the axis of pressures.

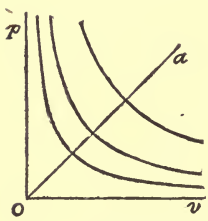

FIG. 1.

EXERCISES.

1. Construct an isothermal for air considered as a perfect gas.

Assume a temperature of 60° F. or call it 520° F. absolute, then

$$p\,v = 27669$$

$v = 10,$	$p = 2766.9$
$v = 100,$	$p = 276.69$
$v = p,$	$p = 166.4$
$v = 1000,$	$p = 27.669$
&c.	&c.

These numbers are so large we take $\frac{1}{100}$ of their values as inches, or parts

of an inch, and construct the curve as in Fig. 2. But for the equilateral hyperbola it is unnecessary to compute any co-ordinates except for the vertex c; for, having found c by making $p = v = O\,a = a\,c$, bisect $O\,a$ at d and make $d\,e = 2\,a\,c$, &c.; and make $O\,g = 2\,O\,a$ and $g\,h = \frac{1}{2}\,a\,c$, &c.

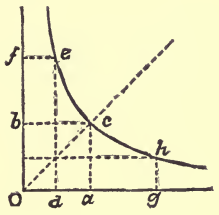

FIG. 2.

2. Construct an isothermal for air at the temperature of 1° F. absolute.

3. Find the vertex of the hyperbola of the isothermal for air whose temperature is $T = 400°$ F.

4. Find R for the following gases:

For hydrogen, $v_0 = 178.83$, $R =$.

nitrogen, $v_0 = 12.75$, $R =$.

oxygen, $v_0 = 11.20$, $R =$.

5. Find the value of R in French units for hydrogen.

6. Find the equation to the isothermal for carbonic acid gas for the temperature $T = 60°$ F.

7. What is the volume of air, considered as a perfect gas, under the pressure of four atmospheres and absolute temperature of $\tau = 800°$?

8. If the heat in one pound of carbon is 14500 B. T. U., how many pounds of carbon completely consumed are necessary to increase the temperature of 2000 pounds of water 45° F.?

9. How many kilogrammes of water would be raised 25° C. by the heat in one pound of carbon?

10. On a diagram of energy draw on the plane $v\,\tau$ the locus of the path of a perfect gas when the pressure is constant.

11. Find the pressure per square inch of two pounds of air when its volume is one half of a cubic metre and its absolute temperature is 500° C.

12. Show that all isothermals of a perfect gas are asymptotic to each other as well as to the co-ordinate axes p and v.

13. What is the temperature of a pound of air when its

volume is 5 cubic feet and pressure 35 pounds per square foot?

14. What is the weight of a cubic foot of air when the pressure is 50 pounds per inch and the temperature 160° F.?

25. Adiabatic or Isentropic lines represent the relations between the volume and pressure of a substance doing work by expansion without transmission of heat. Conceive a gaseous substance to be enclosed in a cylinder having a frictionless piston, it will, by driving the piston, do work. It will be conceived that the external pressure is infinitesimally less than the internal during expansion. The temperature of the enclosed gaseous substance may depend upon several conditions. If heat be properly supplied the temperature may be maintained constant, producing isothermal expansion, which may be represented by the line $A\,b$, Fig. 3. Having performed that operation, bring the substance to its initial state A, and conceive the expansion to take place without any transmission of heat, to do which the vessel must be considered as impermeable to the passage of heat, in which case the external work will be at the expense of the heat-energy of the substance, and therefore the temperature will fall as expansion proceeds, and the pressure will also fall on account of the loss of temperature, as shown by equation (2), and the line $A\,d$ representing the continuous relation between the volume and pressure will be lower than the isothermal $A\,b$, and its slope downward greater for equal volumes. If the substance be compressed from state A, the line $A\,e$ will be above the isothermal $b\,A\,c$. The line $e\,A\,d$, representing the law of expansion or of compression without transmission of heat, is by Rankine called an *Adiabatic* (from *διαβαίνειν*, to pass through), and by Gibbs, Clausius and

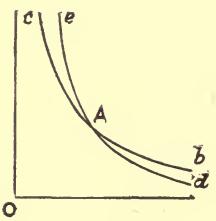

FIG. 3.

others, *Isentropic*, because the entropy (a term to be considered later) remains constant in this kind of expansion. Adiabatics are asymptotic to the axes p and v and also to the isothermals.

26. If a fluid, after a series of changes of pressures and volumes, returns to its initial state, the path of the fluid will be a re-entrant curve, as A and B, Fig. 4, and in such cases the fluid is said to work in a cycle.

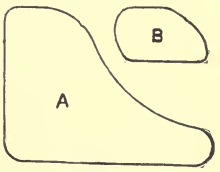

FIG. 4.

27. A heat engine is a machine for continuously transforming heat into work. Such engines in practice work in cycles.

28. Carnot's cycle. This is a cycle performed by an imaginary heat engine, devised by M. Carnot in 1824, and involves the most important fundamental principle of this science. The following is the operation:

Let B, Fig. 5, be a piston moving in a frictionless cylinder, all parts of which are perfectly impermeable to the passage of heat except the base F. Let the base of the cylinder be one square foot, so that the height of the piston will correspond with the number of cubic feet below it, and let the cylinder contain one pound of air, or any other gas. Let H be an indefinitely large vessel containing heat at a given temperature, and L another indefinitely large vessel containing heat at a lower temperature, the initial letters, H and L, indicating the relative temperatures. The vessels are assumed to be indefinitely large, so that, in imparting heat to a finite body, they will maintain a sensibly uniform temperature. Let N and N' be plates, as large or larger than the base F of the cylinder, perfectly impermeable to

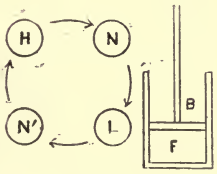

FIG. 5.

the passage of heat. Two of these are used simply for convenience of arrangement, so that the operation to be described, passing in the direction indicated by the arrows, will be the more suggestive of a cycle. Conceive the base F of the cylinder to be placed against the vessel H; the pound of air in the cylinder will quickly become of the same temperature as that of H. While in this condition let the piston move outward against a resistance which is continually infinitesimally less than the pressure within,—the temperature will be constantly that of H, and the expansion will be *isothermal*.

After the piston has been moved outward as far as desired in this manner, transfer the cylinder to the non-conducting cover N and allow the piston to move outward still further by a gradual reduction of the external pressure;—the pressure and temperature of the substance will both fall, and since the walls of the cylinder are impermeable to the passage of heat, the expansion will be *adiabatic*. Let the operation be continued until the temperature of the pound of gas in the cylinder has been reduced to that in the vessel L.

At the end of the preceding operation let the cylinder be removed to the vessel L, and the piston then forced inward; the heat generated by the compression of the pound of air will escape as fast as generated, and is said to be rejected or emitted into the vessel L, the temperature of which will not be sensibly changed; hence the temperature of the pound of air will be constantly that of the vessel L, and the compression will be *isothermal*. Let the operation continue to such a point that when the cylinder is removed to the cover N' and the air compressed adiabatically until the temperature is raised to that in the vessel H, the volume will be the same as that at the beginning of the series of operations.

To show these operations graphically, let $O\,b$, Fig. 6, represent the volume and $b\,B$ the pressure of the pound of gas

in the initial state; then will B on the diagram represent this state.

First operation. When the cylinder is in contact with the vessel H, the expansion of the gas will be represented by the isothermal $B\ C$, $O\ c$ being the final volume and $c\ C$ the final pressure.

Second operation. When the cylinder is in contact with the cover N, the expansion will be represented by the adiabatic $C\ D$, $O\ d$ being the final volume and $d\ D$ the corresponding pressure.

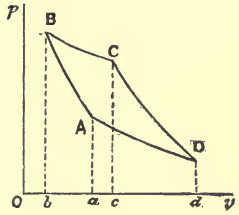

FIG. 6.

Third operation. The compression, when the cylinder is in contact with the vessel L, will be represented by the isothermal line $D\ A$.

Fourth operation. The compression when the cylinder is on the cover N′ will be represented by the adiabatic $A\ B$.

These are the successive operations as indicated by Carnot; but it is more convenient, in describing the process, to begin either at the state C or A, on account of limiting the third operation. Thus, when the cylinder is on the vessel H and in the state C, let it be transferred to N and expanded along $C\ D$ until the temperature is reduced to that of L; then transferred to L and compressed along DA any desired amount; thence transferred to N' and compressed until the temperature is raised to that of H; then transferred to H and expanded along $B\ C$ to the state C.

29. Source. The vessel from which the working substance receives heat, as H in the above operation, is called the *source*. Similarly, the vessel receiving the heat emitted from the working substance, as L in the above operation, is called *the refrigerator*. In engineering science these are called, respectively, the *furnace* and *condenser*.

30. Work done. During the expansion from state B

to state C work is done by the gas while forcing the piston outward, represented by the area $b\ B\ C\ c$, and while expanding from C to D more work is done by the gas, represented by the area $c\ C\ D\ d$; but during the compression from D to A work is done by the piston upon the gas, the amount being represented by the area $d\ D\ A\ a$, and work is still further done upon the gas in compressing it from A to B, represented by the area $a\ A\ B\ b$. The difference between these works will be the *external* work done by the cycle of operations. We have

$$+ b\ B\ C\ c + c\ C\ D\ d - d\ D\ A\ a - a\ A\ B\ b = A\ B\ C\ D.$$

31. Indicator diagram. The diagram $A\ B\ C\ D$ would be described by an indicator on Carnot's imaginary engine; and the area of an actual indicator diagram, taken from any engine, expressed in foot-pounds, is a measure of the heat destroyed in the cycle. It is in this sense that we speak of "foot-pounds of heat."

32. Carnot's cycle is reversible. In a complete cycle, if all the heat taken in is at one uniform temperature, and all the heat rejected is at a uniform lower temperature, the operation is called Carnot's cycle. Such a cycle is *reversible*, for all the operations may be performed in precisely the reverse order, the final result, however, being work done by the piston upon the gas in the cylinder, the energy of the gas thereby being increased by an amount represented by the area $B\ A\ D\ C$, Fig. 6, expressed in foot-pounds. A reversible engine is also called a perfect engine.

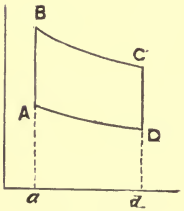

FIG. 7.

Non-reversible cycle. As an example of a non-reversible cycle, after the substance has expanded isothermally while in communication with the source, represented by the line $B\ C$, Fig. 7, let it be transferred directly to the refriger-

ator—heat will be abstracted and the pressure may be reduced at constant volume, and hence without doing work, the operation being represented by the line $C D$. Then compress it isothermally when in communication with the refrigerator along the line DA; then transfer it directly to the source, raising the temperature and pressure to the initial state B. This cycle cannot be performed in precisely the reverse order; for the pressure cannot be reduced from B to A when the engine is in communication with the source, nor raised from D to C when in communication with the refrigerator.

33. Conditions of a reversible cycle. In order that a cycle be reversible, the difference between the external pressure and the internal during a change of volume must be infinitesimal—during expansion the external being infinitesimally less, and during compression infinitesimally greater than the internal; also during the transfer of heat, the difference between the heat of the substance and that of the external body shall also be infinitesimal—during absorption being infinitesimally less than the source, and during emission infinitesimally greater than the refrigerator. The differences being infinitesimal, the quantities will *in finite measures* be equal.

33a. It follows from the conditions of the preceding article, that if a ~~closed~~ cycle be bounded by the isothermals and adiabatics of any substance, the cycle will be reversible when worked with that substance. Thus, if there be an adiabatic compression along $A B$, Fig. 8, an isothermal expansion along $B A'$, adiabatic expansion $A' B'$, isothermal expansion $B' A''$, and so on back to A, the cycle will be reversible.

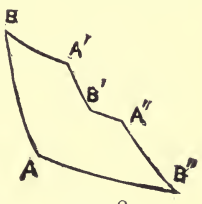

FIG. 8.

Also, the cycle $A B C D$, Fig. 7, may be made reversible by conceiving an indefinite number of sources of heat differing by

$d\tau$, and passing down $C\,D$ by an indefinite number of indefinitely short isothermal compressions and a corresponding number of indefinitely short intermediate adiabatic expansions as indicated in Fig. 9; and a similar reversed operation in ascending from A to B.

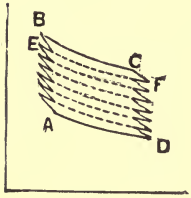

FIG. 9.

34. The heat absorbed *by a substance in working from a state A to state B may be represented on a diagram of energy by the area included between the path of the fluid and the adiabatics passing through A and B respectively, extended indefinitely in the direction of the expansion*, Fig. 10.

Let A be the initial and B the final states for the expansion $v_1\ v_2$, and the line $A\ B$ the path of the fluid. Pass the adiabatics $A\ \varphi_1$ and $B\ \varphi_2$, then will the indefinitely extended area $\varphi_1\ A\ B\ \varphi_2$ represent the heat absorbed by the substance in doing the external work $v_1\ A\ B\ v_2$, in the same units as $v_1\ v_2$ and $v_1\ A$; that is, if $v_1\ v_2$ represents feet, and $v_1\ A$, pounds, the area $\varphi_1\ A\ B\ \varphi_2$ will represent foot-pounds.

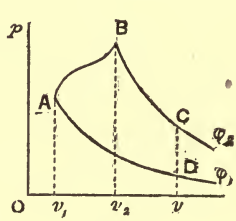

FIG. 10.

From the state B conceive the substance to be expanded adiabatically along $B\ \varphi_2$, doing work as against a piston, to the state C, then will the external work $v_2\ B\ C\ v$ have been done, without the absorption or emission of heat; and hence the reduction of temperature (and pressure) will be due to the transmutation of heat into work. At the constant volume $O\ v$ let sufficient heat be emitted to reduce the pressure to $v\ D$, where D is on the adiabatic $A\ \varphi_1$; from D compress the substance along $A\ \varphi_1$ to A, during which the external work $v_1\ A\ D\ v$ will have been done upon the substance; thence expand from A to B along the path $A\ B$, during which heat must be absorbed. The only heat

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absorbed in this cycle of operations is while working from A to B, and the only heat emitted is in describing the path $C\,D$; and since the cycle is complete the ideally external work $A\,B\,C\,D$ is the exact equivalent of the difference between the heat absorbed and that emitted, or

$$ {}_{a}H_{b} - {}_{c}H_{d} = v_{2}\,B\,C\,v + v_{1}\,A\,B\,v_{2} - v\,v_{1}\,A\,D = A\,B\,C\,D $$
$$ = \varphi_{1}\,A\,B\,\varphi_{2} - \varphi_{1}\,D\,C\,\varphi_{2}, $$

where φ_{1} and φ_{2} may be at a distance indefinitely great.

Let $C\,D$ be moved to the right indefinitely—it will become less and less, and at the limit ${}_{c}H_{d}$, or $\varphi_{1}\,C\,D\,\varphi_{2}$ will be zero, and we will have

$$ {}_{a}H_{b} = \varphi_{1}\,A\,B\,\varphi_{2} $$

for the heat absorbed. At the limit, φ_{1} and φ_{2} being at an indefinite distance to the right may be considered as coinciding, and the path $\varphi_{1}\,A\,B\,\varphi_{2}$ as re-entrant, forming a cycle, in working around which, heat is absorbed only along the path $A\,B$. The enclosed area represents what would be the *external* work done if the substance could be worked in this cycle. If the external work, $v_{1}\,A\,B\,v_{2}$, actually performed *plus* the increased actual energy of the substance equals $\varphi_{1}\,A\,B\,\varphi_{2}$, no *internal* work will be done in working along the path $A\,B$, but if these are unequal, the difference will be the internal work, either done upon the substance in passing from state A to state B, or by the substance in passing from state B to A. This theorem was first given by Rankine, and is very fruitful in the geometrical development of this science.

The heat absorbed in passing from state A to state B may be expended in the three following ways, as stated in article 9:

1. In doing the external work $v_{1}\,A\,B\,v_{2} = U$;
2. In doing internal work $= S$;
3. In increasing the actual energy of the substance $= Q$;

$$ \therefore\ \varphi_{1}\,A\,B\,\varphi_{2} = {}_{a}H_{b} = Q + S + U. \qquad (9) $$

Any of the terms in the last member may be negative.

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35. The mechanical equivalent of heat. The direct determinations of heat have been in terms of thermal units, but on the indicator diagram the work done by heat is in terms of foot-pounds or their equivalent. It is necessary to reduce one of these to the other. The first accurate determination of the mechanical equivalent of the thermal unit was made by Dr. Joule, of Manchester, England, who, after a series of experiments beginning in 1843 and extending over a period of about seven years, concluded that its value was about 772 foot-pounds. To state it otherwise; if a pound of water falling through a height of 772 feet in a vacuum should be suddenly brought to rest, and all the heat thereby generated could be utilized for the purpose, it would increase its temperature one degree Fahrenheit. Joule's experiments gave quite a range of values, and he was inclined to give more weight to the smaller than to the larger ones. Later, in 1876, a committee appointed by the British Association for the Advancement of Science reported that the mean of sixty of the best experiments made by Joule on the friction of water gave 774.1 foot-pounds subject to a small correction, possibly amounting to $\frac{1}{400}$ of its value, on account of the uncertainty of the exact position of the absolute zero on the thermometric scale.

Still later, in 1878, Joule made another set of experiments, giving as results the following values:

Deg. C.	Foot-pounds.
at 12.7,	774.6
15.5,	773.1
17.3,	774.0
Mean 15.1,	773.9

Joule's experiments were made with water at about 60° F., and reducing his results to their equivalent for water at its maximum density, according to the law indicated by Regnault's experiments, reduces the value slightly, though for a

difference of 20° F. it will scarcely affect the first decimal figure, as will be seen when we consider the specific heat of water. Thus, if the mechanical equivalent of heat at 60° F. were 774.1, then at 39.1° F., or say 40° F., it would be 774 (nearly), and reduced from the latitude of Manchester to that of New York it becomes 774.8, nearly; and if the entire margin of error, $\frac{1}{400}$, be positive and applicable to this number, the value would be 776.7, or, to the nearest integer, it would be 777.

More recently, Professor Rowland has made a very critical examination of the specific heat of water at the lower temperatures, and made a more accurate determination of the mechanical equivalent at those temperatures (*On the Mechanical Equivalent of Heat*, Proc. Am. Acad. of Arts and Sc., 1880). The most probable values, as determined by him, are, for the latitude of Baltimore (*ibid.*, p. 196):

Temperatures.		Mechanical Equivalent.		
		For Air Thermometer.		Mercurial Thermometer.
Centigrade.	Fahrenheit.	Kilo.-Metres.	Foot-pounds.	Foot-pounds.
°	°			
4	39.1	(430.0)	(783.4)	(778.3)
5	41.0	429.8	783.0	(778.1)
6	42.8	429.5	782.5	(777.9)
7	44.6	429.3	782.2	(777.6)
16	60.8	427.2	778.4	(775.4)
27	80.6	425.6	775.5	(775.4)
36	96.8	425.8	775.9	(774.7)

The numbers in the parentheses we have computed from those of Professor Rowland's tables, the last column being determined by means of his table on page 41 of the Appendix to his paper. It will here be seen that the equivalent diminishes from 39.1° F. to about 80° F., and, hence, the

specific heat of water diminishes according to the same law. Prior to these experiments, it had been held, in accordance with Regnault's experiments, that the specific heat of liquids increases with the temperature; but according to the above experiments this law is reversed for water from 40° F. to 80° F., being a minimum in the vicinity of the latter value, and increasing for higher temperatures. Regnault's experiments were chiefly for higher temperatures. Rowland's values, even when reduced to the same latitude, all exceed those heretofore used for scientific and engineering purposes, although they agree very nearly with Joule's when reduced to the same thermometer, temperature, and place (*ibid.*, Appendix, 44, 45). The first cause of difference lies in the fact, above stated, that the mechanical equivalent is greater at 39° F. than at 60° F.—amounting to about 3 foot-pounds—instead of less, as given by Regnault's experiments. The second cause is due to the fact that a degree on the air thermometer, from 39° to 40°, is perceptibly larger than on the mercurial thermometer, the difference being about $\frac{1}{150}$ of a degree of the air thermometer, and resulting in an increase of more than 5 foot-pounds above that given by the mercurial thermometer. Joule used a mercurial thermometer.

It is apparent that the old value, 772, so generally used by the scientific world, is much too small, and 774.1, recommended by the committee of the British Association, is not sufficiently large. According to Rowland's experiments, the British Thermal Unit is about 784 foot-pounds per degree on the air thermometer, and nearly 779 on the mercurial thermometer. Scientifically, the air thermometer should be used; while for engineering purposes the mercurial thermometer is almost universally used; but in neither case should the highest numbers be adopted unless the law of change of the substance be known throughout the extent of the investigation. Such a law is not known with scientific

exactness. In refined analysis it has been customary to use an empirical formula representing the experiments of Regnault—especially for water—but in ordinary practice it is customary to consider the specific heat of water as constant at all temperatures, and for this reason it is not advisable to adopt the highest values given by experiment. Before deciding upon the value to be adopted in this work, values were computed by other methods, to be explained hereafter, and that number selected which would, according to our present knowledge of all the elements involved, harmonize with the various methods by which it has been determined. This number is 778. The *exact* value cannot be found, but, like other physical constants, it may be determined within certain limits. The value here adopted is probably within $\frac{1}{300}$ of its own value for the mercurial thermometer at the latitude of New York. The mechanical equivalent we represent by J, and call it *Joule's equivalent.*

36. First Law of Thermodynamics. *Heat and mechanical energy are mutually convertible* in the ratio of about 778 foot-pounds for the British Thermal Unit.

The equivalent in French, or part French and part English units, is

1400 foot-pounds per pound of water per degree centigrade,

426.8 kilogramme-metres per kilogramme of water per degree centigrade.

EXERCISES.

1. How many foot-pounds of heat-energy are there in one pound of coal containing 14500 British thermal units?

Ans.

2. How far will one pound of anthracite coal propel a locomotive weighing 60 tons on a level track, friction 6 pounds per ton, if the entire heat-energy of the coal could

be utilized for this purpose, the coal containing 15000 B. T. U.?

Ans. miles.

3. If in melting one pound of ice 144 B. T. U. become latent, how many foot-pounds of energy are required to change the state of aggregation of the substance—that is, to change ice to water?

Ans. 112032 ft.-lbs.

4. Find the value of the mechanical equivalent of the B. T. U. when expressed in kilogramme-metres.

5. How many thermal units must be transformed into mechanical energy per minute to equal one horse-power?

6. What is the theoretical efficiency of a steam plant that develops one horse-power per hour for every $2\frac{1}{2}$ pounds of coal used, the heat units in a pound of the coal being 13000?

7. Steam plants have been reported as developing a horse-power per hour with 1.5 pounds of coal; what was the theoretical efficiency of the plant, if a pound of the coal contained 15200 B. T. U.? What if it contained 12000 B. T. U.?

Ans. In the latter case, $\frac{10}{71}$ nearly.

8. How many foot-pounds of energy will be required to raise the temperature of five pounds of water from the temperature of melting ice to that of boiling water, the value of J being 778 for each degree?

9. One kilogramme-metre per degree centigrade equals how many foot-pounds per degree Fahrenheit?

Ans. 13.02.

10. How many foot-kilogrammes of heat are necessary in order to raise the temperature of one decigramme of water one degree Fahr.? How many metre-pounds to raise one gramme of water one degree C.?

37. Thermal capacities. The amount of heat nec-

essary to change by unity any quality of unit-mass of a substance under given circumstances is called the *thermal capacity* corresponding to the given change. Three such capacities have received the respective names—*specific heat at constant pressure*, *specific heat at constant volume* and *the latent heat of expansion*. When these capacities are variable, their values at a particular state may be considered as the *rate* at which heat is absorbed per unit of the constant element.

The unit-mass, in English units, is the standard one-pound weight, and in French units is the standard kilogramme.

38. Pressure constant. Specific heat at constant pressure. If the pressure be constant, the path of the fluid will be a right line perpendicular to the p-axis, Fig. 11; and the heat absorbed in working from state A to state B along this line will be, according to Article 34, represented by the area $\varphi_1\ A\ B\ \varphi_2$ in foot-pounds, to find which requires an experiment with the substance in order to determine its thermal capacity under constant pressure.

The specific heat at constant pressure is the amount of heat absorbed in increasing the temperature of a unit-mass of the substance one degree, the pressure being constant and the specific heat constant throughout the degree. In English units, it is the number of thermal units (Art. 6) absorbed in raising the temperature of one pound of the substance one degree Fahrenheit. To represent it on a diagram of energy, the line $A\ B$, perpendicular to the p-axis, must be limited by two isothermals, as τ and $\tau + 1$, differing by unity of temperature; then will the dynamic specific heat at and from τ be represented by the indefinitely extended area $\varphi_1\ A\ B\ \varphi_2$. If the specific heat be variable, the isothermals must differ by $d\ \tau$ only.

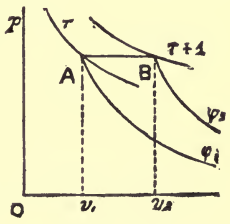

FIG. 11.

For numerical values, see tables of specific heat. To express it algebraically, let

k_p = the ordinary specific heat at the temperature τ and pressure p,

K_p = the equivalent *dynamic* specific heat;

then

$$K_p = J k_p. \tag{10}$$

If $d h_p$ = the thermal units absorbed in raising the temperature of a unit-mass of the substance under a constant pressure an amount $d t = d \tau$ degrees, and $d H_p$ the same expressed in foot-pounds, then

$$d h_p = k_p \, d \tau, \tag{11}$$

$$d H_p = J k_p \, d \tau = K_p \, d \tau; \tag{12}$$

$$\therefore \left(\frac{d H}{d \tau}\right)_p = K_p; \tag{13}$$

hence, after substituting,

$$H_p = \int \left(\frac{d H}{d \tau}\right)_p d \tau. \tag{14}$$

If the specific heat be constant, equation (12) integrated between limits gives

$$H_p = K_p (\tau_2 - \tau_1), \tag{15}$$

and if $\tau_2 - \tau_1 = 1$, we have

$H_p = K_p = \varphi_1 \, A \, B \, \varphi_2$, Fig. 11,

as before stated.

When the path is arbitrary, the heat absorbed may be a function of the three variables p, v, τ, but when p or v is constant $\frac{d H}{d \tau}$ will be a partial differential coefficient, and may be indicated as above with a parenthesis and subscript, or with a parenthesis without a subscript, or by $\frac{d_p H}{d \tau}$ as used by Clausius, or $(H_t)_p$ as by M. Saint-Robert, or even with-

out any distinguishing mark, leaving it for the reader to infer its true character, which may be so easily done in this science as to make it questionable whether any mark is desirable.

39. Volume constant. Specific heat at constant volume is the heat absorbed in raising the temperature of a unit-mass of a substance through one degree when the volume is constant, the specific heat remaining constant throughout the degree. It is the number of heat units necessary to raise the temperature of one pound of the substance one degree F., the volume being constant. When constant its dynamic value may be represented on a diagram of energy by the area between a line $A\ B$, Fig. 12, perpendicular to the v-axis limited by two isothermals differing by one degree, and two indefinitely extended adiabatics $A\ \varphi_1$ and $B\ \varphi_2$, as shown in Article 34.

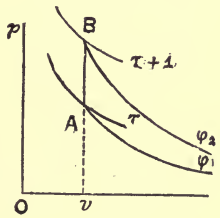

FIG. 12.

Let k_v = the specific heat for a constant volume at the temperature τ in ordinary thermal units,
K_v = its equivalent dynamic specific heat,
$d\,h_v$ = the thermal units absorbed in raising the temperature $d\,\tau$,
$d\,H_v$ = the foot-pounds of heat in $d\,h_v$;
then

$$d\,h_v = k_v\,d\,\tau,$$
$$d\,H_v = J\,k_v\,d\,\tau = K_v\,d\,\tau; \tag{16}$$

$$\therefore\ K_v = \left(\frac{d\,H}{d\,\tau}\right)_v \tag{17}$$

If K_v be constant, then in Fig. 12,

$$K_v = \varphi_1\ A\ B\ \varphi_2.$$

EXERCISES.

1. How much more heat (mechanical energy) is required to raise the temperature of one pound of water 1° F. than of one pound of air, the same amount under constant pressure?

(Find the ratio of the values of their specific heats, as found from a table of specific heats, and express their difference in foot-pounds.)

2. How much more heat is required to raise the temperature of one pound of water 1° C. than of one pound of air the same amount, at constant pressure?

3. How far must a mass of iron fall in a vacuum in order that its resultant energy, if transmuted into heat, would melt it?

4. Assuming that air and hydrogen are perfect gases, how much more heat will be required to increase the temperature of one pound of the latter one degree Fahr. than one pound of the former the same amount? Express the difference in thermal units and in foot-pounds.

5. If oxygen and hydrogen are perfect gases, how many pounds of oxygen will be required in order to contain as much heat as one pound of hydrogen at the same temperature?

6. Show that a right line parallel to the v-axis will be divided into equal parts by a series of isothermals of which the general equation is $p\,v = R\,\tau$, provided $\tau_2 - \tau_1 = \tau_3 - \tau_2$, &c.

7. In the preceding exercise, show that a right line parallel to the p-axis will also be divided into equal parts.

8. Show that a line drawn through the origin of co-ordinates is not divided into equal parts by the successive equilateral hyperbolas of Exercise 6.

40. Let the temperature be constant during expansion. Second Law. In this case the path of the

fluid will be an isothermal, as $A\ B$, Fig. 13; and the heat absorbed during the expansion from v_1 to v_2 will, according to Article 34, be represented by the area $\varphi_1\ A\ B\ \varphi_2$, bounded by the isothermal $A\ B$ and the two adiabatics $A\ \varphi_1$ and $B\ \varphi_2$ indefinitely extended. If H_t be the heat absorbed, the subscript indicating that the entire heat absorbed is to be at one temperature; then

$$H_t = \varphi_1\ A\ B\ \varphi_2.$$

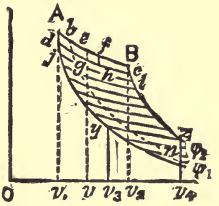

FIG. 13.

Since the temperature of the working substance, in this case, is uniform during expansion, it is assumed that the actual energy of the working substance remains unchanged, and hence Q in equation (9) will be zero, and we have

$$H_t = U + S = \varphi_1\ A\ B\ \varphi_2.$$

That is, during isothermal expansion the heat absorbed equals the entire work done, both external and internal. This heat cannot be directly measured, but it may be computed, as will appear from this and the two following articles. No engine can transmute into external work all the heat absorbed by the working substance, some of it being always rejected at a lower temperature than the source. Experience confirms the following principle, called the

Second Law. *If all the heat absorbed be at one temperature, and that rejected be at one lower temperature, then will the heat which is transmuted into work be to the entire heat absorbed in the same ratio as the difference between the absolute temperatures of the source and refrigerator is to the absolute temperature of the source.* (Appendix ii.)

In other words, *the second law is an expression for the efficiency of the perfect elementary engine.*

The object of the second law is to furnish a basis for the

computation of the heat absorbed during expansion, the heat of the working fluid being maintained at a constant temperature. Thus, if between 600° F. and 500° F. absolute, in a perfect elementary engine, ten thermal units be transmuted into work, then will the heat absorbed at 600° F. have been

$$h = 10 \frac{\tau_1}{\tau_1 - \tau_2} = 60$$

thermal units. If the expansion be isothermal, the equation to the path of the fluid will be that of the gas at constant temperature, and the *external* work may be directly computed, from the equation to the gas, being represented in Fig. 13, by

$$W = v_1 \, A \, B \, v_2 = \int p \, d \, v.$$

It will be observed in Fig. 13 that, if the area $\varphi_1 \, A \, B \, \varphi_2$ be divided into an indefinite number of strips, representing Carnot's cycles, *ultimately* the topmost strip $A \, B \, c \, d$ will equal the topmost strip of $v_1 \, A \, B \, v_2$, representing external work cut off by the second isothermal. If the work done in those Carnot's cycles be equal, the total heat, $\varphi_1 \, A \, B \, \varphi_2$, will, according to the second law, be the area of the topmost one multiplied by the number of cycles. The topmost one, Fig. 13, will be the differential of the external work, or $d \, W$, and $\tau_1 - \tau_2$ becomes $d \, \tau$, and if τ_1 be the constant temperature at which heat is absorbed, which will be the temperature of the isothermal $A \, B$, we have

$$H = d \, W \frac{\tau_1}{d \, \tau} = \tau_1 \frac{d \, W}{d \, \tau}.$$

The second law is the result of observations, experiments,

and deduction. It is not, like a proposition in geometry, capable of a direct, rigid demonstration; but rather, like the axioms of geometry, appeals to our understanding for assent when the terms used and the operations assumed are well understood. Or, perhaps a better parallel will be found in the Newtonian laws of motion, which were first conceived, from the results of experiments, to represent ideally perfect conditions, and later became firmly established by the fact that when applied to the solution of problems in nature the results obtained agreed with those observed. So this law, first conceived to represent what would be the results of experiments if the conditions were perfect, has become firmly established through the fact that it has successfully stood the many crucial tests to which it has been subjected. If the formulas founded upon it had led to results known to be erroneous, they would have disproved the law; but it has been found that all the results so deduced agree with those of experiment at least within the limits of the errors of observation.

Carnot made the first step toward the establishment of the law by showing that the efficiency of his ideal engine was a direct function of the difference of the temperatures of the source and refrigerator, and was independent of the nature of the working fluid. The idea of an absolute temperature had not then entered this science. Later the law became established through the labors of Clausius, and of Joule and Thomson. Rankine virtually deduced it from his theory of molecular vortices. He came to the conclusion that Carnot's law is not an independent principle, but is deducible from the equations of the mutual conversion of heat and expansive power.

Let one pound of any substance having the volume $O\,v_1$, pressure $v_1\,A$, and absolute temperature τ, in constant communication with a source of heat at the same temperature (or at a temperature $\tau + d\tau$), expand from v_1 to v_2 by driv-

ing a piston in a cylinder; then will the indefinitely extended area $\varphi_1 A B \varphi_2$, expressed in foot-pounds, represent the heat absorbed, and divided by 778 will be the value of the heat in British thermal units.

To find the area $\varphi_1 A B \varphi_2$, conceive it to be divided into an indefinite number of strips of equal areas by isothermals of the given substance, as $d c, j i, \ldots y z$, &c.; they will represent equal quantities of heat, and if an elementary engine be worked in the successive cycles $A B c d, d c i j$, &c., the resultant works done will also be equal. These are Carnot's cycles, since all the heat absorbed will be at one temperature, and that which is rejected, at one lower temperature. Let the successive equal quantities of heat thus transmuted into external work constitute a scale of temperatures—known as Thomson's Absolute Thermometric Scale (*Phil. Mag.*, xi. (1856) 216. *Thomson's Papers*, p. 100). The characteristic quality of this scale is — equal quantities of heat when worked in Carnot's cycle will do equal quantities of external work *independently of the nature of the working substance.* At first, any amount of heat or area, as $i j d c$, may be taken, arbitrarily, as a unit, and a repetition of this unit will constitute a scale of natural numbers, as 7, 8, 9, &c., the zero of which may be placed arbitrarily. Having assigned its place and the unit of heat, the quantity of heat involved in any number of such operations becomes known. Thus, the heat necessarily destroyed in performing the operations numbered 7, 8 and 9 will be three times the unit initially assumed. Fractional parts of the scale will correspond to fractional parts of the unit. The scale may be so numbered that the two fixed points shall correspond with 32° F. and 212° F.

Conceive that the zero of the scale corresponds with the total deprivation of heat from the substance, and that in raising the pressure from 0 to $v_1 A$, there are τ of the arbitrary units. Let each unit be divided into an indefinite

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number of equal parts by isothermals, each represented by $d\tau$; then will the number of parts in each unit be $1 \div d\tau$, and the number in τ units will be

$$number\ of\ strips = \frac{\tau}{d\tau}.$$

The area of any one of the infinitesimal strips, as $A\ B\ c\ d$, being known, we have

$$\varphi_1\ A\ B\ \varphi_2 = \frac{\tau}{d\tau} \times A\ B\ c\ d, \tag{18}$$

and the solution is now reduced to that of finding the area $A\ B\ c\ d$. Conceive it to be divided into an indefinite number of parts by vertical lines having between them the constant abscissa $d\ v$ (or, more accurately, let the divisions be made by adiabatics having between their upper ends the abscissa $d\ v$), then

$$e\ f\ h\ g = d\ p\ d\ v,$$

and

$$A\ B\ c\ d = \Sigma_{v_1}^{v_2} d\ p\ d\ v.$$

This summation cannot, generally, be performed by an integration, for, generally, $d\ p$ varies from A to B, and is not simply a function of v. For any assigned value of v, $d\ p$ depends directly upon $d\ \tau$, since it is limited by two consecutive isothermals differing by $d\ \tau$, a condition which, in the language of the calculus, is indicated by the expression

$$g\ e = \left(\frac{dp}{d\tau}\right) d\ \tau,$$

thus, changing from $d\ p$ independent to p dependent upon τ. Substituting this for $d\ p$ above, gives

$$A\ B\ c\ d = \int_{v_1}^{v_2} \left(\frac{dp}{d\tau}\right) d\ \tau\ d\ v = d\ \tau \int_{v_1}^{v_2} \left(\frac{dp}{d\tau}\right) d\ v, \tag{19}$$

where $d\ \tau$ is placed outside the integral sign since it is con-

stant throughout the strip $A\ B$. This value in equation (18) gives

$$H_t = \varphi_1\ A\ B\ \varphi_2 = \tau \int_{v_1}^{v_2} \left(\frac{dp}{d\tau}\right) d\ v. \qquad (19a)$$

If the expansion be infinitesimal and equals $d\ v$, we have

$$d\ H = \tau \left(\frac{d\ p}{d\ \tau}\right) d\ v, \qquad (20)$$

which is the required equation.

In using this equation, $\left(\frac{d\ p}{d\ \tau}\right)$ is to be found by *differentiating* the equation of the gas, given in terms of the *absolute scale of* temperatures, considering v as constant; or by an experiment, finding the change of pressure for a very small change of temperature, but in *integrating* from v_1 to v_2 the temperature must be constant, so that not only will τ, if any in $\left(\frac{d\ p}{d\ \tau}\right)$, be constant, but the τ before the integral sign will also be constant. Indicate this by τ_1. Since an amount of heat equal to that absorbed by any substance during isothermal expansion becomes latent, the preceding equation, more completely expressed, becomes:

$$\left.\begin{array}{l}\text{Latent heat of}\\ \text{expansion from}\\ v_1 \text{ to } v_2\end{array}\right\} = H_{\tau_1} = \tau_1 \int_{v_1}^{v_2} \left[\left(\frac{dp}{d\tau}\right)_v d\ v\right]_{\tau_1} \qquad (21)$$

in which the subscript τ_1 of the bracket implies, as explained above, that τ within the parenthesis is to be considered constant during the integration. As a *thermal capacity*, Article 37, the latent heat of expansion is $\tau \frac{d\ p}{d\ \tau}$, as if $d\ v$ were unity, being the *rate* at which heat is absorbed per unit of volume. Differentiating equation (21), considering τ as constant and v variable, gives

$$\varphi_1\ A\ b\ n = \tau_1 \left(\frac{d\ p}{d\ \tau}\right) d\ v,$$

which is the same as equation (20), where $d\,v$ is the abscissa of b in reference to $v_1 A$, Fig. 13.

The scale of temperatures above used is not practical, except for the purposes of analysis, since heat cannot be actually divided with accuracy by any known means according to the process described; and it remains to be shown how the result can be made of practical value. Conceive a quantity of heat equal to that absorbed by a pound of the substance, $\varphi_1\, A\, B\, \varphi_2$, to be absorbed by such a quantity of a *perfect* gas as will give the same temperature, and let the temperatures be measured by an ideally perfect gas thermometer graduated from absolute zero and having τ equal divisions up to the temperature here considered; then will equal divisions on this scale correspond with equal quantities of actual heat in the perfect gas—so that, if the gas be cooled by abstracting equal, successive quantities of heat, the successive temperatures will be indicated by equal divisions on the scale. In this manner, the heat in a perfect gas might be divided into equal parts. Let the temperature of the given substance be reduced an amount $d\,\tau$ on this scale by working the heat in Carnot's cycle, the same amount of heat will be transmuted into work as must be abstracted from the perfect gas in reducing its temperature the same amount, and so on. Conceive isothermals of the substance to be drawn on the diagram of energy, differing by $d\,\tau$ of the perfect gas thermometer; there will be $\tau \div d\,\tau$ such divisions between zero and τ, as in the former case. These isothermals may be conceived to be described, geometrically, from the equation of the gas given in terms of the scale of the *perfect* gas thermometer, or, physically, by supplying heat to the expanding gas so that the temperature will remain constant as indicated by this thermometer and noting the contemporaneous pressures and volumes. These processes, perfectly done, would give the same isothermals; and since the number is made the same as in the earlier part of this article where the strips

were, arbitrarily, made equal, and since the lowest, or zero-isothermals, coincide, also the highest, or τ-isothermals, it is inferred that the successive isothermals in the two cases coincide. It follows, then, that if the area $\varphi_1 A B \varphi_2$ be intersected by isothermals differing by an absolute constant temperature, the areas between the successive isothermals will be equal; and if the number representing the difference of temperatures be commensurable with the number representing the highest temperature, the entire area $\varphi_1 A B \varphi_2$ will be divided into equal parts. By making the difference indefinitely small, or $d\tau$, the question of commensurability disappears.

But a *perfect* gas is unknown; it has, however, been found, as stated in Articles 14 and 16, that the air thermometer differs but little from that of a perfect gas thermometer, the temperature of melting ice being 491.13° F. above the absolute zero of the air thermometer, and about 492.66° F. above the zero of the absolute scale, a difference of about $\frac{1}{320}$ of the entire 491°, a quantity too small to be measured in actual practice, and can be determined only by the most refined experiments. The position of the zero of the absolute scale cannot be determined exactly, but, accepting the results of Thomson and Joule, if the zero of the air thermometer be made to coincide with the melting point of ice, then by adding 492.66° F. to the reading of the air thermometer, the sum will be the value of the temperature on the absolute scale, almost exactly.

Equations (20) and (21) are *theoretically* exact, and hence are *practically* so for volumes, pressures and temperatures determined by the best methods known.

The following reasoning may aid the reader in satisfying himself of the equality of the strips. Conceive the area $\varphi_1 A B \varphi_2$, Fig. 14, to be divided into an indefinite number of strips by isothermals of the substance, differing by the constant absolute temperature $d\tau$, then will the areas thus formed be equal.

If the areas between equidistant isothermals and the adia-

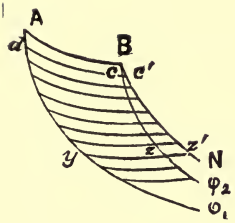

FIG. 14.

batics are not equal, a line, $B\ N$, may be so drawn that they will be equal, but the area $\varphi_1\ A\ B\ N$ (Fig. 14) will be an area exceeding that which represents the heat absorbed; or, if it falls within $B\ \varphi_2$ it will be less than that representing the heat absorbed.

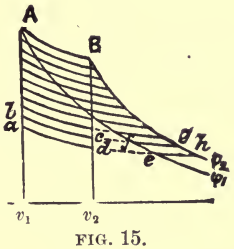

FIG. 15.

If the working fluid is a perfect gas the areas $a\ b\ c\ d$ and $e\ f\ g\ h$ (Fig. 15) will be equal, but if the gas be imperfect all the small areas in $\varphi_1\ A\ B\ \varphi_2$ below the topmost one will exceed those between the corresponding isothermals in $v_1\ A\ B\ v_2$.

41. Thomson and Joule established equation (20) upon a principle of Carnot. Carnot proved that, of the heat absorbed, $\varphi_1 A\ b\ n$, Fig. 13, during isothermal expansion, the part $d\ b$ transmuted into work by working in one of his cycles, was $\mu\ (\tau_1 - \tau_2)$ of the heat absorbed, where μ is a function of the higher temperature only and hence independent of the nature of the working substance, and $\tau_1 - \tau_2$, the fall in temperature of the working substance. In this case, let $\tau_1 - \tau_2 = d\tau$, then will $\mu\ d\ \tau$ be the fractional part of $\varphi_1\ A\ b\ n$ transmuted into work.

Let M be the latent heat of expansion in thermal units, then will $M\ d\ v$ be the heat units in $\varphi_1 A\ b\ n$, and in foot-pounds we have

$$J\ M\ d\ v = \varphi_1\ A\ b\ n,$$

and the heat transmuted into the work $b\ d$ will be

$$b\ d = \mu\ J\ M\ d\ v\ d\ \tau.$$

But we also have

$$b\ d = \left(\frac{d\ p}{d\ \tau}\right) d\ \tau\ d\ v,$$

and making these equal gives

$$J M = \frac{1}{\mu}\left(\frac{d p}{d \tau}\right).$$

Carnot did not find the form of the function μ. In regard to it Thomson says: "It has an absolute value, the same for all substances for any given temperature, but which may vary with the temperature in a manner that can only be determined by experiment" (Thomson's *Papers*, p. 187). Thomson, whose resources ever seem sufficient for the occasion, set about its determination, the processes for which are described in the *Philosophical Magazine*, and more recently in Thomson's *Mathematical and Physical Papers*, covering many pages. Early in the investigation, Joule suggested that the value of μ might be "inversely as the temperature from zero" (Thomson's *Papers*, p. 199); and these experimenters established the truth of this suggestion by that celebrated series of experiments known as "the experiments with porous plugs." Hence, we have

$$\mu = \frac{1}{\tau};$$

$$\therefore J M d v = \tau \left(\frac{d p}{d \tau}\right) d v$$

as already found. The quantity μ is known as "Carnot's function," the title given to it by Sir William Thomson. The value $1 \div \mu = \tau_0$, the absolute temperature of melting ice, was found to be 273.68° C. (*ibid.*, p. 391).

Thomson's absolute scale may be thus defined: *The numbers expressing degrees of absolute temperatures are proportional to the quantities of heat absorbed and emitted at those temperatures in a reversible cycle.* Thus, if $H = \varphi_1$ A B φ_2 = the heat absorbed, Fig. 13, and it be divided into τ equal parts, then will one part be $H \div \tau$; and if h be the

heat emitted $= \varphi_1\, y\, z\, \varphi_2$, then will the number of equal parts in h be

$$h \div (H \div \tau) = \frac{h}{H}\,\tau = t\,(\text{say});$$

$$\therefore \frac{h}{H} = \frac{t}{\tau}.$$

The equal parts of heat in $\varphi_1 A B \varphi_2$ may be conceived to be secured, physically, by a succession of perfect engines in which the refrigerator of one is the source of the next, and so on. It was in this manner that Carnot established his expression for efficiency. The amount of work done by heat depended only upon the difference of the temperatures of the source and refrigerator and some function of the higher temperature, as already given.

42. To express equation (21) in terms of the external work, from Fig. 13, we have

$$U = v_1\, A\, B\, v_2 = \int p\, d\, v\,;$$

$$\therefore d\,U = p\, d\, v\,;$$

$$\therefore \frac{d^2 U}{d\,\tau} = \left(\frac{d\,p}{d\,\tau}\right) d\, v, \text{ also written } \frac{d}{d\,\tau}\, d\, U;$$

hence, substituting in (21),

$$H_t = \tau \int \frac{d^2 U}{d\,\tau} = \varphi_1\, A\, B\, \varphi_2, \text{ Fig. 13}; \quad (22)$$

$$\therefore \frac{d H_t}{\tau} = \frac{d^2 U}{d\,\tau} =$$

$$\frac{d^2 H}{d\,\tau^2}\, d\,\tau = \frac{d^2 U}{d\,\tau^2}\, d\,\tau = e\, f\, h\, g.$$

$$\left(\frac{d\,H_t}{d\,v}\right) d\, v = \tau_1 \frac{d^2 U}{d\,\tau} = \varphi_1\, A\, b\, n.$$

From equation (22) it appears that the heat absorbed may be found from the temperature at which it is absorbed and

the external work regarded as a function of the temperature.

The three preceding articles will be more clearly understood after becoming more thoroughly familiar with the subject as developed in the following pages.

43. In equation (19) the quantity under the integral sign for a given expansion is constant, hence $d\tau$ may be integrated between limits, giving

$$\int_{H_2}^{H_1} dH = H_1 - H_2 = (\tau_1 - \tau_2) \int \left(\frac{dp}{d\tau}\right) dv = AB\,z\,y \quad (23)$$

of Fig. 13, in which τ_1 is the absolute temperature of the isothermal $A\,B$ and τ_2 that of $y\,z$, and H_1 the heat absorbed along $A\,B$ and H_2 that rejected along $z\,y$. Heat absorbed during an operation may be considered positive and that emitted, negative.

EXERCISES.

1. If the equation of the gas be $p\,v = R\,\tau$, find the heat absorbed during expansion at the constant temperature of 500° F. from v_1 = ten cubic feet to v_2 = 30 cubic feet. (Use Eq. (21).)

2. If the equation of the fluid be $p = R\frac{\tau}{v} - \frac{b}{\tau v^2}$ (of which carbonic acid gas is a special case), find the area of one of the strips in Fig. 13 for a difference of temperature $d\tau$, for an expansion from v_1 to v_2 at the temperature τ_1.

$$\text{Ans.} \left[R \log_e \frac{v_2}{v_1} + \frac{b}{\tau_1^2}\left(\frac{1}{v_1} - \frac{1}{v_2}\right)\right] d\tau.$$

3. Find the latent heat of expansion in the preceding exercise.

4. If the equation of the gas be $p = R\frac{\tau}{v} - \frac{b}{\tau v^2}$, find

the external work done in expanding from v_1 to v_2 at the temperature τ_1.

$$\text{Ans. } R\,\tau_1\,log_e\frac{v_2}{v_1} - \frac{b}{\tau_1}\left(\frac{1}{v_1} - \frac{1}{v_2}\right).$$

5. Find the internal work done in exercise 4.

6. In exercise 4 find the area between the two consecutive adiabatics $A\,\varphi_1$ and $b\,n$, Fig. 13.

7. In exercise 4 find the ratio of the internal work to the external for an expansion from $v_1 = 9$ to $v_2 = 18$ cubic feet, at $\tau = 700°$ F., $R = 35$, and $b = 48000$.

8. If the equation of a gas were $p = 4\,v\,\tau$, find the heat absorbed at the temperature $\tau = 600°$ F. in expanding from 20 to 30 cubic feet, and reduce to thermal units.

44. General Case.—Let the path of the fluid be arbitrary, as $A\,B$, Fig. 16, $A\,\varphi_1$, $B\,\varphi_2$, two adiabatics indefinitely extended to the right, then, as shown in Article 34, the area $\varphi_1\,A\,B\,\varphi_2$ will represent the heat absorbed in passing from state A to state B. To find this area, conceive it to be divided into an indefinite number of indefinitely narrow strips, as follows:—Divide the line $A\,B$ into an indefinite number of parts by the isothermals $a\,o$, $b\,p$, &c., differing by $d\,\tau$, the points of division being at a, b, c, &c.; and from these points draw verticals intersecting the isothermals next below in the points n, o, p, q, &c. Through the points a, b, c, &c., and n, o, p, &c., draw adiabatics, as $a\,m$, $o\,m_2$, $b\,m_3$; then will the sum of all the strips $m\,a\,o\,m_2$, $m_2\,o\,b\,m_3$, &c., *ultimately* equal the area $\varphi_1\,A\,B\,\varphi_2$. If τ be the absolute temperature of any isothermal, as $a\,o$, and $d\,v$ the expansion from state a to state o, then, according to equation (20), will the area

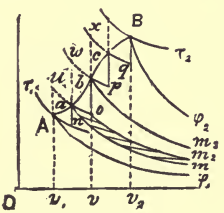

FIG. 16.

$$m\,a\,o\,m_2 = \tau\left(\frac{d\,p}{d\,\tau}\right)d\,v;$$

and, according to Article 39, the area

$$m_2\ o\ b\ m_3 = K_v\ d\ \tau\ ;$$

hence, ultimately,

$$area\ m\ a\ b\ m_3 = d\ H = K_v\ d\ \tau + \tau\left(\frac{d\ p}{d\ \tau}\right) d\ v, \quad (24)$$

which is a GENERAL *differential* EQUATION OF THERMODYNAMICS.

In this solution the polygon $A\ n\ a\ o\ b\ p$, &c., is inscribed in the figure $\varphi_1\ A\ B\ \varphi_2$, but the same result would be reached if the polygon were circumscribed, as indicated in the figure by $A\ u\ a\ w\ b$, &c.

From equation (24) we have

$$\varphi_1 A\ B\ \varphi_2 = {}_A H_B = \int_{\tau_1}^{\tau_2} K_v\ d\ \tau + \int_{v_1}^{v_2} \tau\left(\frac{d\ p}{d\ \tau}\right) d\ v, \quad (25)$$

but the general integral cannot be found since K_v is not a known function of τ, nor τ and $\left(\frac{d\ p}{d\ \tau}\right)$ known functions of v. In equation (24) τ and v are independent variables. The shaded strip $m\ a\ o\ m_2$ represents heat transmuted into work due to an isothermal expansion, and the unshaded strip $m_2\ o\ b\ m_3$ the increased energy of the substance, both actual and potential, due to the change of temperature in passing from a to b.

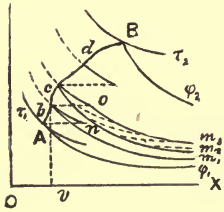

FIG. 17.

45. To make τ and p independent variables. Intersect the path $A\ B$, Fig. 17, with consecutive isothermals differing by the constant $d\ \tau$, as before; and from the points of division b, c, d, &c., draw lines parallel to the axis $O\ v$, intersecting the adjacent isothermals in the points n, o, &c., thus describing an inscribed polygon, $A\ n\ b\ c$, &c. A circumscribed polygon would answer the same purpose. Through the vertices of the polygon, A, n,

b, o, c, &c., draw the adiabatics $b\ m_1$, $c\ m_2$, $o\ m_3$, &c., then, ultimately,

$$m_1\ b\ c\ m_2 = d\ H = m_1\ b\ o\ m_3 - m_2\ c\ o\ m_3$$
$$= K_p\ d\ \tau - m_2\ c\ o\ m_3, \text{ (Eq. (12))},$$
$$= K_p\ d\ \tau - \tau \left(\frac{d\ p}{d\ \tau}\right) d\ v, \text{ (Eq. (20))}.$$

But $d\ v$ in this equation is the abscissa of o in reference to c (Figs. 17 and 18), on an isothermal through c, and hence is not determined directly from the equation of the path. Change this $d\ v$ to $d\ v'$ and let $b\ k = d\ v$, being the difference of the abscissas of two consecutive points of the path; then

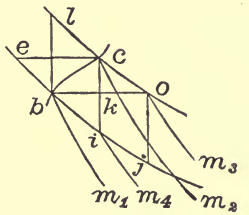

FIG. 18.

$\left(\frac{d\ p}{d\ \tau}\right)_v$ = the rate of change of pressure,

$\left(\frac{d\ p}{d\ \tau}\right)_v d\ \tau = b\ l = c\ i$ = increase of pressure,

$\left(\frac{d\ p}{d\ \tau}\right)_v d\tau dv' =$ area $i\ c\ o\ j$, $\qquad \left(\frac{d\ v}{d\ \tau}\right)_p d\tau = b\ o$,

$$d\ p = k\ c\ ;\ \therefore \left(\frac{dv}{d\tau}\right)_p d\tau dp = b\ o\ c\ e.$$

But $i\ c\ o\ j = b\ o\ c\ e$ having the common base $c\ o$ and between the same parallels. Multiplying the last expression by $\tau \div d\ \tau$, and substituting, gives

$$d\ H = K_p\ d\ \tau - \tau \left(\frac{d\ v}{d\ \tau}\right)_p d\ p. \qquad (26)$$

Equation (26) is a *second* GENERAL EQUATION of thermodynamics in which τ and p are the independent variables. Other forms may be deduced from these, as will hereafter be shown. For convenience of reference these equations are brought together.

46. The two fundamental equations of thermodynamics are :—

$$\left.\begin{aligned} d\,H &= K_v\,d\,\tau + \tau\left(\frac{d\,p}{d\,\tau}\right) d\,v. \\ d\,H &= K_p\,d\,\tau - \tau\left(\frac{d\,v}{d\,\tau}\right) d\,p. \end{aligned}\right\} \qquad (A)$$

The remainder of this work will consist chiefly of a discussion of these equations. Thermodynamics is the science which treats of the ~~mechanical~~ theory of heat.

QUESTIONS FOR EXAMINATION.

(Some of these questions require knowledge outside of this text.)

Give instances of heat generated by mechanical action. Draw inference. What did Count Rumford conclude ? Describe Davy's experiment. Is his experiment conclusive ? Who first made an exact determination of the mechanical equivalent ? Describe his methods. How long did he devote himself to the subject ? What did he consider the most probable value ? What is meant by work ? momentum ? energy ? foot-pound ? horse-power ? metre-kilogram ? heat unit ? caloric ? *rate?* Define exactly the "thermal equivalent." Why does the mechanical equivalent depend upon latitude ? altitude ? thermometer used ? In what respects do the results of Rowland's experiments differ from Regnault's ? Is perpetual motion possible ? Why not ? When a gas expands, why does the temperature fall ? When it expands into a vacuum, does its temperature fall ?

What is an atom ? molecule ? "ether"? What are heat rays ? In what respects do heat and light differ ? agree ? When is a body transparent ? opaque ? athermanous ? diathermanous ? When is a body heated by radiation ? conduction ? When a body is heated, what three effects may be produced ? What is specific volume ? specific pressure ? specific gravity ? specific heat ? real specific heat ? apparent specific heat ? latent heat ? latent heat of expansion ? thermal capacity ?

What is a perfect gas ? imperfect gas ? Does the coefficient of expansion vary with different gases ? For what is it least ? What is the "absolute zero"? Can it be realized ? Of what value is it in theory ? What is thermodynamics ? What is a general equation of thermodynamics ? Eliminate $d\,\tau$ from equations (*A*), and deduce a third equation for $d\,H$.

CHAPTER II.

PERFECT GASES.

47. Difference of specific heats. From the equation of a perfect gas, equation (2), we find

$$\left(\frac{dp}{d\tau}\right) = \frac{R}{v} = \frac{p}{\tau},$$

$$\left(\frac{dv}{d\tau}\right) = \frac{R}{p} = \frac{v}{\tau};$$

and these in equations (A) give

$$\left.\begin{aligned} dH &= K_v\, d\tau + p\, dv. \\ dH &= K_p\, d\tau - v\, dp. \end{aligned}\right\} \tag{27}$$

From equation (2) $p\ d\ v + v\ d\ p = R\ d\ \tau$, which in equation (27), after placing the second members equal, give

$$K_p - K_v = R; \tag{28}$$

hence, *the difference of the two specific heats for a perfect gas is constant.*

48. Specific heat constant. In a perfect gas no internal work is done during a change of state, hence, at constant volume, no work will be done by the absorption of heat, and all the heat absorbed will be *sensible* at all temperatures; hence, the specific heat of a perfect gas at constant volume will be constant, and equation (28) shows that, in this case, the specific heat at constant pressure will also be constant. It is found that the specific heat for sensibly perfect gases at constant volume is independent of the volume.

Let $K_v = C_v$ and $K_p = C_p$ for sensibly perfect gases, and equations (27) become

$$\left.\begin{aligned} dH &= C_v\,d\tau + p\,dv, \\ dH &= C_p\,d\tau - v\,dp, \end{aligned}\right\} \qquad (B)$$

which are the general equations of sensibly perfect gases. Equation (28) becomes

$$R = C_p - C_v. \qquad (29)$$

When Clausius first established the preceding equation, he concluded that both specific heats were constant for perfect gases at all pressures and temperatures, although this view opposed the one then prevalent—that the specific heat was a function of the density of the gas. Soon after, however, the experiments of Regnault confirmed the conclusion of Clausius by showing that it was practically constant for the so-called permanent gases, as air, oxygen, hydrogen and nitrogen.

Regnault found the following results for air at constant pressure (*Relation des Expériences*, ii., 108).

Heat required to raise the temperature of one pound of air 1° *C.* at constant pressure,

		C_p.	
between	− 30° *C.* and + 10° *C.*	0.23771	thermal units,
"	0° *C.* " + 100° *C.*	0.23741	" "
"	0° *C.* " + 200° *C.*	0.23751	" "

which show that it is not strictly uniform, neither is the law of change apparent. There is, apparently, a minimum value, but it is not safe to assert that such is the fact, much less to assign its place. Other experimenters find values differing slightly from these. The departure from the mean is so small, we may, for all ordinary purposes, consider the specific heat as constant.

Regnault also determined the specific heat of air under different pressures from 1 to 12 atmospheres, and of hydro-

gen from 1 to 9 atmospheres, and found the specific heats of each to be sensibly constant within these respective ranges.

49. The perfectness of a gas may also be tested by comparing its agreement with the equation of a perfect gas. Thus, Regnault found for atmospheric air, if the volume be constant, the following:

Density, or pressure in atmospheres at 0° C.	Ratio of pressure at constant volume at 100° C. to that at 0° C.
0.1444	1.36482
0.2294	1.36513
0.3501	1.36542
0.4930	1.36587
1.0000	1.36650
2.2084	1.36760
2.8213	1.36894
4.8100	1.37091

If the gas were perfect we would have for a constant volume v_1 from equation (2),

$$\frac{p_2}{p_1} - 1 = \frac{p_2 - p_1}{p_1} = \frac{R}{v_1 p_1}(\tau_2 - \tau_1) = \frac{\tau_2 - \tau_1}{\tau_1},$$

in which the range, $\tau_2 - \tau_1$, of temperature being constant, and equal 100° C. in the preceding table, and $\tau_1 = 273.7°$, the ratio of $\frac{p_2}{p_1}$ would also be constant. The preceding table shows a slight increase in this ratio with the increase of the density from 0.1444 to 33.3 times that value. The departure, however, from uniformity is so small that, for ordinary purposes, air may be treated as a perfect gas in this respect.

When the pressure was constant, it was found that the volume increased as follows:

Pressure.	Increase of volume for an increase of 100° C., the original volume being unity in each case.		
	Atmospheric air.	Carbonic acid.	Hydrogen.
760*mm.*	0.36706	0.37099	0.36613
2525	0.36944	0.38455	0.36616

In these experiments the increase of volumes was greater for the same range of temperatures when the pressure was greater, although for hydrogen the rate was almost exactly constant. If the gas were perfect we would have for a constant pressure p_1, the equation

$$\frac{v_2}{v_1} - 1 = \frac{\tau_2 - \tau_1}{\tau_1},$$

the left member of which should be constant if the gas were perfect, the range of temperatures and the initial temperature being constant.

In some other experiments the same mass of different gases was subjected to different pressures with the following results, v being the volume of one pound of the mass:

Density or $\frac{1}{v}$	Hydrogen.		Nitrogen.		Atmospheric air.	
	p	pv	p	pv	p	pv
2	2.0008	1.0004	1.9995	0.9992	1.9975	0.998782
4	4.0061	1.0015	3.9918	0.9979	3.9860	0.996490
8	8.0339	1.0042	7.9641	0.9955	7.9457	0.993212
16	16.1616	1.0101	15.8597	0.9912	15.8045	0.987780

This table shows that these three gases follow nearly the gaseous law expressed by the equation $p\,v = R\,\tau_1$, that for hydrogen $p\,v$ increases slightly with increase of pressure, while for nitrogen and atmospheric air this product decreases with increase of pressure.

The following table gives the expansion of several gases under constant pressure from 0° C. to 100° C., and the increased tension for the same range of temperatures under constant volumes, the initial pressure being one atmosphere, as determined by Regnault.

Substance.	Increase in volume under constant pressure for		Increase of pressure under constant volume for	
	100° C.	1° F.	100° C.	1° F.
Hydrogen	0.3661	0.002034	0.3667	0.002037
Atmospheric air	0.3670	0.002039	0.3665	0.002036
Nitrogen	0.3670	0.002039	0.3668	0.002039
Carbonic oxide	0.3669	0.002038	0.3667	0.002037
" acid	0.3710	0.002061	0.3688	0.002039
Protoxide of nitrogen	0.3719	0.002066	0.3676	0.002032
Sulphurous acid	0.3903	0.002168	0.3845	0.002136
Cyanogen	0.3877	0.002154	0.3829	0.002127

50. To find C_v. The specific heat of any substance at constant volume has not been found to any degree of accuracy by direct experiment, but its value for sensibly perfect gases may be computed from equation (29), for we have

$$C_v = C_p - R, \tag{30}$$

which is the required equation.

Regnault found for air, the mean value

$$c_p = 0.2375\ T.\ U.$$
$$\therefore\ C_p = 184.77 = 0.2375 \times 778.$$

Also, equation (3′), $R = 53.37$

difference $= C_v = 131.40$;

$$\therefore\ c_v = 0.1689 = 131.40 \div 778.$$

Equation (3′) is here used because the determinations were made with the air thermometer.

51. Relative specific heats. Since both specific

heats are constant for perfect gases, their ratio will be constant, which we will represent by γ; then

$$\frac{C_p}{C_v} = \frac{c_p}{c_v} = \gamma. \tag{31}$$

For *air* we have

$$\gamma = \frac{0.2375}{0.1689} = 1.4061.$$

This ratio was originally found by means of the velocity of sound in the gas, in a manner soon to be explained, Art. 60.

From equations (30) and (31) we find

$$C_p = \frac{\gamma}{\gamma - 1} R = \frac{\gamma}{\gamma - 1} \cdot \frac{p_o}{\tau_o} v_o = D v_o, \tag{32}$$

where D is a constant for sensibly perfect gases; hence, for another gas we have

$$C_p' = D v_o';$$

$$\therefore \frac{C_p}{C_p'} = \frac{v_o}{v_o'} = \frac{C_v}{C_v'}; \tag{33}$$

that is, *the specific heats of two sensibly perfect gases are directly as their specific volumes.*

But the specific volumes are inversely as the specific weights, or densities, of the gas, or

$$v_o = \frac{1}{w} = \frac{1}{g\delta};$$

hence,

$$\frac{C_p}{C_p'} = \frac{\delta'}{\delta} = \frac{C_v}{C_v'}; \tag{34}$$

that is, *the specific heats of two perfect gases are inversely as their densities.*

52. Let the temperature be constant during expansion, find the heat absorbed.

For this condition

$$d\tau = o$$

in the first and second of equations (B), and from the first we have

$$H = \int p \, d\, v, \tag{35}$$

which may be integrated if p be a known function of v. The equation of the path of the fluid will be equation (2), making $\tau = \tau_1$,

$$p\, v = R\, \tau_1,$$

the value of p from which substituted in the preceding equation gives

$$H = R\, \tau_1 \int_{v_1}^{v_2} \frac{d\, v}{v} = R\, \tau_1 \log_e \frac{v_2}{v_1}. \tag{36}$$

The first member of this equation may be represented by the area $\varphi_1 A\, B\, \varphi_2$, Fig. 19, and the last member by $v_1 A\, B\, v_2$, which is the external work done during the expansion; hence, *in a perfect gas the external work done during an isothermal expansion equals the heat absorbed*—a necessary result, there being no internal work.

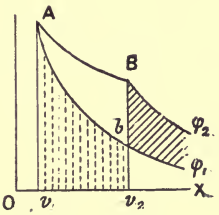

FIG. 19.

Since the area $A\, B\, b$ is common, it follows that, for a perfect gas,

$$\varphi_1\, b\, B\, \varphi_2 = v_1\, A\, b\, v_2.$$

53. Let the volume be constant. Then will $d\, v = 0$ and $v = v_1$, and equations (B) give

$$\begin{aligned} H &= C_v\,(\tau_2 - \tau_1); \\ H &= C_p\,(\tau_2 - \tau_1) - v_1\,(p_2 - p_1). \end{aligned} \tag{37}$$

But the equation of the gas gives

$$v_2\, p_2 = R\, \tau_2, \qquad v_1\, p_1 = R\, \tau_1,$$

and the condition of the problem gives $v_1 = v_2$;

$$\begin{aligned} \therefore H &= C_p\,(\tau_2 - \tau_1) - R\,(\tau_2 - \tau_1) \\ &= (C_p - R)\,(\tau_2 - \tau_1), \end{aligned}$$

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which placed equal to equation (37) gives

$$C_v = C_p - R,$$

as before found in equation (29).

54. Let the pressure be constant. *Significance of R*, equation (29). Let the heat absorbed be

$$d\,H = C_p\,d\,\tau, \tag{38}$$

and equation $(B)_1$ becomes

$$(C_p - C_v)\,d\,\tau = p\,d\,v\,;$$

and since p is constant during the absorption of heat, as indicated by the condition in equation (38), we have by integrating the last equation between the limits τ and $\tau + 1$ for temperature, and v_2 and v_1 for volumes, observing that $v_2 - v_1$ will be the horizontal distance between the isothermals τ and $\tau + 1$ at the upper extremity of the ordinate p, we have

$$(C_p - C_v) = p\,(v_2 - v_1) = R\,;$$

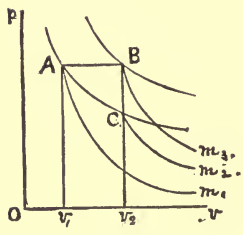

FIG. 20.

that is, *the value of R is the energy exerted by one pound of the gas in expanding at constant pressure while the temperature increases one degree.*

In Fig. 20, if the isothermals through A and B respectively differ by one degree, $A\,B$ being horizontal, $B\,C$ vertical, we have

$$C_p = m_1\,A\,B\,m_3, \qquad C_v = m_2\,C\,B\,m_3,$$

and by the second law,

$$v_1\,A\,C\,v_2 = m_1\,A\,C\,m_2;$$

$$\therefore C_p - C_v = m_1\,A\,B\,m_3 - m_2\,C\,B\,m_3 = m_1\,A\,B\,C\,m_2$$

$$= m_1\,A\,C\,m_2 + A\,B\,C,$$

$$= v_1\,A\,C\,v_2 + A\,B\,C = v_1\,A\,B\,v_2;$$

$$\therefore R = v_1\,A\,B\,v_2,$$

which is the external work done during the expansion at

constant pressure from the state A on one isothermal to the state B on the isothermal one degree higher, as stated above.

For air this becomes 53.21 foot-pounds (Eq. (3)), or $53.21 \div 778 = 0.069$ of a thermal unit.

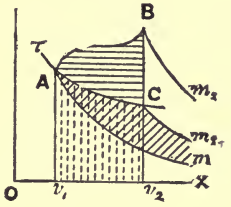

FIG. 21.

55. Let the path be arbitrary. Then will the first of equations (B) give

$$H - \int p\, d\, v = C_v (\tau_2 - \tau_1).$$

The second term is, Fig. 21,

$$\int p\, d\, v = v_1\, A\, B\, v_2,$$

and may be separated into two parts. Through A draw the isothermal $A\ C$, and the adiabatics $A\ m$, $C\ m_\tau$, $B\ m_2$; then

$$v_1\ A\ C\ v_2 = m\ A\ C\ m_\tau;$$

$$\therefore \int p\, d\, v = m\ A\ C\ m_\tau + A\ B\ C.$$

But

$$H = m\ A\ B\ m_2;$$

$$\therefore H - \int p\, d\, v = m\ A\ B\ m_2 - m\ A\ C\ m_\tau - A\ B\ C$$

$$= m_\tau\ C\ B\ m_2 = C_v\ (\tau_2 - \tau_1), \text{ (Eq. (37))};$$

that is, to find the increased energy of the substance in passing from state A to state B due to the absorption of heat, through the initial state A of the substance represented on a diagram of energy pass an isothermal, and note the point C where it intersects the ordinate to the second state, then will the area between $C\ B$ and two adiabatics drawn respectively through C and B indefinitely extended in the direction of increased volume, represent the increased energy of the substance.

Let the isothermal $A\ C$ be prolonged to an intersection

with $B\,\varphi_2$, at D, Fig. 22, then according to the second law the indefinitely extended area $\varphi_3\,C\,D\,\varphi_2$ will equal $v_2\,C\,D\,v'$, to which adding the area $B\,C\,D$, we have

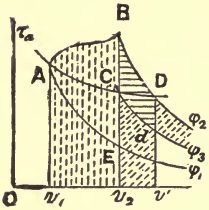

FIG. 22.

$$\varphi_3\,C\,B\,\varphi_2 = v_2\,B\,D\,v',$$

by which means the increased energy of the substance in the state B over that in state A may be represented by the finite area $v_2\,B\,D\,v'$. Thus in working from v_1 to v_2 along the path $A\,B$, Fig. 23, the external work $v_1\,A\,B\,v_2$ will have been done, and the energy of the substance will have been increased by the heat absorbed an amount represented by the area $v_2\,B\,D\,v'$. This mode of representation is due to M. Cazin.

If a piston were driven by the expansion of a fluid without absorbing or emitting heat, it would do the work $v_1\,A\,E\,v_2$, Fig. 22, where $A\,E$ is an adiabatic; but if the heat of the expanding fluid be maintained constant, it will do the work $v_1\,A\,C\,v_2$, where $A\,C$ is an isothermal. In the latter case, the heat absorbed, $\varphi_1 A\,C\,\varphi_3$, according to the second law, equals the entire work done, $v_1\,A\,C\,v_2$; but the work done due to the heat absorbed exceeds that done by adiabatic expansion by the area $E\,A\,C$.

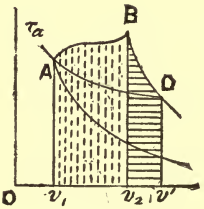

FIG. 23.

EXERCISES.

1. Deduce equation (37) from the second of equations (B), employing any other equation necessary.

2. How many foot-pounds of heat must be absorbed by 2 pounds of air in expanding to double its initial volume at the constant temperature of 100° F.? How

many degrees F. would it raise the temperature of 20 pounds of water? Here, $\tau_1 = 460.66 + 100 = 560.66$, and, equation (36),

$$H = 2 \times 53.21 \times 560.66 \times (2.303 \times 0.301030)$$
$$= 41370 \text{ ft.-lbs.}$$
$$t = \frac{41370}{20 \times 778} = 2.66° \text{ F. nearly.}$$

3. How many B. T. U. of work must be expended in compressing 3 pounds of air at the constant temperature of 15° C. to one fourth its initial volume?

4. By means of equations (3), (29) and (31) reduce equations (B) to the following:—

$$\left.\begin{aligned} dH &= C_p \left[0.711\, d\tau + 0.2887\, \tau \frac{dv}{v}\right]. \\ dH &= C_v \left[1.406\, d\tau - 0.406\, \tau \frac{dp}{p}\right]. \end{aligned}\right\} \quad (39)$$

5. The specific heat of hydrogen at constant pressure being 3.4090 find the specific heat at constant volume. Find the ratio of the specific heat at constant pressure to that at constant volume.

6. The specific heat of oxygen at constant volume having been found to be 0.1551 find the specific heat at constant pressure; the ratio of the two; and their values in foot-pounds.

7. Having

$$C_p - C_v = R,$$

and

$$\frac{C_p}{C_v} = \gamma = 1.406,$$

to find C_p and C_v in terms of R.

$$\text{Ans. } C_p = 3.463\, R = \frac{\gamma}{\gamma - 1} R.$$
$$C_v = 2.463\, R = \frac{1}{\gamma - 1} R.$$

8. Find the value of D in equation (32), and the value of C_v in terms of v_0.

9. The specific heat of air being 0.2375, and the weight of a cubic foot being 0.080728 lbs., and the weight of a cubic foot of hydrogen being 0.005592, find the specific heat of the latter by equation (34) and compare the result with the tabular value.

10. How many foot-pounds of heat must be absorbed in expanding three kilograms of air at the constant temperature of 30° C. from 3 cubic metres to 5 cubic metres?

11. If the equation to the path of the gas be $p = a\,v + b$, the initial volume $v_1 = 10$ cu. ft., initial pressure 2000 lbs. per square foot, the terminal $v_2 = 20$ cu. ft., $p_2 = 5000$ lbs.; how much heat must be absorbed in passing from the initial to the terminal state, how much external work will be done and how much will the energy of the substance be increased? Let the substance be atmospheric air.

Approx. ans. $H = 232000$ ft.-lbs.

$$\int p\,d\,v = 35000 \text{ “ “}$$

$$H - \int p\,d\,v = 197000 \text{ “ “}$$

12. How much heat must be absorbed by a perfect gas in expanding at a constant pressure from v_1 to v_2, the initial temperature being τ_1? What will be the final temperature?

13. How much heat must be absorbed by a perfect gas in expanding from the state p_1, v_1 to p_2, v_2, the equation to the path being $p^2 = m\,(v - b)$? If gas be air, $p_1 = 2000$ lbs. per ft., $p_2 = 6000$, $v_1 = 10$ cu. ft., and $v_2 = 20$ cu. ft.

Ans. 290580 ft.-lbs.

14. In the preceding example, the heat absorbed would raise the temperature of how many pounds of water through three degrees F.?

15. How many B. T. U. will be required to double the

volume of one pound of air at constant pressure from the temperature of melting ice?

16. How much heat will be absorbed in expanding a perfect gas to twice its initial volume, if the equation to the path be $p\,v^{\gamma} = c$ (a constant)?

56. Let the gas expand without transmission of heat. Since no heat is absorbed or emitted, we have, in equations (B),

$$\begin{gathered} d\,H = o; \\ \therefore\ C_v\,d\,\tau = -\,p\,d\,v, \\ C_p\,d\,\tau = v\,d\,p. \end{gathered}$$

Dividing, gives

$$\frac{d\,p}{p} = -\,\frac{C_p}{C_v}\,\frac{d\,v}{v} = -\,\gamma\,\frac{d\,v}{v}.$$

Integrating,

$$log_e \frac{p}{p_1} = log_e \left(\frac{v_1}{v}\right)^{\gamma},$$

where p_1 and v_1 are the initial limits, the other limits being general. From this we find

$$p\,v^{\gamma} = p_1\,v_1^{\gamma} = constant, \tag{40}$$

which is the equation of the projection of the line of no transmission on the plane $p\,v$. To find it on the plane $\tau\,v$, eliminate p from the preceding equation by means of equation (3) and find

$$\frac{\tau}{\tau_1} = \left(\frac{v_1}{v}\right)^{\gamma-1}, \text{ also } = \left(\frac{p}{p_1}\right)^{\frac{\gamma-1}{\gamma}}, \tag{41}$$

which are the equations to the adiabatics for perfect gases.

If p_2, v_2, τ_2 be terminal values, then

$$\frac{\tau_2}{\tau_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \tag{42}$$

EXERCISES.

1. Two cubic feet of air at 60° F., and initial pressure one atmosphere (absolute), is compressed in a cylinder to 5 atmospheres gauge pressure; if there be no transference of heat, required the terminal temperature and volume, and the pounds of water at 50° F. necessary to reduce the temperature to 65° F.

We have, omitting 0.66 in the temperature,

$$\tau_1 = 460 + 60 = 520° \text{ F.}$$

$$\tau_2 = 520\,(\tfrac{6}{1})^{\frac{.406}{1.406}} = 873° \text{ F.}$$

$$v_2 = 2\,(\tfrac{1}{6})^{\frac{1}{1.406}} = 0.559.$$

$$W(65\text{–}50) = 2 \times 0.3375\,(\tau_2 - 525) \times 0.0807 \times \frac{873}{492}$$

$$\therefore W = .89 \text{ lb.}$$

2. If one cubic foot of air expands from a pressure of 4 atmospheres gauge pressure and temperature of 60° F. to an absolute pressure of one atmosphere without transmission of heat, required the terminal temperature.

We have

$$\tau_2 = 520\,(\tfrac{1}{5})^{\frac{.406}{1.406}} = 327°;$$

$$\therefore T_2 = 327 - 460 = -133° \text{ F.},$$

or the terminal temperature will be 133° below the zero of Fahrenheit's scale.

57. It will be seen from exercise 2 that a low temperature may be secured by suddenly expanding a gas from a high pressure and moderate temperature to a low pressure. This principle is used for commercial purposes, one form of which is called "cold storage." A gas—as ammonia—is compressed to a comparatively high tension, thus increasing its temperature, and allowed to cool while under high tension, after which it is expanded to a low tension, thus producing a low temperature of the fluid. A liquid whose

freezing point is lower than that of water—technically called "brine"—is made to circulate in pipes through the cool fluid, thus reducing its temperature, after which it passes to comparatively air-tight compartments containing the articles to be preserved—such as meat, vegetables, fruit, eggs, &c. Any desired temperature may be thus maintained for any length of time at all seasons of the year. Meat is thus stored and kept frozen for months.

Ice machines are constructed on the same principle, by means of which ice may be manufactured during hot weather.

58. An air-compressor is a machine, or engine, for compressing a gas, as atmospheric air, to a higher tension. Air thus compressed is useful for many purposes—especially for driving engines in the place of steam, when the power is to be transmitted a considerable distance, and especially underground. It does not condense like steam. If the heat which is generated in the act of compression could be retained until the air is used in the motor, it would be useful, but a large portion of it generally escapes through the walls of the conducting pipes and storage reservoirs, and hence is energy lost. To avoid this loss efforts are made, in the use of the best compressors, to prevent as much as possible the rise of temperature during the process of compression, by injecting water into the cylinder. The water should enter the cylinder in the form of a fine spray, the elements widely diverging, so as to fill, as nearly as possible, the entire cross-section of the cylinder with a mist.

If compressed air escaping from a vessel suddenly expands, its temperature may be reduced to such an extent as to freeze the water in it and choke the exhaust. This annoyance is reduced, and sometimes prevented, in the case of motors, by causing warm air to circulate about the exhaust, or by gradually increasing the section of the outlet. Refrig-

eration and the principles of the air compressor are discussed on pages 301–306 and in Chapter V.

EXERCISES.

1. An air-compressor whose cross-section is 2 square feet and stroke 3 feet takes in air at a tension of 14 pounds per square inch and temperature 60° F., and compresses it to 60 lbs. gauge pressure without transmission of heat; required the final temperature and the pounds of water at 50° F. necessary to reduce the temperature of the air to 55° F.; to 70° F.; to 90° F.

2. In the preceding exercise, if the air at 60 lbs. gauge pressure and 70° F. expands adiabatically to a final pressure of 20 lbs. gauge pressure; required the final temperature and the pounds of water that it would cool from 60° F. to 32° F.

3. Required the entire amount of heat-energy in one pound of atmospheric air, at the temperature of melting ice, considered as a perfect gas.

Considering that it is brought to its present temperature by being heated from absolute zero at a constant volume, we have, in the first of equations (*B*),

$$d v = o;$$

$$\therefore H = C_v \int_o^{\tau_o} d\tau = C_v \tau_o \qquad (a)$$

$$= 131.40 \times 492.66 = 64735 \text{ ft.-lbs.} \qquad (b)$$

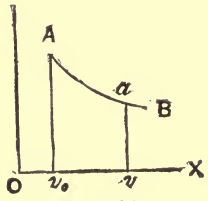

FIG. 24.

We may also consider that the entire heat-energy has been transmuted into external work by an adiabatic expansion, *A B*, Fig. 24, in which case we have, from the first of equations (*B*),

$$d H = 0 = C_v d\tau + p\, d v.$$

But from equation (40),

$$p v^{\gamma} = p_o v_o^{\gamma};$$

$$\therefore p = p_o v_o^{\gamma} v^{-\gamma} = a v, \text{ Fig. 24};$$

$$\therefore - C_v \int_{\tau_o}^{o} d\tau = p_o v_o^{\gamma} \int_{v_o}^{\infty} v^{-\gamma} d v,$$

or $$C_v \tau_0 = \frac{p_0 \nu_0}{\gamma - 1} = \frac{26214}{\gamma - 1} = 64735;$$

$$\therefore \gamma = 1.405 \text{ very nearly.}$$

If $\gamma = 1.406$, as previously found, then

$$\frac{26214}{\gamma - 1} = 64566,$$

which is about $\frac{1}{400}$ of this value less than the preceding.

If the terminal temperature be the zero of the air thermometer, then the value of H in equation (a) would be

$$H = 131.40 \times 491.13 = 64534,$$

which agrees very nearly with the preceding value, where $\gamma = 1.406$.

4. Required the height to which a ball weighing one pound could be projected upward in a vacuum by the heat-energy in one pound of air under the pressure of one atmosphere at the temperature of melting ice. (Use the value in (b) of exercise 3.) How many times the height of a homogeneous atmosphere?

5. Required the entire heat-energy in one pound of hydrogen at sea level at the temperature of melting ice.

6. Required the heat-energy in two pounds of air under the pressure of one atmosphere at the temperature of 100° F.

7. If a gas be forced into or out of a receiver of constant volume, without transmission of heat, can equations (42) be transformed so as to give the relation between weights, temperatures and pressures?

8. For a perfect gas, verify the fact that the external work $v_1 A B v_2$, Fig. 25, equals $\varphi_1 A B \varphi_2$, $A B$ being an isothermal, $A \varphi_1$ and $B \varphi_2$ adiabatics, by finding the area $\varphi_1 b B \varphi_2$.

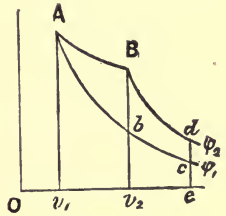

FIG. 25.

The equation of $A \varphi_1$ will be

$$p_1 \overset{\gamma}{v_1} = p \overset{\gamma}{v},$$

of $B\,\varphi_2$, $\quad p_2\,v_2^{\gamma} = p\,v^{\gamma}$;

from the former, $\quad p = e\,c = \dfrac{p_1\,v_1^{\gamma}}{v^{\gamma}}$, Fig. 25,

from latter, $\quad p = e\,d = \dfrac{p_2\,v_2^{\gamma}}{v^{\gamma}}$;

$$\therefore\ d\,c = \frac{p_2\,v_2^{\gamma} - p_1\,v_1^{\gamma}}{v^{\gamma}},$$

and
$$\text{area } \varphi_1\,b\,B\,\varphi_2 = \left(p_2\,v_2^{\gamma} - p_1\,v_1^{\gamma}\right)\int_{v_2}^{\infty}\frac{d\,v}{v^{\gamma}}$$
$$= \frac{p_2\,v_2 - p_1\,v_1^{\gamma}\,v_2^{1-\gamma}}{\gamma - 1}.$$

Also,
$$\text{area } v_1\,A\,b\,v_2 = \int_{v_1}^{v_2} p_1\,v_1^{\gamma}\frac{d\,v}{v^{\gamma}} = \frac{p_1\,v_1 - p_1\,v_1^{\gamma}\,v_2^{1-\gamma}}{\gamma - 1};$$

and since $A\,B$ is an isothermal, we have $p_1\,v_1 = p_2\,v_2$; hence

$$\text{area } \varphi_1\,b\,B\,\varphi_2 = \text{area } v_1\,A\,b\,v_2.$$

Adding $A\,B\,b\,A$ to both, gives

$$v_1\,A\,B\,v_2 = \varphi_1\,A\,B\,\varphi_2.$$

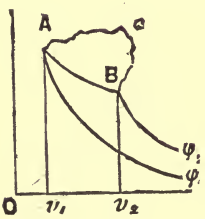

FIG. 26.

9. In a perfect gas, show that if the states A and B, Fig. 26, are on the same isothermal, the intrinsic energies are the same at those states, and hence independent of the path of the fluid between those states.

This follows directly from the analysis of exercise 3; for it is there shown that the intrinsic energy at A will be

$$\frac{p_a\,v_a}{\gamma - 1};$$

and hence at B
$$\frac{p_b\,v_b}{\gamma - 1};$$

but since A and B are on the same isothermal, we have

$$p_a v_a = p_b v_b ;$$

and, therefore, the two preceding expressions are equal, independently of the path between the states A and B.

10. In a perfect gas, verify the principle: The elementary areas between consecutive isothermals and a pair of adiabatics are equal, by using the equations to the adiabatics.

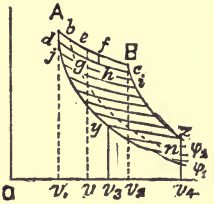

FIG. 27.

Let $A\ B$, Fig. 27, be an isothermal—the topmost one—$y\ z$ any other, both limited by the pair of adiabatics $A\ \varphi_1$ and $B\ \varphi_2$. Let isothermals be drawn under $A\ B$ and $y\ z$ respectively, so that the difference in temperatures will be $d\ \tau$ in each; the length of the one at $A\ B$ measured on the axis of abscissas being $v_2 - v_1$, and of $y z$, $v_4 - v_3$, we are to prove that

$$\int_{v_1}^{v_2} d\ p\ d\ v = \int_{v_3}^{v_4} d\ p\ d\ v.$$

Let the equations to the lines be

$$p\ v = a = R\ \tau_1 \text{ for } A\ B, \tag{1}$$

$$p\ v = d = R\ \tau_2 \text{ for } y\ z, \tag{2}$$

$$p\ v^{\gamma} = b \text{ for } A\ \varphi_1, \tag{3}$$

$$p\ v^{\gamma} = c \text{ for } B\ \varphi_2. \tag{4}$$

To find the abscissa, v_1, of the point of intersection of $A\ B$ and $A\ \varphi_1$, make p common to equations (1) and (3), eliminating which gives

$$O\ v_1 = v_1 = \left(\frac{b}{a}\right)^{\frac{1}{\gamma - 1}}.$$

Similarly, from (1) and (4),

$$v_2 = \left(\frac{c}{a}\right)^{\frac{1}{\gamma - 1}};$$

$$\therefore \frac{v_2}{v_1} = \left(\frac{c}{b}\right)^{\frac{1}{\gamma - 1}}.$$

Similarly, for the points y and z, we have

$$v_3 = \left(\frac{b}{d}\right)^{\frac{1}{\gamma-1}},$$

$$v_4 = \left(\frac{c}{d}\right)^{\frac{1}{\gamma-1}};$$

$$\therefore \frac{v_4}{v_3} = \left(\frac{c}{b}\right)^{\frac{1}{\gamma-1}};$$

$$\therefore \frac{v_4}{v_3} = \frac{v_2}{v_1}.$$

From the equation to a perfect gas, we have

$$p\,v = R\,\tau;$$

$$\therefore d\,p = \frac{R}{v} d\,\tau;$$

$$\therefore \int_{v_1}^{v_2} d\,p\,d\,v = R\,d\,\tau \int_{v_1}^{v_2} \frac{d\,v}{v} = R\,d\,\tau\ log\ \frac{v_2}{v_1},$$

which is the area of the strip $A\ B$.

Similarly,
$$\int_{v_3}^{v_4} d\,p\,d\,v = R\,d\,\tau \int_{v_3}^{v_4} \frac{d\,v}{v} = R d\tau\ log\ \frac{v^4}{v_3},$$

which is the area of the strip at $y\ z$; but since $\frac{v_4}{v_3} = \frac{v_2}{v_1}$, as shown above, it follows that these strips have equal areas.

It appears from the above that the lengths of the strips $A\ B$ and $x\ y$ are unequal, for

$$v_2 - v_1 = \frac{c^{\frac{1}{\gamma-1}} - b^{\frac{1}{\gamma-1}}}{a^{\frac{1}{\gamma-1}}} = \frac{A}{a^{\frac{1}{\gamma-1}}}, \text{ (say),}$$

but

$$v_4 - v_3 = \frac{c^{\frac{1}{\gamma-1}} - b^{\frac{1}{\gamma-1}}}{d^{\frac{1}{\gamma-1}}} = \frac{A}{d^{\frac{1}{\gamma-1}}}.$$

For instance, if the temperature of the isothermal $y\ z$ were 300° absolute, and of $A\ B$, 600° absolute, then from (1) and (2) above we have

$$\frac{a}{d} = 2,$$

and

$$\frac{v_4 - v_3}{v_2 - v_1} = \left(\frac{a}{d}\right)^{\frac{1}{\gamma - 1}} = 2^{2 \cdot 463} = 5.52,$$

or the strip at $y\ z$ for the 300° absolute temperature would be more than $5\frac{1}{2}$ times the length of the topmost, or the 600 degree one. For the depths we have

$$d\ p_1 = \frac{R}{v_1}\ d\ \tau, \text{ at } A,$$

$$d\ p_2 = \frac{R}{v_2}\ d\ \tau, \text{ at } B,$$

$$d\ p_3 = \frac{R}{v_3}\ d\ \tau, \text{ at } y,$$

$$d\ p_4 = \frac{R}{v_4}\ d\ \tau, \text{ at } z,$$

being less and less from A to z. If $v_2 = 2\ v_1$, then $v_4 = 2\ v_3$, and

$$d\ p_1 = 2\ d\ p_2,$$
$$d\ p_3 = 2\ d\ p_4;$$

and further, if $v_2 = v_3$, then

$$d\ p_1 = 2\ d\ p_2 = 4\ d\ p_4.$$

The strips increase in length as the isothermals are lower in temperature, but decrease in depth. In the case of perfect gases the consecutive isothermals cut equal areas from the area representing external work, and the area representing heat, so that in Fig. 15 we have $a\ b\ c\ d = e\ f\ h\ g$.

11. In Fig. 28, show that the area $v_1\ A_1\ B_1\ v_3 = v_2\ A_2\ B_2\ v_4$, for perfect gases.

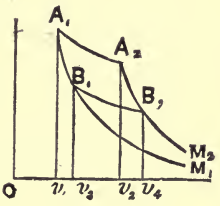

FIG. 28.

Let the equations to the lines be

for $A_1\ A_2$, $p\ v = c_1$,
for $B_1\ B_2$, $p\ v = c_2$,
for $A_1\ M_1$, $p\ v^\gamma = c_3$,
for $A_2\ M_2$, $p\ v^\gamma = c_4$;

from which the co-ordinates of A_1, A_2, B_1, B_2, may be found. Thus,

$O\,v_1 = v_1 = \left(\frac{c_3}{c_1}\right)^{\frac{1}{\gamma-1}}$, and similarly for the other co-ordinates. We have

$$v_1\,A_1\,B_1\,v_3 = \int p\,d\,v = c_3 \int_{c_1}^{v_3} \frac{d\,v}{v^\gamma} = \frac{1}{\gamma-1}(c_1 - c_2),$$

and the area $v_2\,A_2\,B_2\,v_4$ reduces to the same value; hence, in working around Carnot's cycle the work done during expansion without transmission of heat equals that done during compression also without transmission of heat, and they cancel each other. This should have been anticipated, since all the work done is by a transmutation of heat.

12. A cylindrical vessel the area of whose base is one square foot contains 2 cubic feet of air at 60° F. when compressed by a frictionless piston of 2000 pounds resting upon it; required the volume and temperature of the air if the vessel be inverted, there being no transmission of air or heat.

13. A cylindrical vessel the area of whose base is a square feet and height b feet is filled with air at 60° F. and pressure of one atmosphere; a frictionless piston whose weight is w is placed at the upper end and dropped into the cylinder; if there be no escape of air nor of heat how much will the air be compressed, and what will be its temperature at the instant of greatest compression?

Let x = the height of the volume of air when the piston has descended to its lowest point;

p = the pressure of the atmosphere on a square foot;

then will the work done during the descent of the piston be

$$(p\,a + w)\,(b - x);$$

and the opposing work done by the air will be

$$\int p\,d\,v = p_1\,v_1^\gamma \int_x^b \frac{d\,v}{v^\gamma} = \frac{p_1\,v_1^\gamma}{\gamma-1}\left(\frac{1}{x^{\gamma-1}} - \frac{1}{b^{\gamma-1}}\right);$$

$$\therefore (p\,a + w)(b - x) = \frac{a\,p\,b^\gamma}{0.406}\left(\frac{1}{x^{0.406}} - \frac{1}{b^{0.406}}\right),$$

from which the value of x may be found when numbers are substituted

for the other letters, and therefore $b - x$ becomes known. The adiabatic relation also gives

$$\tau_2 = 520.6 \left(\frac{b}{x}\right)^{\gamma - 1}.$$

14. If the heat absorbed varies as τ^n find the path of the fluid.

Here
$$H = a\,\tau^n\,;$$
$$\therefore\ d\,H = a\,n\,\tau^{n-1}\,d\,\tau,$$

and the first and second of equations (B) give

$$a\,n\,\tau^{n-1}\,d\,\tau = C_v\,d\,\tau + p\,d\,v,$$
$$a\,n\,\tau^{n-1}\,d\,\tau = C_p\,d\,\tau - v\,d\,p\,;$$

in which substitute

$$p = \frac{R\,\tau}{v} \text{ and } v = \frac{R\,\tau}{p},$$

and dividing through by τ, thus separating the variables, and integrating, gives

$$\frac{a\,n}{n-1}\left(\tau^{n-1} - \tau_1{}^{n-1}\right) = log\left(\frac{v}{v_1}\right)^R\left(\frac{\tau}{\tau_1}\right)^{C_v} = log\left(\frac{p_1}{p}\right)^R\left(\frac{\tau}{\tau_1}\right)^{C_p}.$$

Eliminate τ by means of equation (2) and find the equation in terms of p and v. Equation (2) may be deduced from the last equation here given.

15. If the heat absorbed varies as v^n find the equation to the path of the fluid.

16. If the heat absorbed varies as p^n find the path of the fluid.

17. If $H = a\,v^n + b\,p^n$ find the equation to the path.

APPLICATIONS.

59. Velocity of a wave in an elastic medium.*

This article is a digression for the purpose of establishing a formula from

* The general problem of wave propagation has received the attention of several of the most eminent mathematicians since the days of Newton, and many problems have been solved in a satisfactory manner. The simple method of Newton, *Principia*, Prob's XLIII.–L., B. II., has not been excelled, and the definite theoretical result obtained is quoted to the present day, although the effect of heat upon the velocity of sound was not then known. La Place, in the *Mécanique Céleste*, tomes II. and V., has treated of the oscillations of the sea and atmosphere ; Lagrange,

which γ may be deduced. Assume that the medium is confined in a prismatic tube of section unity, E the coefficient of elasticity for compression, p_1 a force which will produce a compression $d\,y$ in a length $d\,x$, then from definition we have

$$p_1 = E\,\frac{d\,y}{d\,x}.$$

The lamina $d\,x$ will be urged forward—or backward—by the difference of the elastic forces on opposite sides of it, and as the quantities are infinitesimal, this difference will be $d\,p_1$; or

$$p' = d\,p_1 = E\,\frac{d^2\,y}{d\,x}.$$

Let D be the density of the lamina, then its mass will be $M = D\,d\,x$, and we have from equation (21), page 18, of Analytical Mechanics,

$$D\,d\,x\,\frac{d^2\,y}{d\,t^2} = E\,\frac{d^2\,y}{d\,x},$$

or,

$$\frac{d^2\,y}{d\,t^2} = \frac{E}{D}\,\frac{d^2\,y}{d\,x^2}, \tag{43}$$

which is a partial differential equation of the motion of any lamina, the integral of which is given in works on Differential Equations. One of the methods is as follows: Let $E \div D = a^2$, and adding $a\,\frac{d^2\,y}{d\,x\,d\,t}$ to both members, we have

$$\frac{1}{d\,t}\,d\left(\frac{d\,y}{d\,t} + a\,\frac{d\,y}{d\,x}\right) = \frac{a}{d\,x}\,d\left(\frac{d\,y}{d\,t} + a\,\frac{d\,y}{d\,x}\right).$$

Let

$$V = \frac{d\,y}{d\,t} + a\,\frac{d\,y}{d\,x},$$

---

in the *Mécanique Analytique*, tome II., has discussed the problem of the movement of a heavy liquid in a very long canal; M. Navier published a Mémoire on the flow of elastic fluids in pipes, in the *Académie des Sciences*, tome IX.; and M. Poisson wrote several Mémoires on the propagation of wave movements in an elastic medium, and the theory of sound, for which see *Journal de l'École Polytechnique*, 14th chapter, and of the *Académie des Sciences*, tomes II. and X. These eminent mathematicians established the basis of the analysis for the solution of the problem. More recently we have M. Lamé's *Leçons sur l'Elasticité des Corps solides*, and Lord Rayliegh's *Treatise on Sound*, both of which are works of great merit.

then
$$\left(\frac{d\,V}{d\,t}\right) = \frac{d^2\,y}{d\,t^2} + a\,\frac{d^2\,y}{d\,x\,d\,t}, \tag{44}$$

where the parenthesis indicates a partial differential coefficient, and

$$\left(\frac{d\,V}{d\,x}\right) = \frac{d^2\,y}{d\,t\,d\,x} + a\,\frac{d^2\,y}{d\,x^2}, \tag{45}$$

and equations (43), (44), (45), or, more directly, the last equations on p. 72 give

$$\left(\frac{d\,V}{d\,t}\right) = a\left(\frac{d\,V}{d\,x}\right). \tag{46}$$

The total differential of $V = f(x,\ t)$ is

$$d\,V = \left(\frac{d\,V}{d\,x}\right) d\,x + \left(\frac{d\,V}{d\,t}\right) d\,t,$$

by substituting (46),

$$= \left(\frac{d\,V}{d\,x}\right)(d\,x + a\,d\,t)$$

$$= \left(\frac{d\,V}{d\,x}\right) d\,(x + a\,t),$$

and integrating,

$$V = F(x + a\,t) = \frac{d\,y}{d\,t} + a\,\frac{d\,y}{d\,x}, \tag{47}$$

where F is any arbitrary function.

Similarly subtracting $a\,\dfrac{d^2\,y}{d\,x\,d\,t}$ from (43),

$$V = f(x - a\,t) = \frac{d\,y}{d\,t} - a\,\frac{d\,y}{d\,x}. \tag{48}$$

Adding and subtracting (47) and (48), we have the respective equations

$$\frac{d\,y}{d\,t} = \tfrac{1}{2}\,F(x + a\,t) + \tfrac{1}{2}\,f(x - a\,t),$$

$$\frac{d\,y}{d\,x} = \frac{1}{2\,a}\,F(x + a\,t) - \frac{1}{2\,a}\,f(x - a\,t).$$

But
$$y = f(x,\ t);$$

$$\therefore\ d\,y = \left(\frac{d\,y}{d\,x}\right) d\,x + \left(\frac{d\,y}{d\,t}\right) d\,t,$$

and substituting from above, gives

$$d\,y = \frac{1}{2\,a}\,F(x + a\,t)\,d\,(x + a\,t) - \frac{1}{2\,a}\,f(x - a\,t)\,d\,(x - a\,t);$$

integrating,

$$y = \psi(x + a\,t) - \varphi(x - a\,t), \tag{49}$$

where ψ and φ are any arbitrary functions whatever. Their character and initial values must be determined from the conditions of the problem. The equation represents a wave both from and toward the origin. If the wave be from the origin only, the φ function may be suppressed, and we have

$$y = \psi (x + a\,t), \tag{50}$$

and differentiating,

$$\left(\frac{d\,y}{d\,x}\right) = \psi' (x + a\,t),$$

which is the rate of dilation (the expansion or contraction of a prism of the air), and

$$\left(\frac{d\,y}{d\,t}\right) = a \cdot \psi' (x + a\,t);$$

which is the velocity of a particle, and dividing the latter by the former,

$$\frac{d\,x}{d\,t} = a, \tag{51}$$

which is the velocity of the wave; hence,

$$u = a = \sqrt{\frac{E}{D}}, \tag{52}$$

which is Newton's formula (*Principia*, ii., § 8).

The elasticity of air equals its tension; hence, if p be the pressure per square foot, w the weight of a cubic foot, and H the height of a homogeneous atmosphere, then

$$u = \sqrt{\frac{g\,p}{w}} = \sqrt{g\,H}; \tag{53}$$

hence the *velocity of sound would equal the velocity of a body falling through a height equal to one half the height of a uniform atmosphere of that substance.*

This principle is applicable also to the vibration of elastic cords, and it is found that

The velocity of vibration of an elastic cord equals the velocity of a body falling freely through a height equal to half the length of the same cord whose weight would equal the tension.

Similarly, for long waves, or waves on water whose depth is small compared with the wave-length,

The velocity equals, approximately, the velocity of a body falling freely through a height equal to half the depth of the sea. (Ency. Britannica.)

It has been assumed that E and D remain constant in wave motion; but it has long been known that the results given by equation (53) for gases do not agree with those found by experiment, and La Place showed that the elasticity was increased by the action of the wave due to com-

pression. It is necessary, therefore, to consider equation (52) correct only for ultimate values; or

$$u = \sqrt{\frac{d E}{d D}} = \sqrt{\frac{d p}{d D}}. \tag{54}$$

Since

$$p \propto w^{\gamma}$$

$$d p = \frac{\gamma p}{w} d w = \frac{g \gamma p}{w} d D;$$

$$\therefore u = \sqrt{\frac{g \gamma p}{w}} = \sqrt{g \gamma H}. \tag{55}$$

60. To find the value of γ, we have from equation (55)

$$\gamma = \frac{w}{g p} u^2; \tag{56}$$

by means of which γ may be found when the velocity of sound in a gas of given weight and tension are known. We have

$$\frac{w}{p} = \frac{1}{p v} = \frac{1}{p_0 v_0} \cdot \frac{\tau_0}{\tau};$$

$$\therefore \gamma = \frac{\tau_0 u^2}{g p_0 v_0 \tau}, \tag{57}$$

which reduces the determination of γ to that of the velocity, u, of sound in a gas at known temperature τ; g, $p_0 v_0$, being known.

61. The velocity of sound has been determined by direct experiment with the following results:

In dry air at 0° C.	Velocity per second. Centimetres.	Feet.
MM. Bravais and Martins	33237	1090.5
Hr. Moll, Van Beck, and Kuytenbrouwer	33226	1090.1
The French Academy, 1738	33200	1089.2
" " " 1822	33120	1086.6
M. Regnault	33070	1085.0
M. Le Roux	33066	1084.9
Mean	33153	1087.7

Again, if v be the velocity of sound, λ the wave length and n the number of waves (or the frequency of the waves) in a second, then

$$v = n\lambda.$$

By this means it has been found for the velocity of sound in dry air at 0° C.

	Centimetres.	Feet.
By M. Dulong	33300	1092.5
" Hr. Seebeck	33277	1091.8
" M. Schneebeli	33206	1089.4
" Hr. Wertheim	33313	1093.0
Mean	33274	1091.6

Hr. Kayser determined very accurately the wave length λ, by means of Professor Kundt's dust figures, corresponding to a given tone of vibration, or frequency n, by which means he found for

	Centimetres.	Feet.
the velocity of sound in air at 0° C.	33250	1090.9
Mean of the eleven determinations	33206	1089.4

The height of a homogeneous atmosphere of dry air at 0° C. as determined by Regnault is 26214 feet; hence, the theoretical velocity of sound in air, neglecting the effect of wave condensation, would be

$$v = \sqrt{26214 \times 32.2} = 918.7 \text{ ft. per second};$$

$$\therefore \gamma = \left(\frac{1089.4}{918.7}\right)^2 = 1.4061 = 1.406 \text{ (say)}. \qquad (58)$$

This value of γ is on the supposition that air is a perfect gas, and the error resulting from this hypothesis is scarcely appreciable, and certainly cannot affect the result so much as the errors of observation. The mean of such a large number of good determinations is *probably* more reliable than any single observation taken at random. This was the original mode of finding the value of γ, and the only one known before the determination of Joule's equivalent. It

is nearly the same for all the permanent gases, as air, hydrogen, oxygen and nitrogen.

62. To find the specific heats of the sensibly perfect gases. We found in equation (32) that

$$C_p = \frac{\gamma}{\gamma - 1} R; \tag{59}$$

$$\therefore C_v = \frac{1}{\gamma - 1} R. \tag{60}$$

For air we have

$$C_p = \frac{1406}{406} 53.37 = 184.83;$$

$$\therefore c_p = 184.83 \div 778 = 0.23757.$$

The mean of the values given by Regnault is 0.23751 (*Relation des Exp.*, tome II., p. 101), the first four figures of which agree with the results of our computation. Professor Rankine was the first to make this computation, in 1850, using, however, $\gamma = 1.4$, which gave $c_p = 0.24$. Up to that time this was the most accurate determination of the specific heat of air; and when, soon afterward, the very accurate and entirely reliable experiments of Regnault gave very nearly the same result, Rankine's determination was considered a crucial test of the correctness of the dynamical theory of heat.

63. To find the mechanical equivalent of heat by means of the specific heat of a gas. From equation (59) we have

$$C_p = J c_p = \frac{\gamma}{\gamma - 1} R;$$

$$\therefore J = \frac{\gamma}{\gamma - 1} \cdot \frac{R}{c_p}. \tag{61}$$

Having found γ by means of equation (58) without a knowledge of the value of J, and R and c_p having been found by Regnault, we have, by substitution,

$$J = \frac{1406}{406} \times \frac{53.37}{0.2375} = 778 \text{ foot-pounds}$$

on the scale of the air thermometer. If $R = 53.21$ we find $J = 776$ for the probable mechanical equivalent on the absolute scale. The former equals 426.8 kilogram-metres. This was substantially the method originally used by Mayer of Germany, by which means, in 1842, he found 365 kilogram-metres, and Holtzman, in 1845, found 374. The fact that Dr. Mayer *assumed* air to be a perfect gas, and made no attempt to prove the correctness of the assumption, added to the fact that the value he obtained was scarcely a rough approximation, has, in the eyes of some historians, deprived him of the honor of being the first to determine this important constant. Joule justly has the credit of first determining it accurately. Mayer did not work out the equations as above, but solved the problem in the most elementary manner, the process for which is worthy of special study. Thus, he considered that it required a certain amount of heat to increase the temperature of a given amount of gas at constant volume—all of which simply made the gas hotter, and if the gas expanded against an external resistance it required more heat in order to maintain the higher temperature—thus reviving the idea of Rumford and Davy. Let one pound of air under the pressure of one atmosphere, p pounds per square foot, occupy the volume v. The increase of volume for one degree of temperature, p being constant, will be $v \div \tau$, and hence the external work done will be $\frac{p\,v}{\tau} = R$ (equation (2)). The heat absorbed at constant volume will be c_v, and, at constant pressure, c_p, and the difference of these amounts of heat does the work R, provided no internal work is done; hence,

$$J\,(c_p - c_v) = R;$$

$$\therefore\ J = \frac{R}{c_p - c_v} = \frac{R}{(\gamma - 1)\,c_v},$$

as before.

64. The constants J, R, γ, are so related as to serve as mutual checks upon each other, but this relation does not determine the *exact* values of any one of them. When determined directly they are subject to small errors, due chiefly to errors of observation, but the results are believed to be correct within $\frac{1}{2}$ of one per cent, and in some cases the error is probably much less.

65. Other methods of determining γ. This constant has been found by the principle of adiabatic expansion. Thus, equations (41) give

$$\gamma = \frac{log\ p - log\ p_1}{log\ v_1 - log\ v}$$

$$= \frac{log\ \tau - log\ \tau_1}{log\ v_1 - log\ v} + 1$$

$$= \frac{log\ p - log\ p_1}{log\ p - log\ p_1 - log\ \tau + log\ \tau_1}.$$

To secure data for use in these equations, MM. Clément and Désormes used a 20-litre glass globe closed by a stop-cock A, Fig. 29, and connected with a vertical glass tube B, dipped into water, which acted as a manometer. By means of an air-pump attached at D a partial vacuum was produced in the vessel, after which, by opening the cock at A a very short time, air would rush in and produce the pressure of the external atmosphere, and by compressing that already in the vessel, raise the temperature; and after the cock was closed it cooled to that of the surrounding temperature, and the pressure diminished. In the preceding equations let $p = p_0 =$ the atmospheric pressure, p_1 the initial and p its final pressure; then the temperature being the same at the beginning and end of the experiment,

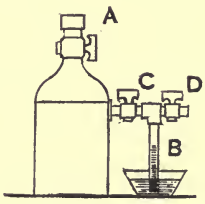

FIG. 29.

we have

$$\frac{v_1}{v} = \frac{p}{p_1},$$

or

$$log\ v_1 - log\ v = log\ p - log\ p_1;$$

$$\therefore \gamma = \frac{log\ p_0 - log\ p_1}{log\ p - log\ p_1}.$$

It was found in one of the experiments that

$$p_0 = 1.0136, p_1 = 0.9953, p = 1.0088;$$

$$\therefore \gamma = 1.3524.*$$

MM. Gay Lussac and Wilter modified this experiment by forcing air into the vessel and allowing it to escape adiabatically until the pressure in the vessel equalled that of the external air. They found

$$p_0 = 1.0096, p_1 = 1.0314, p = 1.0155;$$

$$\therefore \gamma = 1.3745.†$$

In this manner M. Hirn found 1.3845, M. Dupré, 1.399, Hr. Weisbach, 1.4025, M. Masson, 1.419 for air. For carbon dioxide M. Masson found 1.30.

The discrepancy in the results arises chiefly from the fact that the changes in pressure are not adiabatic, but the inertia of the inflowing gas produced a compression exceeding the normal value, resulting in a reaction tending to force a portion of the air out again, producing an oscillating effect, as shown by M. Cazin,‡ who also found, by similar means, the same value, 1.41, for air, oxygen, nitrogen, hydrogen and carbon monoxide.

Hr. Kohlrausch substituted an aneroid for the manometer used above, because more sensitive to pressure, and found $\gamma = 1.296$ (*Pogg. Annalen*, 1869, CXXXVI., 618). This

* *Jour. de Physique*, LXXXIX. (1819), 428.

† *Ann. de Ch. et de Phys.*, 1821, XIX., 436.

‡ *Ibid.*, 1862, LXVI., 206.

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result was not considered good on account of the small quantity of air used in the experiment, although the method is considered an improvement on the preceding.

Dr. Röntgen in 1872 made a series of experiments with a more perfect apparatus, containing a much larger quantity of the gas, the mean of ten good experiments giving $\gamma = 1.4053$ for air (*ibid.* (1873), CXLVIII., 580).

The difficulty in these experiments of obtaining the observations for strictly adiabatic changes generally results in too small a value for this constant.

66. Flow of gases. The flow of *perfect gases* as affected by the principles of thermodynamics was investigated by Messrs. Joule and Thomson (*Proc. Roy. Soc.*, 1856) and Weisbach (*Civilingeneur*, 1856). See the author's *Analytical Mechanics*, page 389.

Let w = the weight of a unit of volume,
p = the pressure at any point of the issuing jet,
V = the velocity at the point where p is measured;

then, for a unit of section and distance $d\,s$ the mass moved will be $w\,d\,s \div g$ and the work done by $d\,p$ will be

$$d\,p\,d\,s = \tfrac{1}{2}\frac{w\,d\,s}{g}\,d(V^2);$$

$$\therefore \frac{V^2}{2\,g} = \int \frac{d\,p}{w}.$$

If the cooling due to the expansion during discharge follows the adiabatic law, then from equations (41) we have

$$\left(\frac{v_1}{v}\right)^{\gamma-1} = \left(\frac{w}{w_1}\right)^{\gamma-1} = \left(\frac{p}{p_1}\right)^{\frac{\gamma-1}{\gamma}};$$

$$\therefore w = w_1\left(\frac{p}{p_1}\right)^{\frac{1}{\gamma}},$$

which substituted above gives

$$\frac{V^2}{2g} = \frac{p_1^{\frac{1}{\gamma}}}{w_1} \int_{p_2}^{p_1} \frac{dp}{p^{\frac{1}{\gamma}}} = \frac{\gamma}{\gamma - 1} \frac{p_1}{w_1} \left[1 - \frac{\tau_2}{\tau_1}\right]; \qquad (62)$$

where p_1 is the tension just within the reservoir, and p_2 that just outside. But the equation of a perfect gas reduces to

$$\frac{p_1}{w_1} = \frac{p_0}{w_0} \frac{\tau_1}{\tau_0} = p_0 v_0 \frac{\tau_1}{\tau_0};$$

$$\therefore V = \sqrt{2g \cdot \frac{\gamma}{\gamma - 1} \cdot \frac{p_0 v_0}{\tau_0} \tau_1 \left(1 - \frac{\tau_2}{\tau_1}\right)},$$

which in English units becomes

$$V = \sqrt{2 \times 32.2 \times \frac{1406}{406} \frac{p_0 v_0}{\tau_0} \tau_1 \left(1 - \frac{\tau_2}{\tau_1}\right)}$$

$$= 14.933 \sqrt{\frac{p_0 v_0}{\tau_0} \tau_1 \left(1 - \frac{\tau_2}{\tau_1}\right)}; \qquad (63)$$

and this for air becomes

$$V = 14.933 \sqrt{53.21 \, \tau_1 \left(1 - \frac{\tau_2}{\tau_1}\right)}$$

$$= 108.93 \sqrt{\tau_1 \left(1 - \frac{\tau_2}{\tau_1}\right)}, \text{ nearly.}$$

If the flow be into a vacuum, $p_2 = o$; $\therefore \tau_2 = o$, and

$$V = 108.93 \sqrt{\tau_1};$$

which at the temperature of melting ice becomes

$$V = 108.93 \sqrt{492.66} = 2417 \text{ feet per second.}$$

Making $\tau_2 = o$ in equation (62) and comparing with equation (56) of Article 60, shows that the velocity of discharge into a vacuum will be

$$\sqrt{\frac{2}{\gamma - 1}}$$

times the velocity of sound in the gas at the melting point

of ice, which for air becomes $2.214 \times 1089.4 = 2417$ feet per second, which is less than half a mile per second.

67. The weight of gas escaping per second will be

$$w_2 Q = k p_1 S \left(\frac{\tau_2}{\tau_1}\right)^{\frac{1}{\gamma-1}} \sqrt{\frac{2g\gamma}{\gamma-1} \cdot \frac{\tau_0}{p_0 v_0 \tau_1}\left(\frac{\tau_1-\tau_2}{\tau_1}\right)}, \quad (64)$$

in which

Q = the volume escaping measured outside the reservoir,
w_2 = the weight of unity of volume outside the reservoir,
S = the section of the orifice, and
k = the coefficient of efflux.

Equation (64) is a maximum for

$$\left(\frac{\tau_2}{\tau_1}\right)^{\frac{\gamma}{\gamma-1}} = \frac{p_2}{p_1} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{w_2}{w_1}\right)^{\gamma} = \left(\frac{\delta_2}{\delta_1}\right)^{\gamma}; \quad (65)$$

which for air becomes

$$\frac{\tau_2}{\tau_1} = 0.831, \qquad \frac{p_2}{p_1} = 0.527, \qquad \frac{v_1}{v_2} = 1.577$$

The values of k as found by Professor Weisbach are:

Conoidal mouthpieces, of the form of the contracted vein, with effective pressures
of from 0.23 to 1.1 atmospheres.....0.97 to 00.99
Circular orifices in thin plates.....0.55 to 0.79
Short cylindrical mouthpieces.....0.73 to 0.84
The same rounded at the inner end.....0.92 to 0.93
Conical converging mouthpieces, the angle of convergence being 7° 9′.....0.90 to 0.99

EXERCISES.

1. For a perfect gas, if the temperature of the gas at the outside of the orifice equals that of the reservoir, what will be the velocity of exit?

2. What is the initial velocity with which hydrogen will flow into a vacuum from a vessel in which the temperature is 60° F.?

3. What weight of air will flow from a very large vessel in one second in which the internal pressure is 4 atmospheres and temperature 100° F., the external, one atmosphere and temperature 40° F., flowing through a short cylindrical tube $\frac{1}{4}$ inch in diameter, the coefficient of discharge being 0.8? Consider the vessel so large that the pressure may be considered constant during the discharge.

SUGGESTIONS FOR REVIEW.

What does R represent? In Fig. 20, may A be taken anywhere on the τ isothermal? Draw several verticals between two isothermals differing by unity and show what areas must be equivalent if C_v be constant. In Fig. 22, if the gas be compressed from state B to state A, show what changes take place in the heat. Do the principles applicable to expansion also hold for compression? What is a perfect gas? What is a thermal capacity? Define the two more common specific heats. Can there be more than two specific heats? Illustrate. If a pound of air occupies 10 cubic feet, and another pound 40 cubic feet, both at the same temperature, which will absorb the more heat in having its temperature raised one degree? Describe Mayer's method of determining the mechanical equivalent of heat. What gas has the greatest specific heat? If the mechanical equivalent were the heat necessary to raise the temperature of one pound of air one degree, about what would be its numerical value? Describe methods of finding the value of γ. What is the smallest value of γ given in the text? the largest? the value adopted? How was the last one determined? Will the greatest *volume* of a gas escape from an orifice when the velocity of exit is greatest? Will the greatest *weight* of gas escape when the velocity is greatest? Why not?

CHAPTER III.

IMPERFECT FLUIDS.

68. General discussion. Equations (A) are the general equations for the heat absorbed by an imperfect fluid, and for convenience are brought forward. They are

$$\left.\begin{aligned} dH &= K_v\, d\tau + \tau\left(\frac{dp}{d\tau}\right) dv. \\ dH &= K_p\, d\tau - \tau\left(\frac{dv}{d\tau}\right) dp. \end{aligned}\right\} \qquad (A)$$

In the first of these equations the last term is the entire work, both external and internal, due to an expansion dv, so that if p' be such a pressure that when multiplied by dv would equal the entire work done, we have

$$p' = \tau\left(\frac{dp}{d\tau}\right), \qquad (66)$$

which we call a *virtual pressure*. In Fig. 30, if $v_1\, A = p$ be the external pressure at the volume v and temperature τ, then will some ordinate, as $v_1\, a$, represent p', and hence

$$A\, a = p' - p = \tau\left(\frac{dp}{d\tau}\right) - p$$

will be the *real virtual* pressure, being such an *ideal* pressure as would when multiplied by dv give the *internal* work due to expansion only. If the path $A\,B$ be arbitrary, we have, generally,

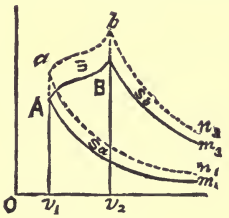

FIG. 30.

$$v_1 A B v_2 = \int p \, dv.$$

$$v_1 a b v_2 = \int \tau \left(\frac{dp}{d\tau}\right) dv + \varphi(\tau).$$

$$A a b B = \int_{v_1}^{v_2} \left[\tau \left(\frac{dp}{d\tau}\right) - p \right] dv + \varphi(\tau), \qquad (67)$$

the indicated integral being the latent heat due to expansion, and $\varphi(\tau)$ a function of the temperature, being the latent heat due to an increase of temperature.

If K_v in the first of equations (A) is variable, then will a part of the heat absorbed do internal work due to a change of temperature. It appears, then, that the *internal* work may be considered in two distinct parts: one due entirely to change of volume, the other entirely to change of temperature.

69. Let the temperature be constant during expansion. This is a case of isothermal expansion, and we have $d\tau = o$ and $\tau = \tau_1$, and the first of equations (A) becomes

$$H = \tau_1 \int \left(\frac{dp}{d\tau}\right)_v dv, \qquad (68)$$

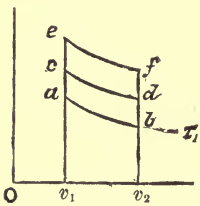

FIG. 31.

in which $\left(\frac{dp}{d\tau}\right)_v$ is to be found from the equation to the gas. In Fig. 31 let $a\,b$ be the path of the fluid, which will be an isothermal of the substance, then will $v_1 a b v_2$ be the external work done during expansion, and let $a\,e\,f\,b$ represent the internal work, then will $v_1 e f v_2$ represent the total work done, and will be *the latent heat of expansion;* and we have

ordinate to $e f = p' = \tau_1 \left(\frac{dp}{d\tau}\right)$,

$$v_1 e f v_2 = \tau_1 \int \left(\frac{dp}{d\tau}\right) dv,$$

$$v_1\, a\, b\, v_2 = \int p\, d\, v_2 \tag{69}$$

$$a\, e\, f\, b = \int \left[\tau_1 \left(\frac{d\, p}{d\, \tau} \right) - p \right] d\, v. \tag{70}$$

EXERCISES.

1. If equation (7) be the equation of an imperfect gas, find the total work done during an isothermal expansion from v_1 to $2\ v_1$. (Use equation (68).)

2. If $a\ b$, Fig. 31, be the isothermal of the gases in equation (7), and $c\ d$ what it would be when α, β and γ are each zero, give the equations of $a\ b$, $c\ d$, $e\ f$, and the values of $v_1\ a$, $a\ c$ and $c\ e$.

3. Given equation (7) to find the external work $v_1\ a\ b\ v_2$, Fig. 31, and the internal, $a\ e\ f\ b$. (Equations (69) and (70)).

4. If the equation of the gas be $p\ v = R\ \tau - \frac{b}{\tau\ v}$, show that $a\ c = c\ e$, Fig. 31, $a\ b$ being the isothermal of the gas, and $c\ d$ what the isothermal becomes when b is zero.

If one pound of carbonic acid gas at 300° F. expand isothermally from 10 cubic feet to 20 cubic feet, find the total work done, also the external and internal work. (Use equations (6), (69) and (70).)

5. What will be the total internal work of expanding two pounds of carbonic acid gas indefinitely at the constant temperature of 200° F., the initial volume being 8 cubic feet? (The limits of integration in equation (70) will be ∞ and 8.)

Ans. 364 *foot-pounds.*

6. The initial volume of a pound of carbonic acid gas being 8 cubic feet, how much must it be compressed at the constant absolute temperature of 600° F., so that the internal work shall equal the external work?

7. If equation (4) be the equation of the gas, in which

$a_0 = \frac{1}{v}$, $a_1 = \frac{2}{v^2}$, $a_2 = \frac{3}{v^3}$, find the equation of an isothermal, the external work done during an isothermal expansion, and the *total* work done.

Ans. for the total work,

$$R\,\tau_1\,log\,\frac{v_2}{v_1} - \left[\frac{1}{\tau_1 v_2^2} - \frac{1}{\tau_1 v_1^2} + \frac{2}{\tau_1^2 v_2^3} - \frac{2}{\tau_1^2 v_1^3}, \text{ \&c.}\right]$$

70. Change of state *in regard to aggregation. Let the temperature and pressure be constant*, required the heat absorbed.

For this case $d\tau = o$, and $\tau\left(\frac{dp}{d\tau}\right)$ will be independent of v, hence

$$H = \tau\left(\frac{dp}{d\tau}\right)\int_{v_1}^{v_2} dv = \tau\left(\frac{dp}{d\tau}\right)(v_2 - v_1). \qquad (71)$$

These conditions are realized during three physical changes—fusion, vaporization and sublimation.

71. Latent heat of fusion, *or of liquefaction.* Substances may be melted—changing from a solid to a liquid state—under the constant pressure of the atmosphere, or other pressure, and at a fixed temperature for that pressure; and during this change of state heat is absorbed which does not affect the thermometer, and hence, according to the definition, is called *latent.* Its value can be found only by direct experiment. Having this value of H for any substance, which, for distinction, call H_f (noticing that f is the initial letter of *fusion*), we may find

$$\frac{d\tau}{dp} = \frac{\tau(v_2 - v_1)}{H_f}, \qquad (72)$$

for which the rate of change of temperature per unit of pressure may be calculated. If the volume v_1 of the substance in the initial, or solid state, exceeds that in the terminal, or liquid state, v_2, then will $\frac{d\tau}{dp}$ be negative, and the

temperature of fusion will be lowered by an increase of pressure, a principle first pointed out by Professor James Thomson (*Edinburgh Trans.*, Vol. XVI.). Water, antimony, cast iron and some other substances, are more bulky in the solid than in the liquid state; and the melting point of all such substances is lowered by pressure.

The latent heat of fusion of ice is 144 B. T. U., as determined by experiment or $144 \times 778 = 112032$ foot-pounds; and this is the work which must be expended upon one pound of ice at 32° F. in reducing it to liquid water at the same temperature, which work is necessary to completely break down the crystalline structure of the ice. Conversely, it is the equivalent of the heat-energy which must be emitted from a pound of water and absorbed by surrounding objects in changing water from the liquid to the solid state at 32° F. Solids, under a definite pressure, have a corresponding definite melting point, or point of fusion.

The following are some examples of the latent heat of fusion:

Substance.	Melting point Deg. Fahr.	Latent heat of fusion.	
		B. T. U.	Foot-pounds *Hf.*
Ice	32	144*	112032
Zinc	793	50.6	39367
Sulphur	224	16.9	13148
Lead	635	9.7	7547
Mercury	−40	5.1	3968
Tin	455	500 as given by Rankine. 26.6 " " " Box on Heat, p. 13†	
Spermaceti	124	148 (Rankine). 46.4 (Box).	
Cast iron	2000	233 as given by Clements.	

* *Phil. Mag.*, 1871, XLI., 182. Peclet found 135, Person, 144.04, and 144 appears to be the most reliable.

† We have not determined which (if either) is correct.

If the specific heat of water were constant, 144 pounds of water at any temperature above 33° F. would have its temperature reduced one degree in just melting one pound of ice at 32°. The mixture *after* melting would reduce the temperature a little more.

The expansive force resulting from congealing water was well illustrated by Major Williams in 1786, at Quebec, Canada, by filling an iron shell with water and driving an iron plug weighing over 2½ pounds into the fuse hole, and subjecting it to an out-door temperature of — 18½° F.; when, upon freezing, the plug was fired out and projected over 400 feet (Trans. Roy. Soc. Edinburgh, II., 23).

EXERCISES

1. If for water we have

$$\tau = 492.66° \text{ F.}$$
$$v_2 = 0.01602 \text{ cu. ft. per pound;}$$

and for ice

$$\tau = 492.66° \text{ F.}$$
$$v_1 = 0.0174 \text{ cu. ft. per pound,}$$

and $H_f = 112032$;

how much will the melting point of ice be lowered by a pressure of one atmosphere, 2116.2 pounds per square foot? (Use equation (72).)

Ans. 0.0128° *F.*
0.0071° *C.*

2. Required the pressure per square foot necessary to lower the melting point of ice 1° F.

We have

$$-\frac{dp}{d\tau} = \frac{H_f}{\tau(v_1 - v_2)} = \frac{112032}{492.66 \times 0.00138} = 164784 \text{ lbs.}$$

In this exercise $d\tau$ is considered unity and $dp = 164784$; or the second member may be considered constant and the left member integrated between limits, giving $\frac{p_2 - p_1}{1} = 164784$. The notation of the preceding exercise may be treated in a similar manner.

3. If 1140 pounds pressure per square inch will lower the melting point of ice from 32° F. to 31° F., diminishing the volume of one pound 0.00138 of a cubic foot; required the latent heat of fusion of one pound of the ice.

Ans. 143.2.

4. Required the external and internal work in melting ice at 32° F. at atmospheric pressure.

The external work will be that done in lowering the atmosphere through a distance equal to the decrease of volume in changing the state of aggregation, or $2116.3 \times 0.00138 = 3$ foot-pounds, nearly. The total work will be by using the result in exercise 2,

$$\tau \frac{dp}{d\tau} dv = 492.66 \times 164784 \times 0.00138 = 112032 \text{ ft.-lbs., nearly,}$$

and 3 pounds more due to atmospheric pressure.

From this it appears that the work is nearly all internal, and is more than 36500 times the external work.

5. The pressure required to reduce the melting point of ice 1° F. being 164784 pounds per square foot when the initial temperature is $\tau = 492.66°$ F.; find the diminution of volume of one pound in changing from congealed to liquid water.

6. Required the pressure necessary to reduce the melting point of ice to — 18° C., assuming that the above formula is valid so far below 0° C.

7. What is the highest temperature at which ice can exist indefinitely in a vacuum?

72. Experimental verification. Sir William Thomson, by a delicate and beautiful experiment, proved that the melting point of ice was lowered by pressure (*Phil. Mag.* (1850), III., XXXVII., 123). A delicate thermometer, constructed for the purpose, was enclosed in a vessel with water and lumps of clear ice and an air gauge for measuring the pressure. At atmospheric pressure the ice would not melt if below 32° F., but it was found that when the con-

tents of the vessel was subjected to pressure the thermometer fell as the water assumed the temperature of the melting point of ice corresponding to the increased pressure; and the observed results corresponded well with those calculated.

Professor Mousson made the following experiment: A prism of steel, Fig. 32, was used, having a cylindrical bore 0.71 *cent.* (0.28 inch), closed at the lower end by a copper cone forced in by a strong screw, and the upper end by a long slightly conical copper plug *a* pressed down by a steel piston by means of a strong screw, and when in an inverted position a small brass rod *b* was dropped in and the bore filled with water. After being exposed to cold at — 9.5° C. the protruding ice was removed, the copper cone inserted and screwed up, and the whole reversed and put into a freezing mixture at — 18° C., after which the upper plug was forced in at a pressure roughly estimated at not less than 13250 atmospheres. When the lower plug was removed the brass rod dropped out first, showing that the ice had been melted, permitting the rod to fall to the lower end. The pressure was more than five times that required by theory to melt the ice, but the temperature at which it melted is unknown.

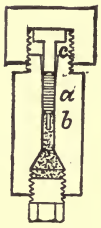

FIG. 32.

73. It is natural to infer the *opposite principle*—that the melting point of those substances which expand when fused will be raised by compression, and this principle has been verified by Mr. Hopkins (*Rep. B. A.* (1854), II., 56), as well as by others. In Mr. Hopkins's experiments the instant of fusion was determined by means of a small iron ball supported by the substance when solid, but which fell when the substance liquefied; and when supported it deflected a needle which was suspended just outside the vessel, but the deflection ceased when the ball fell. The temperature was determined by that of the oil in a bath in which the whole was im-

mersed; and the effective pressure was taken as the half sum of the pressures which forced the piston inward and that required to just permit it to return to its initial position, thus elminating the effect of friction.

The following results were found :—

Pressure Atmospheres.	Melting points in degrees centigrade.			
	Spermaceti.	Wax.	Sterrane.	Sulphur.
1	51.1	64.7	67.2	107.2
538.4	60.0	74.7	68.3	135.3
820.5	80.3	80.3	73.9	140.6

The melting point of wax will be given nearly by the empirical formula

$$t_c = 64.68 + 0.0188\,p.$$

M. Person gives the following empirical formula as the results of his experiments on the latent heat of fusion of non-metallic substances (*Ann. de Chem. et de Phys.*, Nov., 1849).

$$l = (c' - c)\,(\mathrm{T} + 256°) \qquad (73)$$

in which

c = the specific heat of the substance in the solid state,
c' = " " " " " " liquid state,
T = its temperature of fusion in degrees Fahr., and
l = the latent heat of fusion of one pound in B. T. U. at the pressure of the atmosphere.

EXERCISES.

1. If the specific heat of water be unity, of ice 0.504, and the temperature of fusion, T, be 32° F., required the latent heat of fusion.

2. If ice be subjected to such a pressure that it will melt

at 0° F., find its latent heat of fusion, the law being assumed to hold good to that point; and the amount less than at 32° F.

3. What must be the temperature of ice that there will be no latent heat of fusion, and to what pressure must it be subjected?

Ans. $T = -256°$ F.

$$Pressure = \frac{256 + 32}{0.0128} + 1 = 22500 \text{ atmospheres.}$$

If this could be realized there would be no mechanical distinction below this temperature between solid and liquid water.

4. Assuming that experimental results may be used to the extent implied in the questions here given, find the increase of volume due to the fusion of spermaceti at 124° F.

For the ratio of dp to $d\tau$ considered as finite, see the preceding table; then we have

$$v_2 - v_1 = \frac{H_t}{\tau \frac{dp}{d\tau}} = \frac{148 \times 778}{584 \times \frac{537.4 \times 2116}{140 - 124}}.$$

5. The latent heat of fusion of solid water at 32° F. being 144 B. T. U. and its specific heat in the liquid state being unity, find its specific heat in the solid state. (Equation (73).)

6. The specific heat of ice being 0.504 at 32° F., and the latent heat of fusion 144 B. T. U., find the specific heat of water.

7. The specific heat of solid spermaceti being 0.32 and considering its latent heat of fusion as 46.4 B. T. U. at 120° F., find its specific heat in the liquid state. (Equation (73).)

74. Latent heat of evaporation. It is found by experiment that there is a definite boiling point for liquids corresponding to the pressure to which they are subjected, and from this condition they will pass into a vapor at that temperature and pressure. Hence, equation (71) is directly applicable to this case, and indicating this particular latent

heat by H_e (e being the initial letter of *evaporation*), we have

$$H_e = \tau \left(\frac{d\,p}{d\,\tau}\right)(v_2 - v_1), \tag{74}$$

and $$J\,h_e = H_e.$$

By determining the factors of equation (74) by experiment, the value of H_e may be computed. M. Regnault determined the value of h_e directly for water at a series of boiling points from its freezing point to about 375° F., which may be represented with great precision by the empirical formula, in English units,

$$H_e = [1091.7 - 0.695\,(T - 32) - 0.000000103\,(T - 39.1)^3] \times 778, \tag{75}$$

or in French units,

$$H_e = [606.5 - 0.695\,T - 0.00000033\,(T - 4)^3]\,426.8. \tag{76}$$

(*Mémoire Académie des Sciences*, 1847. *Trans. Roy. Soc., Edinburgh*, Vol. XX.)

In practice it will be sufficiently exact to use the following:—

$$\begin{aligned} h_e &= 966 - 0.7\,(T - 212) \\ &= 1092 - 0.7\,(T - 32) \\ &= 1114.4 - 0.7\,T \\ &= 1436.8 - 0.7\,\tau, \text{ B. T. U.,} \end{aligned} \tag{77}$$

or its equivalent,

$$\begin{aligned} H_e &= 751548 - 544.6\,(T - 212) \\ &= 867003 - 544.6\,T \\ &= 1117880 - 544.6\,\tau, \text{ foot-pounds,} \\ &= a - b\,\tau. \end{aligned} \tag{78}$$

The latent heat of evaporation of some other substances is given in the Addenda.

The exact value of the latent heat of evaporation of one pound of water at the pressure of one atmosphere, as found by Regnault, is 966.1 B. T. U. = 751624 foot-pounds. This is the *work* necessary to simply change water from its liquid state when at 212° F. under the pressure of one at-

mosphere to the condition of vapor at the same temperature and pressure. Since water in the form of steam occupies more space than as a liquid, the molecules must be farther apart in the former than in the latter state, and hence, without considering their *exact* condition in the two cases, it appears that it requires 751624 foot-pounds of energy to simply separate the molecules of a pound of water sufficiently to produce steam. This amount of heat is absorbed and *disappears* without affecting the temperature; and the same amount reappears, or passes to other bodies, when it returns to a liquid at that pressure.

Latent heat of evaporation of one pound of certain substances at the pressure of one atmosphere.

Substance.	Boiling point. Deg. F.	Latent heat.	
		B. T. U.	Foot-pounds.
Water	212	966.1	751624
Alcohol	172.2	364.3	283425
Ether	95.0	162.8	125658
Bisulphide of carbon	114.8	156.0	121368
Oil of turpentine	316	184.0	143152
Naphtha	141	236.0	183608

75. Vapor is any substance in a gaseous condition at the *maximum density for that temperature or pressure.* If a vapor be in contact with the liquid, as steam in the same vessel as water, there is a certain pressure which is at once the least pressure under which the substance can exist in the liquid state and the greatest pressure at which it can exist in the gaseous state at that temperature. Under such conditions the vapor is called *saturated vapor* (or *saturated steam*), and the pressure, the *pressure of saturation.* If at the temperature of saturation the pressure be slightly increased some

of the vapor will be condensed until equilibrium be restored, and, conversely, if the pressure be diminished more vapor will be generated. To illustrate, if a cylinder containing a piston were placed directly over a steam boiler having one end open to the boiler, and the piston be forced inward, a portion of the steam would be condensed, and if the piston be drawn outward more steam would be generated—the temperature and pressure remaining constant while the volume varied. This is known as Dalton's law (Stewart on *Heat*, p. 143).

76. The relation between pressure and temperature of a vapor can be determined only by experiment, as has been done by Regnault (*Mémoire de l'Académie des Sciences*, 1847; *Comptes Rendus*, 1854). His results are represented quite accurately by the following empirical formula, given by Rankine, and first published in the *Edinburgh New Philosophical Journal* for July, 1849 (*Phil. Mag.*, Dec., 1854).

$$com.\ log\ p = A - \frac{B}{\tau} - \frac{C}{\tau^2} \tag{80}$$

where A, B, C, are constants to be determined by experiment. That author remarks that this formula is sufficiently accurate for temperatures from —22° F. to 446° F. From (80) we find

$$\tau = \frac{1}{\sqrt{\frac{A - log\ p}{C} + \frac{B^2}{4\,C^2}} - \frac{B}{2\,C}}, \tag{81}$$

$$\frac{dx}{d\tau} = p\left(\frac{B}{\tau^2} + \frac{2\,C}{\tau^3}\right) \times 2.3026, \tag{82}$$

$$\tau\,\frac{dp}{d\tau} = p\left(\frac{B}{\tau} + \frac{2\,C}{\tau^2}\right) \times 2.3026. \tag{83}$$

The following are the values of the constants for the

vapor of water (*saturated* steam) for degrees Fahrenheit and pressures in *pounds per square foot:*

$$A = 8.28203,\ log\ B = 3.441474,\ log\ C = 5.583973.$$

These values differ slightly from those given by Rankine, because he used $\tau_0 = 493.2°$ F., while here $\tau_0 = 492.66°$ F. Regnault's experiments also furnish the data for determining the constants for Alcohol, Ether, Bisulphide of Carbon and Mercury.

77. Volume of vapor. From equations (74) and (83) we have

$$v_2 - v_1 = \frac{H_e}{\tau \dfrac{dp}{d\tau}} \tag{84}$$

$$= \frac{H_e}{p\left(\dfrac{B}{\tau} + \dfrac{2C}{\tau^2}\right) \times 2.3026},$$

which for saturated steam becomes, equation (78),

$$v_2 - v_1 = \frac{1117880 - 544.6\,\tau}{p\left(\dfrac{B}{\tau} + \dfrac{2C}{\tau^2}\right) \times 2.3026}, \tag{85}$$

from which v_2, the volume of a pound of the steam, may be determined. It will be found hereafter that the volume of a pound of saturated steam at 212°, the pressure of one atmosphere being 29.9218 inches of mercury, is 26.50 cubic feet, nearly, and the volume of one pound of water at its maximum density, 39.1° F., is 0.01602 cubic feet; hence, a pound of saturated steam under the pressure of one atmosphere occupies 1654 times the volume of one pound of water at its maximum density. This increase is illustrated by Fig. 33, in which the small square at the upper left-hand corner represents the volume of one pound of water, $v_1 = 0.01602$, and the entire large square the volume occu-

pied by the steam produced from it at 212° F., $v_2 = 26.58$, and the large square *minus* the small one represents the increase of volume in changing the water to vapor, $v_2 - v_1$. If the boiling point increases in temperature, the *volume* of steam decreases; still within the range of ordinary practice the volume of water is so small compared with that of the steam generated from it, the former may be neglected, and we have with sufficient accuracy

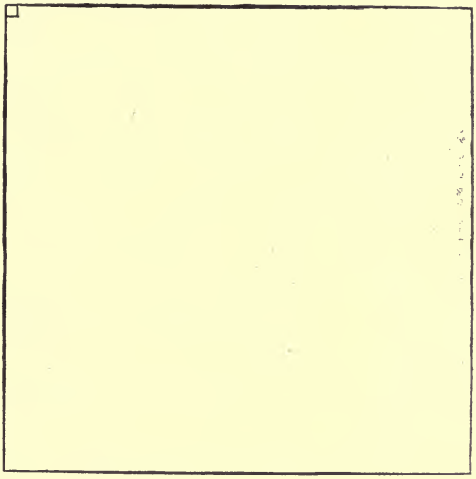

FIG. 33.

$$v_2 = \frac{H_e}{\tau \frac{dp}{d\tau}}. \tag{86}$$

78. Weight of vapor *per cubic foot*, sometimes called the *density of the vapor*. Since a pound of the vapor occupies v_2 cubic feet, the *weight* of a cubic foot of the vapor will be

$$w = \frac{1}{v_2}.$$

If L be the latent heat of evaporation per cubic foot, then

$$L = \frac{H_e}{v_2} = \tau \frac{dp}{d\tau};$$

$$\therefore w = \frac{L}{H_e}, \tag{87}$$

which is the reciprocal of equation (84).

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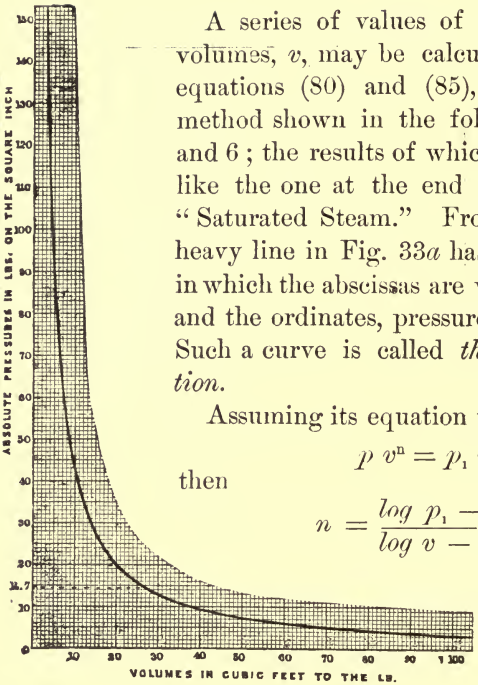

FIG. 33*a*.

A series of values of pressures, p, and volumes, v, may be calculated by means of equations (80) and (85), according to the method shown in the following exercises 5 and 6 ; the results of which may form a table like the one at the end of this volume for "Saturated Steam." From these results the heavy line in Fig. 33*a* has been constructed, in which the abscissas are volumes per pound, and the ordinates, pressures per square inch. Such a curve is called *the curve of saturation.*

Assuming its equation to be

$$p\,v^n = p_1\,v_1^{\,n};$$

then

$$n = \frac{log\ p_1 - log\ p}{log\ v - log\ v_1}.$$

If $p_1 = 14.7$, $v_1 = 26.59$,
and $p = 60$, $v = 7.100$, then $n = 1.06470$;
or, $p = 100$, $v = 4.403$, $n = 1.065837$;
or, $p = 160$, $v = 2.830$, $n = 1.065954$;
or, $p = 200$, $v = 2.294$, $n = 1.06551$.
Mean $n = 1.06550$;

$$\therefore p\,v^{1.06550} = constant, \qquad (88)$$

or,
$$p\,v^{1\frac{1}{16}} = 484, \qquad (89)$$

p being pounds per square inch and v cubic feet per pound.

EXERCISES.

1. Required the latent heat of evaporation of water at 20° F., 60° F., 200° F., 400° F., and explain why it is less

at the higher temperatures. (Equation (78), or tables of *saturated* steam.)

2. If Regnault's law can be trusted so far, find the temperature at which the latent heat of evaporation will be zero for steam.

Ans. $\tau = 2052°$ F. or $T = 1592°$ F.,

or about the temperature of the melting point of brass, above which there would be no difference between the liquid and vaporous forms. This is called the *critical temperature*, and has been determined experimentally for some substances (*Phil. Trans.* (1869), CLIX., 575).

3. If the latent heat of evaporation of one pound of water at the melting point of ice could be utilized in projecting that pound vertically upward, how many miles high would it ascend in a vacuum, considering gravity constant, and $g = 32.2$?

4. Through what height must a 100-pound ball descend in a vacuum so that its energy if entirely utilized for the purpose would just evaporate one pound of water at and from 212° F.

5. Find the value of $\frac{dp}{d\tau}$ for saturated steam at 212° F.

If we resort to tables of saturated steam, we find that Rankine's Table VI. (*Steam Engine*) is not suitable for this purpose, because the temperatures are for differences of 9° F. Several other tables give the pressures for every degree. From one of these we find that at

211° F., the pressure is 14.406 lbs. per sq. in.
212° " " " " 14.696 " " " "
213° " " " " 14.991 " " " "

Hence, from 211° to 212° we have

$$\frac{dp}{d\tau} = 0.290 \times 144 = 41.76, \text{ approximately,}$$

from 212° to 213°, we have

$$\frac{dp}{d\tau} = 0.295 \times 144 = 42.48, \text{ approximately,}$$

the mean of which gives

$$\frac{d\,p}{d\,\tau} = 42.12,$$

which is nearer correct, but cannot be exact. To find the correct value directly, use equation (82), and we have

$$\begin{aligned}\frac{d\,p}{d\,\tau} &= \frac{2116.2}{672.66}\left[\frac{B}{672.66} + \frac{2\,C}{(672.66)^2}\right] \times 2.3026 \\ &= 2116.2 \times 2.3026 \times \frac{5.80530}{672.66} \\ &= 42.06,\end{aligned}$$

which is the value required.

6. Find the volume of a pound of saturated steam at 212° F., and the weight of a cubic foot of it.

From equation (84) we have

$$v_2 = v_1 + \frac{H_e}{\tau \dfrac{d\,p}{d\,\tau}}$$

Water increases in volume 0.04775 per unit from that at its maximum density to 212° F.; hence,

$$v_1 = 0.01602 \times 1.04775 = 0.01678 = 0.017$$

with sufficient accuracy.

Then

$$v_2 = 0.017 + \frac{966 \times 778}{672.66 \times 42.06} = 26.585 \text{ cu. ft.},$$

and

$$w = \frac{1}{v_2} = 0.03762 \text{ lbs. per cu. ft.}$$

(Rankine gives the following empirical formula for the volume of liquid water at any absolute temperature.

$$v_1 = 0.0080\left(\frac{\tau}{500} + \frac{500}{\tau}\right).)$$

If steam followed the law of perfect gases, we could now

find the volume of a pound of it at any temperature. For we would have

$$R = \frac{p\,v}{\tau} = \frac{2116.2 \times 26.5}{672.66} = 83.37\,;$$

$$\therefore\ v = 83.37\,\frac{\tau}{p}. \qquad (89a)$$

7. Find the volume of one pound of saturated steam at 1200° F.

8. Find the volume of saturated steam at 212° F. generated from one cubic foot of water.

If $v_2 = 26.6$, as in Exercise 6, then will the required volume be 1659 cubic feet. Heretofore writers have used Rankine's results, giving 1644 or 1645. Observation has not fixed the *exact* value.

9. Find the external and internal work in changing one pound of water into steam at and from 212° F., and their ratio.

The volume of one pound of steam will be $v_2 = 26.585$,
" " " " " " water at 212° F., $v_1 = \quad 017$,

$$v_2 - v_1 = 26.568.$$

The entire work will be

$$\tau\,\frac{d\,p}{d\,\tau}\,(v_2 - v_1) = 672.66 \times 42.06 \times 26.568 = 751672.$$
$$966.1 \times 778 = 751626.$$

These results would be identical if all the quantities were determined with precision.

The external work consists in forcing the pressure of one atmosphere through 26.568 feet, or

$$2116.2 \times 26.568 = 56219 \text{ foot-pounds},$$

which deducted from the former, gives

$$751672 - 56219 = 695453 \text{ foot-pounds}$$

for the *internal* work ; hence, the *internal* work will be

$$\frac{695453}{56219} = 12.37$$

times the *external* for the conditions given in the problem.

78*a*. **Isothermal of a liquid, its vapor and its gas.**

To illustrate, conceive that one pound of the liquid is subjected to an immense pressure at some moderate temperature, T,—say 212° F. Let A, Fig. d, represent this state, the ordinate to which will represent the pressure, and the abscissa the volume. Let the pressure be gradually removed while the temperature is maintained constant—the volume will increase slightly and the path of the fluid may be represented by $A\ B$, the abscissa of B exceeding that of A. Let B be the state at which the liquid will boil at the temperature τ; then at that pressure the volume will increase, the temperature remaining constant, and $B\ C$ will represent the path. At C let the pound be entirely vaporized and the pressure gradually removed, while the temperature, τ, is maintained constant; then will the path be the isothermal $C\ \tau$. The entire broken line $A\ B\ C\ \tau$ is an isothermal of the fluid. Let the operation be repeated at a higher temperature—the boiling point will be reached at a state above and a little to the right of B, so that a curve $F\ E$ passed through such points may be called *the locus of boiling points.* Continuing the operation as before, and the state at which the pound will be evaporated will be represented by a point above and to the left of C, and the curve traced through all such points will be *the curve of saturation* $E\ C\ G$, already described. These curves will meet at some point E, and the temperature of that state is called *the critical temperature.*

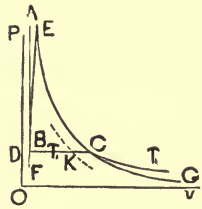

FIG. *d.*

79. An experimental determination of the *density of saturated steam* was first made by Fairbairn and Tate in 1860 (*Phil. Trans.* (London), CL., (1860), 185; CLII., (1862), 591). The densities as thus found differed from those previously found by Rankine from $\frac{1}{126}$ to $\frac{1}{13}$ of the experimental values, thereby giving larger values above 242°

F., and below some were larger and others smaller than the experimental ones (*Miscellaneous Scientific Papers*, p. 423). In these experiments, the steam was in a statical condition, while in Regnault's experiments the steam was in rapid motion from the boiler to the condenser—differences of condition which would naturally affect the results. The proper value of J would also affect the result; but the results obtained by the two methods agree as nearly as one might expect under the circumstances. Rankine's Tables are in his "Prime Movers," and those of Fairbairn and Tate, above the pressure of one atmosphere, are in Richard's *Steam Indicator*, Weisbach on the *Steam Engine*, and other works.

The apparatus employed by Fairbairn and Tate for determining the temperature of saturation consisted of two glass globes connected by a bent tube below them. The tube was filled with mercury, above which in the globes was the liquid, one containing more than the other, then the globes and tube were placed inside of a small boiler containing the same liquid, and the whole heated. So long as the steam is saturated the mercury in the tube will remain stationary, but the instant that the smaller volume of water is all changed to vapor (some water still remaining in the other), the mercury will rise in that end of the tube *nearest the globe in which all the water has been evaporated*, as after that, that steam becomes superheated, and the *rate* of increase of pressure is *greater* for saturated than for superheated vapor. At the instant of change the volume of the steam will be the volume of the space above the mercury, and the temperature and pressure of the steam in the globes will be the same as that in the boiler, and hence may be readily measured.

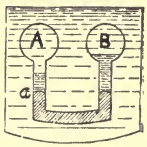

FIG. 33*a*.

80. Measurement of heights. The principles

* *Théorie Mécanique de la Chaleur*, 3me *éd.* (1875), II.

here discussed furnish the means of determining altitudes. Thus, if at any point on a hill or mountain the temperature of boiling water be observed, the pressure of the atmosphere at that place and time may be computed by means of equation (80), then will the height above the level of the sea be

$$h = 60346 \log \frac{p_0}{p} \text{ feet}, \qquad (90)$$

in which p_0 is the pressure of the atmosphere at sea-level, 2116.2 pounds per square foot. If p_1 be the pressure at another station, the difference in elevation will be

$$h' = 60346 \log_{10} \frac{p_1}{p}.$$

Errata

(See the author's *Elementary Mechanics*, p. 328.)

81. Sublimation consists of a change from the solid to the vaporous state without passing through the liquid state. Experimental data in regard to this change are wanting, so that we are unable to make use of any analysis representing this change.

82. Evaporation without ebullition. Experiment indicates that evaporation takes place at all temperatures for a great variety of substances, if not for all. Snow and ice evaporate at temperatures below freezing, and many solids emit sufficient vapor to be detected by the sense of smell. The evaporation of water in the atmosphere is the most important part of this subject. Water is elevated in the form of mist in the atmosphere, forming clouds, by which means water is redistributed over the earth. Evaporation takes place *at* the surface of bodies of water and is unaffected by the conditions below, except so far as they modify the surface. It is generally greatest at higher temperatures, although other conditions modify this law. It is modified by the hygrometrical conditions of the atmosphere, and is greater during a wind than during a calm. It is greater during summer than winter, and generally greater during June and

July than for any other months of the year, but it is sometimes, though rarely, greater during August than for any other month. It is greatly affected by locality, there being sometimes a difference of several inches within a few miles. It has been observed that one half an inch in depth has been evaporated from a large pond in twenty-four hours in latitude about 42° north, but this is an extreme case. The amount is also very different for different years, the maximum exceeding the minimum by more than fifty per cent. Twenty to thirty-five inches of evaporation per year is very common, in which cases the amount

for June would range from about 2.8 inches to 5.4 inches,
" July " " " " 3.0 " " 5.8 "
" August " " " " 2.3 " " 5.3 "

It is recorded that in July, 1875, near Boston, the evaporation was 7.21 inches. (Much valuable information on this subject was collected and published for the use of the court in The Case in the Supreme Court of the State of New York—General Term—Fifth Department—for an Application by the City of Rochester to acquire certain rights to draw water from Hemlock Lake, Vol. II., about 1884. Also certain *Transactions* of the American Society of Civil Engineers, New York, particularly a paper by Mr. D. Fitzgerald, *Transactions*, Vol. XV., 581, (1886).)

83. Assume the pressure and volume both constant during the absorption of heat. These conditions make $dp = o$, and $dv = o$, in equations (A); hence,

$$dH = K_v\, d\tau = K_p\, d\tau;$$
$$\therefore K_v = K_p.$$

Strictly speaking, these conditions are realized for only a few exceptional cases, as water at its maximum density. Generally, the volume changes during the absorption of heat under constant pressure, but for solids and liquids, the change of volume under constant pressure, and of pressure at constant volume is so small they may be considered as

constant, and more especially so when it is considered that the work due to these changes for these conditions is usually very small compared with the energy expended in making the substance hot. If C be the mean specific heat of a solid or a liquid for a large range of temperatures, we have practically,

$$K_v = K_p = C.$$

Tables of specific heats of solids and liquids give values for C only.

Specific heat of a few solids and liquids.

Substance.	Specific heat C.	Authority.
Wrought iron.....	0.11379	Regnault.
Cast iron.....	0.12983	"
Copper.....	0.09511	"
Ice.....	0.504	Person.
Spermaceti.....	0.320	Irvine.
Alcohol.....	0.622	Dalton.
Sulphur.....	0.20259	Regnault.

84. Mechanical mixtures may produce a change of the state of aggregation, as when warm water melts ice. The general equations of thermodynamics are discontinuous for such cases; but having determined expressions for the heat absorbed by fusion and evaporation, we may write an expression for the heat absorbed in passing from the solid to the gaseous state. Thus, let c_1 be the specific heat of the substance in the solid state, c_2 of the liquid, c_3 of the vapor, w the weight of the substance, T_0 the temperature of the melting point, T_1 of the boiling point, T' the initial and T'' the final temperatures; then will the heat absorbed in passing from a temperature T' below freezing to T'' above boiling be

$$h = w[c_1(T_0 - T') + h_f + c_2(T_1 - T_0) + h_e + c_3(T'' - T_1)], \quad (91)$$

which for one pound of water at the pressure of one atmosphere, becomes

$$h = 0.504(32 - T') + 144 + 180 + 966 + 0.48(T'' - 212°)$$
$$= 1204.4 - 0.504\,T' + 0.48\,T'' \quad (92)$$
$$= 1200 + 0.5\,(T'' - T'), \text{ nearly.}$$

EXERCISES.

1. Required the temperature after mixing 3 pounds of water at 90° F., 10 pounds of alcohol at 30° F., and 20 pounds of mercury at 60° F.

2. Required the temperature of a mixture of 3 pounds of ice at 10° F. with 12 pounds of water at 60 F., after the ice melts, there being no loss of heat.

3. Required the resultant temperature of a mixture of 6 pounds of ice at 20° F. and one pound of steam at 225° F.

Here

$$0.504 \times 6 \times 12 + 864 + 6\,(t - 32) = 0.48 \times 13 + 966 + (212 - t).$$

4. Desiring to determine the approximate temperature of the gases at the base of a chimney, a mass of iron weighing 8 pounds was placed in them, and after remaining a considerable time was removed and submerged in 100 pounds of water at 50° F., when it was found that the temperature of the water was raised to 55° F.; required the temperature of the gases.

We have, nearly,

$$\tfrac{1}{9} \times 8\,(t - 55°) = 1 \times 100 \times 5;$$
$$\therefore\ t = 617° \text{ F., nearly.}$$

5. How many pounds of water at 200° F. will be necessary to reduce one pound of steam at 212° F. to water, and leave the final mixture of water 212° F.

6. Required the temperature of a mixture of one pound of ice at 32° F., one pound of water at 32° F., and one pound of steam at 212° F.

Proceeding in the ordinary way, we have

$$144 + 2\,(t - 32) = 966 + (212 - t);$$
$$\therefore\ t = 366° \text{ F.,}$$

an absurd result, since the mixture would have a higher temperature than that of the hottest substance.

The proper explanation is—only a part of the steam was condensed producing water at 212° F. By a sufficient pressure, however, it may all be reduced to water.

7. In the preceding exercise how much steam was condensed forming water at 212° F. and how much steam remained uncondensed?

8. Required the resultant temperature of a mixture of 5 lbs. ice at 28° F., 20 lbs. H_2O at 50° F. and 1 lb. steam at 212° F.

9. A mixture of i pounds of ice at $32° - t_1$, x pounds of water at 60° F. and s pounds of steam at $t°$ F., produces a temperature of $T°$ F.; required x, and discuss the result.

We have

$$0.504\, i\, t_1 + 144\, i + i\,(T - 32) = I\,(\text{say}),$$
$$0.48\, s\,(t_2 - 212°) + 966\, s + s(212 - T) = S,$$
$$x\,(T - 60) = W;$$

$$\therefore x = \frac{S - I}{T - 60}.$$

Now assume that

$S = I$; $S > I$ and $T > 60$;
$S > I$, $T = 60$; $S > I$, $T < 60°$ (impossible), $S < I$, $T < 60°$, &c.

10. How many pounds of water at 212° will be necessary by mixing with 5 pounds of alcohol at 40° F. to just make the latter boil under a pressure of one atmosphere?

85. Total heat of evaporation is a *conventional* term to indicate the heat absorbed in raising a liquid from a fixed temperature in the liquid state to the boiling point and evaporating it at the latter temperature. It is the sum of the sensible and latent heats above the *fixed* temperature. It is also called *the total heat of the vapor*, and in reference to water *the total heat of steam*. To find it, we use the first of equations (A), or

$$d\,H = K_v\, d\,\tau + \tau \left(\frac{d\,p}{d\,\tau}\right) d\,v,$$

in which the first term of the second member is to be integrated from τ_2 to τ_1, while the second term of that mem-

ber is zero during that operation, and then the value of the last term is found while τ_1 remains constant, and hence is H_e, *the latent heat of evaporation.* The total value of H will be the sum of these values; hence, making $K_v = C$, the dynamic specific heat, we have

$$H = C(\tau_1 - \tau_2) + H_{e1}. \quad (93)$$

The lower *fixed* temperature is that of melting ice, unless otherwise specified; hence, in English units

$$\tau_2 = \tau_0$$
$$\tau_1 = \tau_0 + T - 32^\circ,$$

where T is the temperature on Fahrenheit's scale, and

$$H = C(T - 32) + H_e, \quad (94)$$

and for water $C = 778$, and substituting the value of H_e, (page 95), we have

$$\begin{aligned} H &= 778\,[1091.7 + 0.305\,(T - 32)] \\ &= 849342 + 237\,(T - 32) \\ &= 841758 + 237\,T. \end{aligned} \quad (95)$$

By means of this formula a table may be computed that will give the "total heat of steam" above the melting point of ice.

86. Evaporative power.—If the temperature at which water is fed to a boiler be T_2° F., the foot-pounds of heat which must be supplied in order to evaporate it will be

$$H = 778\,[1091.7 + 0.305\,(T - 32) - (T_2 - 32)]. \quad (96)$$

In determining the efficiency of a boiler, or the amount of water evaporated by a pound of fuel, it is customary to reduce the amount of evaporation which actually takes place *from* the temperature of the feed water *at* the temperature of the steam to an equivalent amount *at* and *from* 212° F. (100° C.). In the latter case 966 heat units are absorbed, and making this the *unit* of evaporative power, the evaporative power in any other case will be, nearly,

$$\frac{1092 + .3\,(T - 32) - (T_2 - 32)}{966}$$

$$= 1 + \frac{0.3\,(T - 212) + (212 - T_2)}{966},$$

a form due to Rankine, who properly called the expression a *factor of evaporation*. By assuming a series of values for T and T_2, a table may be formed of the factors by means of which the given conditions may readily be reduced to that of the above *unit*, and the actual evaporative power will be, in foot-pounds,

$$778 \times 966 \times \text{tabular number}.$$

The preceding expression reduces to

$$1 + \frac{148.4 + 0.3\ T - T_2}{966}, \tag{97}$$

by means of which the following table has been computed.

FACTORS OF EVAPORATION.

Boiling point. $T°$ F.	Initial temperature of the feed water, T_2 degrees F.									
	40	60	80	100	120	140	160	180	200	212
212	1.18	1.15	1.13	1.11	1.09	1.07	1.05	1.03	1.01	1.00
230	1.18	1.16	1.14	1.12	1.10	1.08	1.06	1.04	1.02	1.01
250	1.19	1.17	1.15	1.12	1.10	1.08	1.06	1.04	1.02	1.01
270	1.19	1.18	1.15	1.12	1.10	1.08	1.06	1.04	1.02	1.02
290	1.19	1.19	1.16	1.14	1.12	1.10	1.07	1.04	1.03	1.02
310	1.20	1.20	1.16	1.14	1.12	1.10	1.08	1.05	1.04	1.03
330	1.21	1.21	1.17	1.15	1.13	1.11	1.09	1.07	1.05	1.03
350	1.22	1.21	1.19	1.17	1.15	1.12	1.10	1.08	1.06	1.04
370	1.23	1.21	1.19	1.17	1.15	1.12	1.10	1.08	1.06	1.04
390	1.24	1.22	1.19	1.17	1.15	1.13	1.11	1.09	1.07	1.05
410	1.24	1.22	1.20	1.18	1.16	1.14	1.12	1.10	1.08	1.06
430	1.24	1.22	1.21	1.19	1.17	1.15	1.13	1.11	1.09	1.07

87. Superheated steam.—When the temperature of a vapor for a given pressure is higher than the boiling point for that pressure, the vapor is said to be *superheated*, and is sometimes called "steam gas." Vapor may be superheated by separating it from its liquid and subjecting it to a still higher temperature. Let the vapor be generated at τ_1 degrees and afterward heated to τ_2 degrees, then will the heat absorbed above τ_1 degrees be,

$$H = H_e + \int_{\tau_1}^{\tau_2} K_p \, d\tau = H_e + K_p (\tau_2 - \tau_1). \quad (98)$$

If the vapor be steam and H_{e_1} be generated at T_1 degrees F., we have (Eq. [10], p. 409),

$$\begin{aligned} H &= 778 (1123.7 - 0.695 \, T_1 + 0.48 [T_2 - T_1]) \\ &= 874239 + 373 \, T_2 - 914 \, T_1. \end{aligned} \quad (99)$$

See Errata.

EXERCISES.

1. If one pound of coal will evaporate 10 pounds of water *at* and *from* 212°, how many pounds would it evaporate *from* 80° F. *at* 310° F.?

2. If, when the feed water is at 32° F. and the boiling point at 410° F., one pound of coal evaporates 7 pounds of water, how much ought it to evaporate *at* and *from* 212° F.?

3. Experiment proves that one pound of good coal, completely burned, will develop 14500 heat units (B. T. U.), how many pounds of water could one pound of such coal evaporate *at* and *from* 212° F. if all its heat of combustion were utilized for that purpose, under the pressure of one atmosphere?

Ans. 15.0 lbs.

4. Under what physical conditions could one pound of

such coal as mentioned in Exercise 3 evaporate 20 pounds of water?

5. If the feed water be 32° F., what must be the temperature and pressure that the "factor of evaporation" shall be 2.0?

6. Find the B. T. U. required to produce one pound of saturated steam at 212° from water at 32°; and steam gas at the same temperature from the same water, and compare the results.

(Those who have not time to pursue the more abstruse part of the subject may omit to Art. 98, p. 143, except Art. 95, p. 126.)

88. Free expansion of gases. When the external pressure is much less than the expansive force of the gas during expansion, the expansion is said to be *free*. This principle has been used for determining the difference between the absolute zero of the perfect scale and that of the air thermometer. When a gas rushes freely from a vessel under pressure into another of lower pressure, the only work done will be that of the friction through the passage and among its own particles, which process will generate heat; but during the expansion in the second vessel the gas will be cooled, and if there were no transmission of heat to or from external bodies and the gas were *perfect* the final temperature should be the same as the initial.

Joule made an experiment upon air, by immersing in a vessel of water two other vessels connected by a pipe, one of which was filled with air at 22 atmospheres and the other exhausted of air; after which, by opening a stopcock in the connecting pipe, the air rushed from one vessel to the other, but no apparent change of temperature was observed (*Phil. Mag.* (3), (1845), XXVI., 376). M. Hirn, in 1865, made a more delicate experiment for the same purpose, but without detecting any change in temperature (*Théorie Mécanique de la Chaleur*, 3ème, I., 298).

Sir William Thomson in 1851 executed a much more

delicate experiment. A porous plug, composed of a bunch of fine silk—or, in some cases, of cotton—was inserted in a long tube, and the difference of pressures on either side was regulated by the amount of silk or cotton in the plug. The air was forced through a box with a perforated cap stuffed with cotton-wool, so as to prevent fluctuations, and this pump was worked some time, so as to secure steady action before records were made. The pressure and temperature were kept nearly constant during the experiment. The part of the tube beyond the plug was immersed in a vessel of water, observations upon which determined the amount of cooling (Thomson's *Mathematical Papers*, pp. 333–455; *Phil. Mag.* (1852), (4), IV., 481). Since heat will generally be abstracted, H will be negative, and the second of equations (A) becomes

$$-dH = K_p\, d\tau - \tau \frac{dv}{d\tau} dp.$$

Adding and subtracting $v\,dp$, we have

$$-dH = K_p\, d\tau - \left(\tau \frac{dv}{d\tau} - v\right) dp - v\,dp;$$

$$\therefore H = K_p(\tau_2 - \tau_1) + \int \left(\tau \frac{dv}{d\tau} - v\right) dp + \int v\,dp. \quad (100)$$

considering K_p as constant.

But integrating $v\,dp$ by parts, between the limits of $p_2\,v_2$ and $v_1\,p_1$, gives

$$\int_{p_2}^{p_1} v\,dp = v_1 p_1 - v_2 p_2 + \int_{v_1}^{v_2} p\,dv.$$

The last term would be negative if the order of the limits were reversed. But the work done upon the gas in forcing it through the plug will be nearly the external work for the sensibly perfect gases; or

$$H = \int_{v_1}^{v_2} p\,dv;$$

and substituting these in equation (100), gives

$$\int \left(\tau \frac{d\,v}{d\,\tau} - v \right) dp = K_p (\tau_1 - \tau_2) + v_2 p_2 - v_1 p_1. \quad (101)$$

But this cannot be reduced, since τ depends upon the zero of the perfect scale, which we do not know; we, however, know by experiment that it is not far from the zero of the air thermometer; hence, if t be the temperature from the absolute zero of the air thermometer, we have from equation (2)

$$\frac{p\,v}{t} = \frac{p_0\,v_0}{t_0} = R, \text{ very nearly};$$

$$\therefore \frac{d\,v}{d\,t} = \frac{R}{p},$$

$$v = \frac{R\,t}{p},$$

$$v_1 p_1 = v_2 p_2,$$

$$\frac{d\,v}{d\,\tau} = \frac{d\,v}{d\,t}, \text{ very nearly},$$

$$\tau_1 - \tau_2 = t_1 - t_2, \text{ almost exactly},$$

for the range of temperatures in an experiment, and considering the specific heat as constant, or $K_p = C_p$; these will reduce equation (101) to

$$\tau = t + \frac{C_p (t_1 - t_2)}{R \log \frac{p_1}{p_2}}, \quad (102)$$

which shows that the absolute zero of the perfect scale is lower than the zero of the air thermometer.

89. The absolute zero, found by experiments upon air, oxygen and nitrogen, by the aid of equation (102), has a mean value of

$$492.66° \text{ F.} = 273.7° \text{ C.}$$

below the melting point of ice (Thomson's *Papers*, p. 392), while the zero of the air thermometer is 491.13° F. below, so that the absolute zero is about 1.53° F. below that of the

air thermometer. The difference is so small compared with the distance from the melting point of ice as to render it probable that the approximation is very close, supposed to be within $\frac{1}{400}$ of the exact value.

Specific Heat.

90. General expression for specific heat. The first of equations (A) gives

$$\frac{d\,H}{d\,\tau} = K_v + \tau\left(\frac{d\,p}{d\,\tau}\right)\left(\frac{d\,v}{d\,\tau}\right), \tag{103}$$

which is a general expression for the sp. heat of any substance *at* the volume v and temperature τ, the *path of the fluid* being arbitrary. Unless otherwise stated, the path is assumed to be either parallel to, or perpendicular to, the v-axis, giving rise to the conditions p constant or v constant; hence, $d\,p = o$ or $d\,v = o$, which in equations (A) give the respective equations

$$\left(\frac{d\,H}{d\,\tau}\right)_v = K_v,$$

$$\left(\frac{d\,H}{d\,\tau}\right)_p = K_p,$$

as previously established in Articles 38 and 39.

91. Specific heat at a change of state of aggregation. It has been shown that for fusion, evaporation and sublimation the change of state takes place *at* a fixed temperature corresponding to a given pressure for each substance; hence, for these cases we have

$$d\,\tau = o, d\,H > 0;$$

$$\therefore K_p = \frac{d\,H}{0} = \infty, \tag{104}$$

similarly,

$$K_v = K_p - R = \infty;$$

or *the specific heats* AT *the change of state are infinite.*

92. Modified expression for the specific heat. The specific heat of substances which are capable of a large expansion—conceived to be indefinitely large—admit, according to the theoretical views of Rankine, of being expressed in two parts—a constant and a variable part. To show this, let b, Fig. 34, be the initial state of one pound of the substance, $b\,e$ an isothermal for the substance, $c\,d$ a consecutive isothermal, K_v the specific heat at the volume v, $K_{v''}$ the specific heat at the volume v''. From the state b let the temperature be increased $d\,\tau$, thus increasing the pressure an amount $b\,c$; from the state c expand it, doing the external work $v\,c\,d\,v''$, at d abstract heat, so as to reduce the temperature $d\,\tau$, reducing the pressure to $v''\,e$, thence compress it isothermally to b, doing work $v''\,e\,b\,v$ upon the substance, then will the resultant external work be

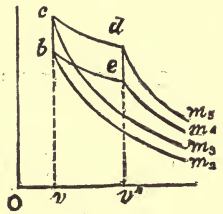

FIG. 34.

$$b\,c\,d\,e = \int_v^{v''} dp\,dv;$$

and, according to the second law, omitting the parentheses indicating partial differentials,

$$\text{area } m_2\,b\,e\,m_4 = \tau \int_v^{v''} \frac{dp}{d\tau}\,dv$$

$$m_3\,c\,d\,m_5 = (\tau + d\,\tau) \int_v^{v''} \frac{d\,(p + dp)}{d\,\tau}\,dv$$

$$= \tau \int \frac{dp}{d\tau} dv + \tau \int \frac{d^2p}{d\tau^2} dv d\tau + d\tau \int \frac{dp}{d\tau} dv + d\tau \int \frac{d^2p}{d\tau^2} dv d\tau.$$

Also,

$$m_2\,b\,c\,m_3 = K_v\,d\,\tau,$$
$$m_4\,e\,d\,m_5 = K_{v''}\,d\,\tau;$$

And,

$$m_2 b c m_3 + m_3 c d m_5 - m_2 b e m_4 - m_4 e d m_5 = b c d e;$$

hence, substituting and reducing,

$$K_v = K_{v''} - \tau \int_v^{v''} \frac{d^2 p}{d \tau^2} d v - d \tau \int_v^{v''} \frac{d^2 p}{d \tau^2} d v$$

$$= K_{v''} + \tau \int_{v''}^{v} \frac{d^2 p}{d \tau^2} d v.$$

Removing v'' indefinitely to the right, we have $v'' = \infty$, and then $K_{v''}$ becomes K_∞, which, according to a theory of Rankine, is constant for a constant state of aggregation, according to which the preceding equation becomes

$$K_v = C + \tau \int_\infty^v \left(\frac{d^2 p}{d \tau^2}\right)_v d v, \tag{105}$$

in which the last term is the rate at which the *internal* work is done at the volume v and temperature τ, due to a change of temperature. It may be found more directly from equation (67) by differentiating it, considering τ as the independent variable, giving

$$\int \left[\tau \left(\frac{d^2 p}{d \tau^2}\right) d \tau + d \tau \left(\frac{d p}{d \tau}\right) - \left(\frac{d p}{d \tau}\right) d \tau \right] d v = \tau \int \left(\frac{d^2 p}{d \tau^2}\right) d v d \tau,$$

and dividing by $d \tau$ gives the heat doing *internal* work per unit of temperature, which is the same as that above when the corresponding limits are assigned. Equation (105) being the specific heat at the volume v and temperature τ, we have, for the heat absorbed at *constant volume*,

$$H = \int K_v d \tau = C(\tau_2 - \tau_1) + \int_{\tau_1}^{\tau_2} \int_\infty^v \tau \frac{d^2 p}{d \tau^2} d \tau d v, \tag{106}$$

the last term of which may be integrated when the equation of the gas is given.

Similarly, if p and τ be the independent variables, we

would find

$$K_p = C + R - \tau \int_o^p \left(\frac{d^2 v}{d\tau^2}\right)_p dp, \qquad (107)$$

and for an increase of temperature $\tau_2 - \tau_1$, the heat absorbed at constant pressure would be

$$H = \int K_p d\tau = (C+R)(\tau_2 - \tau_1) - \int_{\tau_1}^{\tau_2}\int_o^p \tau\left(\frac{d^2 v}{d\tau^2}\right) d\tau dp. \qquad (108)$$

93. The apparent specific heat is the total heat absorbed by unity of weight in producing an increase of temperature one degree, and includes that necessary to make the substance hot as well as that doing the internal work, and also the external in the case of constant pressure. It is represented, for expansible gases, by the second members of equations (105) and (107).

The real specific heat is that part of the apparent specific heat which directly makes the substance hotter. It is represented by C in equation (105), and is called actual energy.

The apparent specific heat is the sum of the actual and potential energies in the particular specific heat. Equations (105) and (107) are the *dynamic* specific heats, and to find them in ordinary units they must be divided by J.

The theory of Rankine, referred to above, is "The real specific heat of each substance is constant at all densities so long as the substance retains its condition, solid, liquid, or gaseous" (*Prime Movers*, p. 307). The correctness of this theory has been questioned by Clausius, and in one place Rankine says "it is *probably* constant" (*ibid.*, p. 250). The theory is useful in showing the different effects probably produced by the absorbed heat; but it is of no service except in expansible gases, and cannot be used in those cases except where the equation of the fluid is known—and it is known for only a few substances.

EXERCISES.

1. Find the specific heat at volume v and temperature τ for the gases represented by the equation $p\,v = R\,\tau - \frac{a}{\tau\,v}$.

We have

$$p = R\frac{\tau}{v} - \frac{a}{\tau\,v^2},$$

$$\left(\frac{d\,p}{d\,\tau}\right) = R\frac{1}{v} + \frac{a}{\tau^2\,v^2},$$

$$\tau\left(\frac{d\,p}{d\,\tau}\right) = R\frac{\tau}{v} + \frac{a}{\tau\,v^2}$$

$$= p + \frac{2\,a}{\tau\,v^2},$$

$$\left(\frac{d^2\,p}{d\,\tau^2}\right) = -\frac{2\,a}{\tau^3\,v^2};$$

$$\therefore \int_\infty^v \frac{d^2\,p}{d\,\tau^2}\,\tau\,d\,v = -\frac{2\,a}{\tau^2}\int_\infty^v \frac{d\,v}{v^2} = \frac{2\,a}{\tau^2\,v}$$

$$\therefore K_v = C + \frac{2\,a}{\tau^2\,v}.$$

From this result it appears that the specific heat at constant volume decreases as the temperature increases, for all gases represented by the above equation, and approaches C as a limit.

2. Find the heat necessary to raise the temperature of one pound of the gas of the preceding exercise from τ_1 to τ_2 at the constant volume v.

Equation (106) becomes

$$H = C\,(\tau_2 - \tau_1) + \frac{2\,a}{v\,\tau_2\,\tau_1}\,(\tau_2 - \tau_1)$$

$$= \left(C + \frac{2\,a}{v\,\tau_2\,\tau_1}\right)(\tau_2 - \tau_1).$$

From this it appears that $C + \frac{2\,a}{v\,\tau_2\,\tau_1}$ is the mean specific heat between τ_2 and τ_1.

3. Find the heat absorbed by one pound of CO_2 in rais

ing its temperature from 500° F. to 600° F. at the volume 8.5 cubic feet.

From equation (6), page 13, we have

$$R = 35, \qquad a = 481600,$$

and the preceding equation gives (C being 132),

$$H = \left(132 + \frac{963200}{2550000}\right) 100 = (132 + 0.377)\, 100$$
$$= 13238 \text{ foot-pounds.}$$

It will here be observed that the term due to internal work is very small compared with the actual energy, and may properly be omitted, especially when we consider in addition thereto that it is less than the errors of observation determining the number 132.

4. Find the apparent specific heat of the gases represented by the equation $p\,v = R\,\tau$. (Use equations (105) and (107).)

94. General expression for the difference of specific heats.

In equation (103) the left member is the specific heat at constant pressure, if p be constant; hence,

$$K_p = K_v + \tau \left(\frac{d\,p}{d\,\tau}\right)\left(\frac{d\,v}{d\,\tau}\right).$$

To make this more apparent, use both of equations (A), which are

$$d\,H = K_v\,d\,\tau + \tau \left(\frac{d\,p}{d\,\tau}\right) d\,v,$$

$$d\,H = K_p\,d\,\tau - \tau \left(\frac{d\,v}{d\,\tau}\right) d\,p;$$

subtracting, gives

$$0 = (K_p - K_v)\,d\,\tau - \left[\tau \left(\frac{d\,p}{d\,\tau}\right) d\,v + \tau \left(\frac{d\,v}{d\,\tau}\right) d\,p\right].$$

In Figure 35 let $A\,\tau$ and $B\,C$ be two consecutive isothermals and $A\,b$ the path of the fluid, $A\,B$ perpendicular to the

axis of volumes, $A\ C$ parallel thereto, then, as shown by equation (20), will

$$\tau \left(\frac{d\,p}{d\,\tau}\right) d\,v = m_2\ B\ b\ m_3\,;$$

$$\tau \left(\frac{d\,v}{d\,\tau}\right) d\,p = m_3\ b\ C\ m_4\,;$$

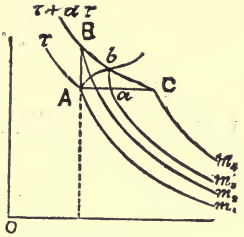

FIG. 35.

in which $d\,v$ in the former expression is the abscissa of b from $A = A\ a$, and $d\,p = a\ b$ in the latter is the ordinate of b from $A\ C$. If $d\,v$ in the former be made the abscissa of C in reference to A, that is, $d\,v = A\ C$, then would $\tau \left(\frac{d\,p}{d\,\tau}\right) d\,v$ be the area $m_2\ B\ C\ m_4$, which is the sum of the two preceding expressions, and hence the value of the expression in the [ ] of the former equation. To indicate that $d\,v$ extends thus far, it is necessary to express v as a function of τ, since it will be limited by two isothermals, and will be written thus:

$$\left(\frac{d\,v}{d\,\tau}\right)_{\mathrm{p}} d\,\tau\,;$$

then

$$\tau \left(\frac{d\,p}{d\,\tau}\right) \left(\frac{d\,v}{d\,\tau}\right) d\,\tau = m_2\ B\ C\ m_4\,;$$

which, substituted, gives

$$K_{\mathrm{p}} - K_{\mathrm{v}} = \tau \left(\frac{d\,p}{d\,\tau}\right) \left(\frac{d\,v}{d\,\tau}\right), \qquad (109)$$

as before.

In solids and liquids it is difficult to find the variation of pressure with temperature at constant volume, $\left(\frac{d\,p}{d\,\tau}\right)_{\mathrm{v}}$, and that factor may be eliminated as follows:—

$$v = f\,(p, \tau)\,;$$

$$\therefore d\,v = \left(\frac{d\,v}{d\,p}\right) d\,p + \left(\frac{d\,v}{d\,\tau}\right) d\,\tau.$$

But for v constant $d\,v = o$;

$$\therefore \frac{d\,p}{d\,\tau} = -\frac{\left(\frac{d\,v}{d\,\tau}\right)}{\left(\frac{d\,v}{d\,p}\right)};$$

which in (109) gives

$$K_p - K_v = -\tau\,\frac{\left(\frac{d\,v}{d\,\tau}\right)^2}{\left(\frac{d\,v}{d\,p}\right)}, \qquad (110)$$

an expression equally true for gases.

EXERCISES.

1. Find the difference between the specific heats of a perfect gas, the equation being $p\,v = R\,\tau$. (Equation (109) or (110).)

2. Find the difference between the specific heats of a gas whose equation is $p\,v = R\,\tau - \frac{a}{\tau\,v}$.

$$\text{Ans. } K_p - K_v = \frac{v}{\tau}\left(p + \frac{2\,a}{\tau\,v^2}\right)\left(\frac{\tau\,p\,v^2 + 2\,a}{\tau\,p\,v^2 - a}\right).$$

3. Find the difference between the specific heats of carbonic acid gas at the melting point of ice at the pressure of one atmosphere.

4. Find the difference between the specific heat of water at constant pressure and at constant volume when at its maximum density.

It is at its maximum density under the pressure of one atmosphere at the temperature of 39.1° F., and at this state

$$\left(\frac{d\,v}{d\,\tau}\right)_p = o\,;\ \therefore K_p = K_v$$

5. Find the difference between the specific heats of water at 39° F. and 77° F. under the pressure of one atmosphere,

the coefficient of expansion at 77° F. being 0.00014 of its volume per degree, and the coefficient of compression being 0.000046 of its volume for one atmosphere.

The volume of one pound of water at maximum density being 0.016 cubic feet, we have

$$\left(\frac{d\,v}{d\,\tau}\right)_p^2 = (0.016 \times 0.00014)^2 = \frac{5}{10^{12}}.$$

Assuming that the rate of expansion for 38° (= 77 − 39) is uniform, the volume at 77° F. will be

$$0.016 + 0.016 \times 0.00014 \times 38 = 0.01609, \text{ nearly};$$

hence, for a pressure of one pound per square foot we have

$$\left(\frac{d\,v}{d\,p}\right) = -\frac{0.01609 \times 0.000046}{2116.2} = -\frac{351}{10^{12}};$$

$$\therefore K_p - K_v = 537.6 \times \tfrac{5}{3\,5\,0} = 7.68;$$

$$\therefore c_p - c_v = 0.0099.$$

A similar computation at 122° gives

$$c_p - c_v = 0.0360.$$

Adopting Regnault's values for the specific heat at constant pressure, we find

$$\text{at } 39° \begin{cases} c_p = 1.0000 \\ c_v = 1.0000 \end{cases}$$

$$\text{at } 77° \begin{cases} c_p = 1.0016 \\ c_v = 0.9917 \end{cases}$$

$$\text{at } 122° \begin{cases} c_p = 1.0042 \\ c_v = 0.9682 \end{cases}$$

It will be observed that while the value of c_p increases with the temperature c_v decreases, and hence the *difference* increases more and more as the temperature increases. The exact numerical results here found are not to be relied upon, but they are approximately correct, and indicative of a general law.

6. Show, both analytically and geometrically, that the specific heat for constant pressure exceeds that at constant volume.

7. Show that the term $\tau \int \frac{d^2 p}{d\,\tau^2}\, d\,v$ disappears for gases whose equations (if any) are $p\,v = R\,\tau + \varphi\,(v)$.

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95. Water is the only substance whose specific heat has been accurately found for a large range of temperatures. According to Regnault's experiments, there is a small increase in the specific heat with every increase of temperature above 32° F., but according to the very accurate experiments of Professor Rowland, the specific heat of water decreases from 4° C. to 27° C., and may be approximately represented by the empirical formula

$$c = 1 - 0.00052\,(t - 4) + 0.000003\,(t - 4)^2, \quad (111)$$

in which t should not exceed 30° C.

Rankine's formula for representing Regnault's experiments is

$$c = 1 + 0.000000309\,(T - 39.1)^2 \text{ (Fah.)}, \quad (112)$$
$$= 1 + 0.000001\,(t - 4)^{2*} \text{ (Cent.)}.$$

M. Bosscha represents Regnault's experiments by the formula

$$c = 1 + 0.00022\,t \text{ (Cent.)}. \quad (113)$$

When the law of the specific heat is known, the number of thermal units absorbed in raising a pound of the substance from T_1 to T_2 degrees will be

$$h = \int_{T_1}^{T_2} c\,d\,T,$$

and the mean specific heat will be

$$\frac{h}{T_2 - T_1}.$$

96. Another form of the general equation. Substituting the value of K_v, equation (105) in the first of equations (A) gives

$$d\,H = C\,d\,\tau + \tau \int_{\infty}^{v} \frac{d^2 p}{d\,\tau^2}\,d\,\tau\,d\,v + \tau\,\frac{d\,p}{d\,\tau}\,d\,v. \quad (114)$$

* *Trans. R. S. E.*, XX. (1851), 441.

This is analyzed by Rankine as follows (*Prime Movers*, p. 312):—

"I. The variation of the actual heat of unity of weight of the fluid $C_v\, d\,\tau$.

"II. The heat which disappears in producing work by mutual molecular actions depending on change of temperature and not on change of volume.

"III. The latent heat of expansion, $\tau \dfrac{d\,p}{d\,\tau}\, d\,v$, that is, heat which disappears in performing work, partly by the forcible enlargement of the vessel containing the fluid and partly by mutual molecular actions depending on expansion."

The integral of the last equation would if determined, give the heat absorbed, in foot-pounds, in producing the changes of temperature and volume; but the integral cannot be performed without the equation of the path of the fluid and the equation of the fluid.

Still another form. Subtracting $p\,d\,v$, the external work done, from both members of the preceding equation gives

$$H - \int p\,d\,v = C\,(\tau_2 - \tau_1) + \int_{\tau_1}^{\tau_2}\int_{\infty}^{v_2} \tau \frac{d^2 p}{d\,\tau^2}\, d\tau dv + \int_{v_1}^{v_2} \left(\tau \frac{d\,p}{d\,\tau} - p\right) d\,v, \quad (115)$$

in which each member is the excess of the heat absorbed above the external work done. The several terms may be represented by Fig. 36, in which $A\ B$ is the path of the fluid, $A\ C$ an isothermal through A, and $A\ a\ c\ C$ represents the internal work done during the isothermal expansion $A\ C$. Then

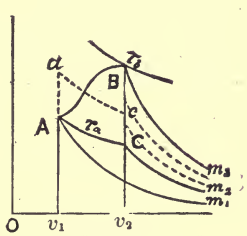


FIG. 36.

$$H = m_1\ A\ B\ m_3,$$

$$\int p\ d\ v = v_1\ A\ B\ v_2 = v_1\ A\ C\ v_2 + A\ B\ C.$$

$$\int\int \tau \frac{d^2 p}{d\,\tau^2}\ d\ \tau\ d\ v = \textit{areas represented by the dotted lines between } C\ m_2 \text{ and } B\ m_3,$$

$C\ (\tau_2 - \tau_1) = m_2\ C\ B\ m_3$ — *areas represented by the dotted lines,*

$$C\ (\tau_2 - \tau_1) + \int\int \tau\ \frac{d^2 p}{d\tau^2}\ d\tau\ dv = m_2\ C\ B\ m_3,$$

$$\int \left(\tau\ \frac{dp}{d\tau} - p\right) dv = A\ a\ c\ C,$$

$$v_1\ a\ c\ v_2 = m_1\ A\ C\ m_2;$$

$$\therefore H - \int p\ dv = m_1\ A\ B\ m_3 - v_1\ A\ B\ v_2$$

$$= A\ a\ c\ C + m_2\ C\ B\ m_3.$$

It is apparent that the second term of the second member of equation (115) is the $\varphi\ (\tau)$ of equation (67).

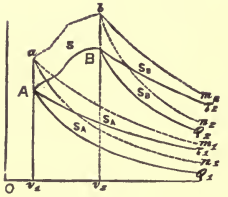


FIG. 37.

The internal work may be represented in another form. In Fig. 37 let $A\ \tau_1$ and $B\ \tau_2$ represent isothermals extended indefinitely, and conceive that $\tau_1\ A\ B\ \tau_2$ forms a closed cycle, then will the resultant internal work in passing around the cycle be zero. Let ${}_aS_b$ be the internal work done in passing from A to B, ${}_bS_\infty$ in passing from B to infinity, ${}_\infty S_a$ in passing from infinity back to A; then

$${}_aS_b + {}_bS_\infty - {}_aS_\infty = 0. \tag{116}$$

Let τ_1 be the isothermal through A, τ_2 through B, Figs. 36 and 37, then, equation (115),

$${}_aS_b = \int_{v_1}^{v_2} \left(\tau_1 \left(\frac{dp}{d\tau}\right) - p\right) dv + \int_{\tau_1}^{\tau_2}\int_{\infty}^{v_2} \tau \left(\frac{d^2 p}{d\tau^2}\right) d\tau\ dv, \tag{117}$$

and from Fig. 37, S_a being the potential energy at A, etc.

$$S_a = \int_{\infty}^{v_1} \left(\tau_1 \left(\frac{dp}{d\tau}\right) - p\right) dv, \tag{118}$$

$$S_b = \int_{\infty}^{v_2} \left(\tau_2 \left(\frac{dp}{d\tau}\right) - p\right) dv, \tag{119}$$

these combined with equation (116) reduces (115) to

$$H - \int p\, d\, v = (C\, \tau - S)_b - (C\, \tau - S)_a. \quad (120)$$

in which $(C\, \tau - S)_b$ is called the intrinsic energy of the gas in the state B, and depends upon the state of the gas only; hence, the heat absorbed above that necessary to perform the external work equals the increase of the *intrinsic energy.*

EXERCISES.

1. If the equation of the gas be $p\, v = R\, \tau - \frac{a}{\tau\, v}$, verify the statement that the internal work is the same whether the path be $A\ C$ and $C\ B$, Fig. 36, or the indefinitely extended isothermals τ_a and τ_b. (Equations (117), (118), (119).)

2. If the equation be the general one given by Rankine,

$$p\, v = R\, \tau - a_0 - \frac{a_1}{\tau\, v} - \frac{a_2}{\tau^2\, v} - \text{\&c.},$$

a_0, a_1, a_2, &c., being constant, verify the fact that the internal work is the same, whether $A\ C$ and $C\ B$ be the path, or whether the path be along two isothermals through A and B, respectively, indefinitely extended.

3. Test the same principle for the ideal gas whose equation is $p\, v^2 = c\, \tau$, c being a constant.

4. Will the principle stated in Exercise 2 be true if the equation of the gas were $p = v\, \tau$?

5. Find the internal work done by expansion at constant pressure from v_1 to v_2 when the equation of the gas is $p\, v = R\, \tau - \frac{a}{\tau\, v}$. ((117), or (118) and (119).)

Ans. $2\, a \left[\frac{1}{\tau_1\, v_1} - \frac{1}{\tau_2\, v_2} \right]$.

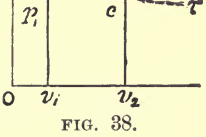


FIG. 38.

This is the value of the last term of equation (108). The last term of equation (108) is not easily found directly, since

it contains internal work due both to a change of volume and temperature. Since τ_2 increases with v_2 when p is constant, it follows from the preceding Answer that the internal work increases with the expansion and approaches the limit $\frac{2\,a}{\tau_1\,v_1}$, which is a function of the initial state only, and decreases with the increase of temperature. It may be inferred from this—although we have by a more complete analysis found—that the last term of equation (107) is not only essentially negative but decreases, numerically, with increase of temperature; hence, the internal work of all expansible substances whose equation approximates to the form $p\,v = R\,\tau - \frac{a}{\tau\,v}$ increases with increase of temperature under constant pressure.

6. Find the *real* dynamic specific heat of carbonic acid gas at the pressure of one atmosphere.

We have

$$C_p = C_v + R = 132 + \frac{17264}{492.66} = 167 \text{ foot-pounds.}$$

7. Find the internal work of expanding $C\,O_2$ from $v_1 = 8.5$ cu. ft. at $\tau_1 = 500°$ F. to $\tau_2 = 600°$ F. at a constant pressure.

From equation (6) we have

$$p_1 = 35\,\frac{500}{8.5} - \frac{481600}{500 \times (8.5)^2} = 2047 \text{ lbs.},$$

$$p_2 = p_1 = \frac{35 \times 600}{v_2} - \frac{481600}{600\,v_2^2};$$

$$\therefore v_2 = 10.22 \text{ cubic feet.}$$

Substituting in the Answer of Exercise 5, gives

$$2 \times 481600 \left[\frac{1}{500 \times 8.5} - \frac{1}{600 \times 10.22}\right] = 70 \text{ foot-pounds, nearly.}$$

But this is the work for an increase of 100 degrees of temperature; hence, the average will be 0.70 foot-pounds per degree of temperature. This is less than $\frac{1}{250}$ of the heat producing actual energy of the substance

per degree of temperature, as will be found by comparing this result with that given in the preceding Exercise.

From the preceding analysis it appears that for ordinary engineering purposes the specific heat of all substances may be considered constant for a constant state of aggregation; and the most important element involving the imperfection of the fluid is that due to a change of state of aggregation. It, however, furnishes a wide field for scientific investigation.

97. Other forms of the general equations. In Fig. 39 let the path $A\ b$ be intersected by equidistant isothermals, of which τ through A and $\tau + d\ \tau$ through b are consecutive. Through A draw the horizontal $A\ a$ and through b the vertical $b\ a$; then will the heat absorbed in passing from A to a at constant pressure be

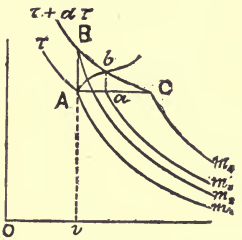


FIG. 39.

$$m_1 A\ a\ m_3 = K_p\ d\ \tau,$$

excepting that $d\ \tau$ is not independent, but is dependent upon $d\ v$, the abscissa of b in reference to A, and hence we have

$$m_1 A\ a\ m_3 = K_p \left(\frac{d\ \tau}{d\ v}\right) d\ v.$$

Similarly the heat absorbed from a to b at constant volume will be

$$K_v \left(\frac{d\ \tau}{d\ p}\right) d\ p;$$

hence, ultimately, the heat absorbed in working from A to b will be the sum of these, or

$$d\ H = K_p \left(\frac{d\ \tau}{d\ v}\right) d\ v + K_v \left(\frac{d\ \tau}{d\ p}\right) d\ p,$$

a form given by Zeuner (*Théorie Mécanique de la Chaleur*, p. 547).

Substituting $\left(\frac{d\ \tau}{d\ v}\right)$ found from equation (109) and $\left(\frac{d\ \tau}{d\ p}\right)$

from the same, reduces this to

$$d\,H = \frac{\tau}{K_p - K_v}\left[K_p\left(\frac{d\,p}{d\,\tau}\right)d\,v + K_v\left(\frac{d\,v}{d\,\tau}\right)d\,p\right], \quad (121)$$

which may be found directly from equations (A) by eliminating $d\,\tau$ between them. This form is given by Clausius (on *Heat*, p. 179).

Again, from equation $(A)_1$ we have, for τ constant,

$$\left(\frac{d\,H}{d\,v}\right)_\tau = \tau\left(\frac{d\,p}{d\,\tau}\right);$$

$$\therefore \frac{d}{d\,\tau}\left(\frac{d\,H}{d\,v}\right)_\tau = \tau\left(\frac{d^2\,p}{d\,\tau^2}\right) + \left(\frac{d\,p}{d\,\tau}\right);$$

and from equation (105)

$$\left(\frac{d\,H}{d\,\tau}\right)_v = K_v = C + \tau\int\left(\frac{d^2\,p}{d\,\tau^2}\right)d\,v;$$

$$\therefore \frac{d}{d\,v}\left(\frac{d\,H}{d\,\tau}\right)_v = \tau\left(\frac{d^2\,p}{d\,\tau^2}\right),$$

$$\frac{d}{d\,\tau}\left(\frac{d\,H}{d\,v}\right)_\tau - \frac{d}{d\,v}\left(\frac{d\,H}{d\,\tau}\right)_v = \left(\frac{d\,p}{d\,\tau}\right) = \frac{1}{\tau}\left(\frac{d\,H}{d\,v}\right);$$

in which τ and v are the independent variables. Similarly, from equation $(A)_2$ and (107), we find

$$\left(\frac{d\,H}{d\,p}\right)_\tau = -\,\tau\left(\frac{d\,v}{d\,\tau}\right),$$

$$\frac{d}{d\,p}\left(\frac{d\,H}{d\,\tau}\right)_p = -\,\tau\left(\frac{d^2\,v}{d\,\tau^2}\right),$$

$$\frac{d}{d\,\tau}\left(\frac{d\,H}{d\,p}\right)_\tau - \frac{d}{d\,p}\left(\frac{d\,H}{d\,\tau}\right)_p = -\left(\frac{d\,v}{d\,\tau}\right);$$

in which τ and p are the independent variables. These forms are given by Clausius, though obtained in a very different manner (*Mechanical Theory of Heat*, Clausius (Browne's translation), pp. 118, 119).

Again, from equation (109), we have

$$\frac{K_p}{K_v} = \gamma = \frac{1}{1 - \frac{\tau}{K_p}\left(\frac{d\,p}{d\,\tau}\right)\left(\frac{d\,v}{d\,\tau}\right)},$$

a form used by Professor Rowland in his paper on the *Mechanical Equivalent of Heat*, page 146.

Again, in Fig. 40 let τ_1 be the temperature of the isothermal $A\,B$, τ_2 that of $j\,i$, $d\,H_1$ the heat absorbed at τ_1 between the consecutive adiabatics $A\,\varphi_1$ and $b\,n$; then, according to Article 40, equation (20), we have

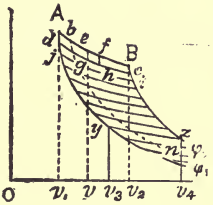


FIG. 40.

$$\frac{d\,H_1}{\tau_1} = \left(\frac{d\,p}{d\,\tau}\right) d\,v,$$

$$\frac{d\,H_2}{\tau_2} = \left(\frac{d\,p}{d\,\tau}\right) d\,v;$$

from which, the second members being the same, we have

$$\frac{d\,H_1}{\tau_1} - \frac{d\,H_2}{\tau_2} = 0.$$

Similarly, if τ_3 be the temperature of the isothermal $d\,c$, τ_4 of $y\,z$, we would have

$$\frac{d\,H_3}{\tau_3} - \frac{d\,H_4}{\tau_4} = 0;$$

$$\therefore \frac{d\,H_1}{\tau_1} - \frac{d\,H_2}{\tau_2} + \frac{d\,H_3}{\tau_3} - \frac{d\,H_4}{\tau_4} = 0,$$

or, considering heat emitted as essentially negative, all the terms may be written with the *plus* sign, and, generally, *when a succession of operations are performed in Carnot's cycle*, we have

$$\Sigma \frac{d\,H}{\tau} = 0,$$

or, ultimately,

$$\int \frac{d\,H}{\tau} = 0.$$

But any cycle may be divided into an indefinite number of strips by adiabatics drawn across it, and by drawing isothermals from their intersections with the path to adjacent adiabatics, an inscribed polygon may be constructed whose area may be made to differ from that of the given cycle by less than any assignable quantity; hence, ultimately, if the integral extend throughout the entire cycle,

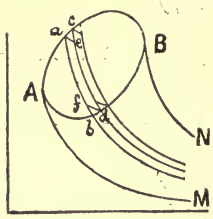


FIG. 41.

$$\int \frac{d\,H}{\tau} = 0. \qquad (122)$$

If the integral be separated into two parts, one along $A\ a\ B$, during which heat is absorbed, the other along the $B\ b\ A$, during which heat is emitted, we have

$$\int_A^B \frac{d\,H}{\tau} + \int_B^A \frac{d\,H}{\tau} = 0.$$

Equation (122) is Thomson's generalization of the second law. It was first published by Clausius in 1854 (*Pogg. Ann.*, Vol. XCIII., p. 500; Clausius on *Heat*, p. 90).

(The differential of a function of two or more independent variables is said to be an *exact differential*. Let

$$M\,dx + N\,dy$$

be such an expression, in which M and N may be functions of x and y; then it is shown by the calculus that the differential of M in regard to y equals the differential of N in regard to x, or,

$$\frac{d\,M}{d\,y} = \frac{d\,N}{d\,x}.$$

This principle has been successfully applied to many problems on heat, and of the early investigators, Thomson led in this mode of analysis.)

Since it has been shown that the integral of $\frac{d\,H}{\tau}$ is zero

for a complete cycle, it is an exact differential, and may be represented by a single symbol, as φ, and we have

$$\frac{d\,H}{\tau} = d\,\varphi;$$

hence, from $(A)_1$ we have

$$d\,\varphi = \frac{d\,H}{\tau} = K_v \frac{d\,\tau}{\tau} + \left(\frac{dp}{d\,\tau}\right) d\,v. \qquad (123)$$

Applying the preceding principle, we have

$$\frac{d}{d\,v}\left(\frac{K_v}{\tau}\right) = \frac{d}{d\,\tau}\left(\frac{dp}{d\,\tau}\right),$$

or

$$d\,K_v = \tau\left(\frac{d^2 p}{d\,\tau^2}\right) d\,v,$$

which is the differential of equation (105). In differentiating the left member, τ is not a function of v, since v is constant during the change of temperature.

Finally, let $\mathcal{E}'$ = the internal energy of the substance both actual and potential,

L = the latent heat of expansion as a thermal capacity, and other notation as previously given, then

$$d\,H = d\,E + p\,d\,v = K_v\,d\,\tau + L\,d\,v; \qquad (124)$$

$$\therefore d\,E = K_v\,d\,\tau + (L - p)\,d\,v. \qquad (125)$$

But when any substance is worked in a complete cycle the resultant internal work is zero; hence, $d\,E$ is an exact differential, and we have, omitting the parentheses of the partial differentials,

$$\frac{d\,K_v}{d\,v} = \frac{d}{d\,\tau}(L - p);$$

$$\therefore \frac{d\,K_v}{d\,v} = \frac{d\,L}{d\,\tau} - \frac{d\,p}{d\,\tau}. \qquad (126)$$

Similarly, for τ and p independent variables

$$\frac{d\,K_p}{d\,p} = \frac{d\,L'}{d\,\tau} + \frac{d\,v}{d\,\tau}. \qquad (127)$$

Since $\frac{d\,H}{\tau}$ is an exact differential, therefore will its value

$$\frac{K_v}{\tau} d\,\tau + \frac{L}{\tau} d\,v \tag{128}$$

also be an exact differential; hence,

$$\frac{d}{d\,v} \cdot \frac{K_v}{\tau} = \frac{d}{d\,\tau} \cdot \frac{L}{\tau}; \tag{129}$$

$$\therefore \frac{d\,K_v}{d\,v} = \frac{d\,L}{d\,\tau} - \frac{L}{\tau},$$

observing that τ in the left member is not a function of v; hence, $d\,\tau = 0$ in that member, and by (126)

$$\frac{d\,p}{d\,\tau} = \frac{d\,L}{d\,\tau} - \frac{d\,K_v}{d\,v} = \frac{L}{\tau}. \tag{130}$$

97a. The Thermodynamic Function, or **Entropy.** The function φ Rankine calls *the thermodynamic function. The differential of* φ, *or* $d\,\varphi$, *is the heat absorbed for each degree of absolute temperature between zero*

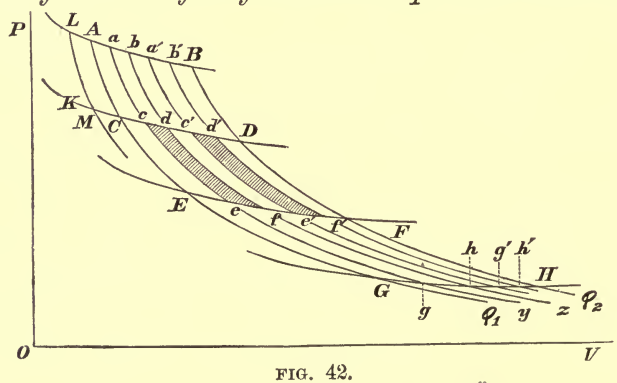


FIG. 42.

and τ, *while the substance is worked along any path from a point on one adiabatic to a point on the adjacent adiabatic.*

First let the path be an isothermal, as $A\,B$, Fig. 42, whose temperature is τ.

Intersect the path by an indefinite number of ordinates having between them the constant distance $d\,v$, and from

the points of intersection a, b, a', b', &c., draw adiabatics; and across these draw the isothermals $K\,D$, $E\,F$, $G\,H$, &c., the successive ones differing by one degree, in which case the temperature of $C\,D$ will be $\tau - 1$, of $E\,F$, $\tau - 2$, &c. Then will $d\,\varphi$, for an expansion from state a to state b, be

$$d\,\varphi = c\,d\,f\,e;$$

for we have previously shown (Article 40) that the heat absorbed in working from a to b isothermally will be

$$d\,H = y\,a\,b\,z = \tau\left(\frac{d\,p}{d\,\tau}\right)d\,v;$$

$$\therefore \frac{d\,H}{\tau} = \left(\frac{d\,p}{d\,\tau}\right)d\,v;$$

but it was shown in the same article that isothermals equidistant in temperature divide the heat into equal parts; hence, $c\,d\,f\,e$ in the figure will represent one of those equal parts;

$$\therefore \frac{d\,H}{\tau} = c\,d\,f\,e = d\,\varphi = \left(\frac{d\,p}{d\,\tau}\right)d\,v. \qquad (131)$$

Also,

$$d\,\varphi = a\,b\,d\,c = c\,d\,f\,e = e\,f\,h\,g, \text{ \&c.}$$

And for an expansion from state A to state a, we have

$$d\,\varphi = A\,a\,c\,C = C\,c\,e\,E = E\,e\,g\,G, \text{ \&c.},$$

and similarly for other expansions. But $C\,c\,e\,E$ does not equal $c\,d\,f\,e$, &c. For the sensibly perfect gases equation (131) becomes

$$d\,\varphi = \frac{R}{v}\,d\,v,$$

according to which $d\,\varphi$ diminishes as v increases, R and $d\,v$ being constant; hence

$$C\,c\,e\,E > c\,d\,f\,e > d\,c'\,e'\,f, \text{ \&c.},$$

and for v indefinitely large the area representing $d\,\varphi$ vanishes; and for v zero, $d\,\varphi$ will be indefinitely large. The indefinite integral of the preceding equation is

$$\varphi = R\,log\,v + C,$$

and for $v = 1$, we have

$$\varphi = C = \varphi_1 \text{ (say)};$$
$$\therefore \varphi - \varphi_1 = R \log v.$$

But the value of φ_1, or C, is unknown, and it is impracticable to find the value of φ; a condition, however, which is of little consequence, since it is only the difference of the thermodynamic functions that is of any importance. Although this illustration is for perfect gases, yet the principle is the same for all substances. This principle may be partially illustrated by our notation for indicating heat absorbed during isothermal expansion. Thus, in Fig. 42, let H_A be the heat absorbed in expanding from some unknown state L on the isothermal $A\ B$ to state A, and H_B the heat absorbed in expanding isothermally from the same unknown state to B; then

$$H_B - H_A = {}_AH_B,$$

will be the heat absorbed in expanding isothermally from A to B. A similar notation will apply to expansion according to any law, the points L, A and B being on the same path. Similarly, whatever be the unknown initial value of φ, we may write

$$\varphi_B - \varphi_A = {}_A\varphi_B.$$

The sum of all the areas $C\ e$, $c\ f$, &c., between $C\ E$ and $D\ F$ represents the heat absorbed for one degree of absolute temperature for an expansion from state A to state B. From equation (131), if τ_1 be the constant temperature of $A\ B$, we have by integration

$$\frac{H_B - H_A}{\tau_1} = \varphi_B - \varphi_A = C\ D\ F\ E = A\ B\ D\ C, \text{ \&c.}$$

But it is customary to compute the heat absorbed from the initial state, A, in which case $H_A = 0$, and we have

$$\frac{{}_AH_B}{\tau_1} = \varphi_B - \varphi_A = {}_A\varphi_B.$$

The value of the thermodynamic function for B will be

$$\varphi_B = L\,B\,D\,M,$$

and for A, $\varphi_A = L\,A\,C\,M;$

$$\therefore \varphi_B - \varphi_A = A\,B\,D\,C = \int_{v_1}^{v_2} \frac{d\,p}{d\,\tau}\,d\,v.$$

Thus far we have considered v as the independent variable, and deduced and interpreted $d\,\varphi$ under that hypothesis. Now we will make φ the independent variable, in which case $d\,\varphi$ will be constant. This condition will be accomplished by drawing the successive adiabatics $A\ \varphi_1$, $a\ e$, $b\ f$, &c., in such a manner that the areas $A\,a\,c\,C$, $a\,b\,d\,c$, &c., shall be equal, in which case we not only have

$$A\,a\,c\,C = C\,c\,e\,E = E\,e\,g\,G, \text{ \&c.},$$

but also

$$A\,a\,c\,C = a\,b\,d\,c = b\,a'\,c'\,d = c\,d\,f\,e, \text{ \&c.},$$

so that if the entire space, $\varphi_1\,A\,B\,\varphi_2$, be divided by isothermals of which the successive ones shall differ by one degree of temperature, and adiabatics drawn in the manner just described, *all the small areas thus formed will be* equal.

Since Fig. 42 is used to illustrate two modes of division, it should be observed that the two sets of adiabatics will not coincide, but that there will be a less number between the initial and terminal adiabatics, $A\ \varphi_1$ and $B\ \varphi_2$, in the latter mode of division, if the area $A\,a\,c\,C$ be the same in both cases. When $d\,\varphi$ is constant, the distance between pairs of contiguous ordinates passing through the points $A, a, b,$ &c., of the intersection of successive adiabatics, with any isothermal, $A\,B$, will be governed by the law

$$\left(\frac{d\,p}{d\,\tau}\right) d\,v = \left(\frac{d\,p'}{d\,\tau'}\right) d\,v';$$

$$\therefore \frac{d\,v}{d\,v'} = \left(\frac{d\,p'}{d\,\tau'}\right)\left(\frac{d\,\tau}{d\,p}\right),$$

which for sensibly perfect gases becomes, observing that, for this case, $\tau = \tau'$,

$$\frac{dv}{dv'} = \frac{p'}{\tau'} \cdot \frac{\tau}{p} = \frac{p'}{p};$$

hence, *the increments of the volume vary inversely as the pressures for those volumes, or directly as the volumes.*

The increments, then, become indefinitely large as the adiabatics approach indefinitely near the axis $O\,v$ and diminish indefinitely as they approach the axis $O\,p$.

Let these adiabatics be *numbered* in the order of the natural numbers, beginning with any arbitrary number, as 7 for $A\,\varphi_1$, then will $a\,e$ be 8; $b\,d$, 9, &c., and if the terminal one, $B\,\varphi_2$, be 12, the *number* of spaces between $A\,\varphi_1$ and $B\,\varphi_2$ will be

$$12 - 7 = 5,$$

and the area

$$A\,B\,D\,C = 5 \times A\,a\,c\,C;$$

similarly,

$$C\,D\,F\,E = 5 \times A\,a\,c\,C;$$

&c. &c.

Generally, if φ_A be the *number* of the initial adiabatic $A\,\varphi_1$, counting from any arbitrary zero, φ_B of $B\,\varphi_2$, then

$$A\,B\,D\,C = (\varphi_B - \varphi_A).\ A\,a\,c\,C = C\,D\,F\,E;$$

and if τ_1 be the *number* indicating the temperature of the isothermal $A\,B$, and τ_2 of any lower isothermal, as $E\,F$, then will the area

$$A\,B\,F\,E = (\tau_1 - \tau_2)\,(\varphi_B - \varphi_A).\ A\,a\,c\,C.$$

Let $A\,a\,c\,C$ be the unit of measurement, which may be arbitrarily chosen, and we finally have

$$A\,B\,F\,E = (\tau_1 - \tau_2)\,(\varphi_B - \varphi_A) = {}_2\tau_1 \times {}_A\varphi_B, \qquad (132)$$

in which, if the foot-pound be the unit of heat, the *thermodynamic function*, ${}_A\varphi_B$, will represent *the* NUMBER *of foot-pounds of heat absorbed* by the substance per degree

of temperature while expanding isothermally from any adiabatic, $A\ \varphi_1$, to another, $B\ \varphi_2$.

If $\tau_2 = o$, we have

$$\varphi_1\ A\ B\ \varphi_2 = \tau_1\ (\varphi_B - \varphi_A). \qquad (132a)$$

The *natural* zero-adiabatic is the ordinate $O\ p$, but between that and the initial adiabatic of any problem, as $A\ \varphi_1$, there will be an infinite number of adiabatics including areas equal to $A\ a\ c\ C$ between the isothermals $B\ A$ and $D\ C$; hence from this zero the number between A and B would be expressed by the difference between two infinites, thus,

$$\infty - \infty,$$

which is indeterminate. An arbitrary zero-adiabatic may be assigned, but it is unnecessary so to do, since it is only the *difference* of the *thermodynamic functions* that is sought.

The form of the expression in the second member of equation (132) is similar to that expressing the area of a rectangle. Thus, suppose that in measuring a rectangular field, $A\ B\ D\ C$, a point is arbitrarily assumed in the side $A\ B$ prolonged, from which the corner A is a feet, and B, b feet; then will the length of that side be $b - a$ feet long independently of the position of the point. Similarly, if the corner D be x yards from a point in the line of $B\ D$, and B, y yards from the same point, the area of the rectangle will be

$$A\ B\ D\ C = (b - a)\ (x - y) \text{ ft.-yds.},$$

the unit area being one foot wide and one yard long. If differences of temperature only were used the position of the absolute zero would be of little consequence.

Equation (132a) furnishes the following definition: *The difference between the numerical values of the thermodynamic functions corresponding to two adiabatics is equal to the quotient of the number of foot-pounds of heat absorbed*

or rejected in passing along any isothermal from one of these adiabatics to the other, divided by the absolute temperature corresponding to that isothermal.

If the path of the fluid is not an isothermal, the same principle is applicable, but τ will vary, and $\tau\, d\,\varphi$ cannot be integrated, but the difference of the thermodynamic functions may be determined in some cases. For instance, in Fig. 12, we have

$$d\,H = \tau\, d\,\varphi = K_v\, d\,\tau;$$

$$\therefore \frac{d\,H}{\tau} = d\,\varphi = K_v \frac{d\,\tau}{\tau};$$

in which, if K_v be constant, we have

$$\varphi_B - \varphi_A = K_v \log \frac{\tau_B}{\tau_A};$$

while the heat absorbed will be

$$H = \int \tau\, d\,\varphi = K_v\,(\tau_B - \tau_A).$$

When the path is arbitrary, the differential expression for the thermodynamic function is

$$\frac{d\,H}{\tau} = d\,\varphi = K_v \frac{d\,\tau}{\tau} + \left(\frac{d\,p}{d\,\tau}\right) d\,v.$$

Clausius calls the expression

$$\int \frac{d\,H}{\tau}$$

"the *entropy* of the body,"* from the Greek word $\tau\rho o\pi\eta$, *Transformation*, since it is a measure of the rate of the transformation of heat from the source to the body per unit of absolute temperature of the source, in passing from one adiabatic to another. It is identical with Rankine's *thermodynamic function.*

It will be observed that the spaces $A\ B\ D\ C$, $C\ D\ F\ E$, &c., are divided into the same *number* of strips by the adiabatics through A, a, b, &c.; hence, $\varphi_B - \varphi_A$ has the same value for the space between any pair of isothermals,

* *Mechanical Theory of Heat*, p. 102.

whether equidistant or not. If the state a approaches A; ultimately coinciding with it, the spaces $A\ a\ c\ C$, $C\ c\ e\ E$, &c., vanish; in other words, if the expansion be adiabatic, no heat will be absorbed or emitted, and for this case

$$d\,\varphi = 0;$$
$$\therefore\ \varphi = constant;$$

that is, *the entropy—or the thermodynamic function—of an adiabatic is constant*, and this is the characteristic equation of an *adiabatic*. For this reason we have frequently used the symbol φ to mark the adiabatic.

This property may be put in contrast with a property of an isothermal in the following manner:

That property of a substance which remains constant throughout such changes as are represented by an isothermal line is the *temperature*. The constant property is that of *constant heat*.

That property of a substance which remains constant throughout the changes represented by an adiabatic line is the *Thermodynamic function*, or *Entropy*. This constant property is the *constant rate* at which heat must be absorbed by a substance per unit of absolute temperature when the path of the fluid is from *any* point on an adiabatic to a point on the adjacent one.

98. Liquid and its vapor, combined. To find the differential expression for the heat absorbed, we first find the heat necessary to evaporate the $d\,x$ part of one pound, that is, a weight $d\,x$. For $d\,v$ volume we have

$$\tau \frac{d\,p}{d\,\tau} d\,v;$$

and if the volume of a pound of saturated vapor be v_2, then

$$\frac{d\,v}{v_2} = d\,x.$$

If W be the pounds of fluid of which $d\,W$ be evaporated, then if $d\,H$ be the heat absorbed for an increase $d\,\tau$ of

temperature of the liquid and dx the weight evaporated, we have

$$W\,dH = W\,C\,d\tau + dW \cdot H_e,$$

or

$$dH = C\,d\tau + \frac{dW}{W} H_e$$

$$= C\,d\tau + dx \cdot H_e, \qquad (133)$$

where x is the fractional part of a pound of the substance vaporized. Integrating this, observing that the conditions of the problem require that the higher temperature τ_1 must be that at which the quantity x is evaporated, we have

$$H = C(\tau_1 - \tau_2) + x \cdot H_{e_1}. \qquad (134)$$

If the substance be water, we have

$$C = J = 778 \text{ foot-pounds},$$

$$H_e = 1117880 - 544.6\,\tau.$$

Equation (134) is sometimes used in calorimeter tests for determining the amount of water in steam. Thus, to find the per cent of water, we have from equation (134)

$$100\,(1 - x) = 100\,\frac{H_e + C(\tau_1 - \tau_2) - H}{H_e}. \qquad (135)$$

(See Addenda.)

EXERCISES.

1. By condensing the steam from a boiler into a reservoir of water it was found that 600000 foot-pounds of heat had been imparted to one pound of the steam above the temperature of the feed water; the temperature of the feed water being 100° F. and the steam from the boiler 320° F., how much liquid water did the steam contain?

Here we have

$\tau_1 - \tau_2 = 220°$,

$C(\tau_1 - \tau_2) = 171160$ ft.-lbs.,

$H_e = 692000$ " " nearly, at 320° F.,

sum = 863160 " "

which is the foot-pounds of heat that the steam should have contained, above 100° F. if it had all been evaporated, but the test showed that

$$H = 600000;$$

$$\therefore \textit{difference} = 263160;$$

$$\therefore 100\,(1 - x) = \frac{26316000}{692000} = 38 \text{ per cent.}$$

2. If the feed water be 100° F. and the temperature of the steam be 338° F. and the heat absorbed above that of the feed water, $H = 900000$ foot-pounds, required the amount of water suspended in the steam.

Here,

$$C\,(\tau_1 - \tau_2) = 238° \times 778$$
$$= 185000$$
$$H_e = 683000 \text{ ft.-lbs. at } 338°,$$
$$\textit{sum} = 868000 \text{ " "}$$

which not being so much as was measured, the steam must have been superheated.

99. The specific heat of saturated vapor is not that at constant pressure nor that at constant volume, but it is the heat necessary to raise the temperature of one pound of the substance one degree when the steam remains continually at the point of saturation. Conceive the temperature of the entire mass to be increased an amount $d\,\tau$ and the volume an amount $d\,v$; then will the heat exist in the three following parts :—

1°. The heat absorbed by the liquid. The liquid not evaporated will be $W - w$, using the notation of the preceding article, and the heat absorbed by it will be

$$(W - w)\,C\,d\,\tau.$$

2°. The heat absorbed by the vapor. Let S be the dynamic specific heat of the saturated vapor of constant weight, then will the heat absorbed by it be

$$w\,S\,d\,\tau.$$

3°. An additional amount, $d\,w$, of the liquid will be evaporated both on account of the enlargement, $d\,v$, and the

increase of temperature, $d\tau$, and the amount of heat absorbed will be

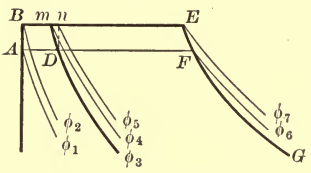
$$d w \cdot H_e.$$

Hence, equation $(A)_1$ becomes

$$W \cdot d H = (W - w) C d \tau + w S d \tau + d w \cdot H_e;$$

$$\therefore d H = (1 - x) C d \tau + x S d \tau + H_e d x, \quad (35a)$$

which will be the heat absorbed by one pound of the entire substance under the conditions imposed. All the quantities in this equation have been determined except S.



To find S, let a pound of liquid (water, for instance) be evaporated at state A; $A\,D$ being the volume when the xth part of the pound is evaporated, neglecting the volume of the liquid; $A\,F$ the volume when the entire pound is evaporated, for which $x = 1$. Let state B be $d\tau$ higher than A, $B\,m$ the volume when the xth part is vapor, $m\,n$ be the arbitrary increment of increase of the xth volume, and $B\,E$ the volume of a pound of the vapor for the pressure $O\,B$, then $d x = m\,n \div B\,E$. Draw the adiabatics and join $n\,D$. It is proposed to find the heat absorbed along the path $n\,D$. In working around the area $A\,B\,n\,D\,A$ (or any other cycle) the heat absorbed minus the heat emitted in foot pounds will equal the area of the cycle.

We have

$$A\,D = x v,$$

$$\varphi_1 A\,B\,\varphi_2 = C d \tau \text{ (heat absorbed)},$$

$$\varphi_3 D\,A\,\varphi_1 = x H_e \text{ (emitted)},$$

$$\varphi_2 B\,m\,\varphi_4 = x H_e + \left(\frac{d H_e}{d \tau}\right) d \tau \text{ (absorbed)},$$

$$\varphi_4 m\,n\,\varphi_5 = d x \cdot H_e + \left[\frac{d x \cdot d H_e}{d \tau}\right] d \tau, \text{ (in which the}$$

last term being a difference of a lower order than the preceding term will be neglected), and we have

$$\varphi_4\, m\, n\, \varphi_5 = d\,x \cdot H_e \text{ (absorbed)}.$$

$\varphi_3\, D\, n\, \varphi_5 = d\,H$ (which may be absorbed or emitted depending upon the slope of $n\,D$; we will consider it as *emitted*, then if in any case it is *absorbed* the algebraic sign will change).

$$\frac{x\,H_e}{\tau}\,d\,\tau = A\,D \cdot d\,p = x\,v \cdot d\,p = \text{the area } D\,A\,B\,n\quad D$$

$$\therefore C\,d\,\tau + x\,H_e + \frac{x\,d\,H_e}{d\,\tau}\,d\,\tau + H_e\,d\,x - d\,H - x\,H_e = \frac{x\,H_e}{\tau}\,d\,\tau;$$

$$\therefore d\,H = C\,d\,\tau + x\left(\frac{d\,H_e}{d\,\tau} - \frac{H_e}{\tau}\right)d\,\tau + H_e \cdot d\,x; \qquad (136)$$

$$\therefore \frac{d\,H}{d\,\tau} = C + x\left(\frac{d\,H_e}{d\,\tau} - \frac{H_e}{\tau}\right) + H_e\,\frac{d\,x}{d\,\tau}; \qquad (137)$$

which is the specific heat of a fluid in which the $(1 - x\text{th})$ part is liquid and the xth part of it is vapor, the path being arbitrary.

If the weight of vapor remain constant during the change of temperature, then $d\,x = 0$. If the entire pound be dry saturated vapor during the change of state, then $x = 1$ and $d\,x = 0$, and $d\,H \div d\,\tau$ will be the specific heat of the vapor kept at the point of saturation throughout the change of state; and the resulting value will be S in equation (~~135~~ 135a);

$$\therefore \frac{d\,H}{d\,\tau} = S = C + \frac{d\,H_e}{d\,\tau} - \frac{H_e}{\tau} \qquad (138)$$

or in heat units

$$S = c + \frac{d\,h_e}{d\,\tau} - \frac{h_e}{\tau}* \qquad (139)$$

substituting S from (138) in (135a) will give (136).

For water $c = 1$, $h_e = 1436{\cdot}8 - 0{\cdot}7\,\tau$;

$$\therefore s = 1 - \frac{1436{\cdot}8}{\tau}, \qquad (140)$$

which will be negative for all values of τ less than 1436° F. above absolute zero, or 976° F. above the zero of Fahrenheit's scale. The negative value may be thus explained:— If saturated steam be expanded in a non-conducting cylinder, a portion of it will condense, giving up its

* For an analytical solution see Sir William Thomson's *Math. and Phys. Papers*, Vol. I., pp. 145–207; *Phil. Mag.* (1852), IV.; *Trans. R. Soc. Ed.*, 1851.

heat to the remainder of the steam, thus maintaining the temperature corresponding to the pressure of saturation; and if it be compressed in such a cylinder, heat must be abstracted if the pressure and temperature continually correspond to those of saturation. If heat be not abstracted in the latter case the steam will be superheated, and the temperature will exceed that corresponding to the pressure of saturated steam.

In regard to this Rankine said: "This conclusion (the liquefaction of steam) was arrived at contemporaneously and independently by M. Clausius and myself. Its accuracy was subsequently called in question, chiefly on the ground of experiments which show that steam after being wire-drawn, that is to say, by being allowed to escape through a narrow orifice, is superheated, or at a higher temperature than that of liquefaction at the reduced pressure. Soon afterward, however, Professor William Thomson proved that these experiments are not relevant against the conclusion in question, by showing the difference between the *free expansion* of an elastic fluid, in which all the power due to the expansion is expended in agitating the particles of the fluid, and is reconverted into heat, and the expansion of the same fluid *under a pressure equal to its own elasticity*, when the power developed is all communicated to external bodies, such, for example, as the piston of an engine" (*Misc. Sc. Papers*, p. 399).

Professor Clausius said: "The conclusion that the specific heat of saturated steam is negative was drawn by Rankine and by myself independently at about the same time (*Theory of Heat*, p. 135).*

100. Adiabatics of imperfect gases. This condition requires that $H = 0$, $\therefore dH = 0$ in equations (A), giving

* Both papers were read in February, 1850—Rankine's in Edinburgh, and Clausius' in Berlin.

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$$0 = K_v\, d\tau + \tau \left(\frac{dp}{d\tau}\right) dv,$$

$$0 = K_p\, d\tau - \tau \left(\frac{dv}{d\tau}\right) dp;$$

$$\therefore \frac{K_p}{K_v}\, dv = -\left(\frac{dv}{d\tau}\right)\left(\frac{d\tau}{dp}\right) dp.$$

In order to integrate the first of these, K_v, and $\frac{dp}{d\tau}$ must be known functions of τ and v. K_v not only depends upon the volume but is not a known function of τ. Even granting that its general expression is given by equation (105), its determination requires a knowledge of the equation of the fluid, and that can be known only empirically, and hence would apply only for the range of the experiments upon which the formulas were based. We have, however, found for carbonic acid gas, and for all other fluids investigated, that the specific heat at constant volume for a constant state of aggregation is, without a large error, constant within the range of ordinary experience; and similarly for K_p; hence, representing these by C_v and C_p, respectively, we have

$$\left.\begin{aligned} C_v\, d\tau &= -\tau \left(\frac{dp}{d\tau}\right) dv. \\ C_p\, d\tau &= \tau \left(\frac{dv}{d\tau}\right) dp. \\ \gamma\, dv &= -\left(\frac{dv}{d\tau}\right)\left(\frac{d\tau}{dp}\right) dp; \end{aligned}\right\} \qquad (141)$$

in which γ must be constant for the range through which the specific heats are considered constant. Assuming equation (4) as the general equation of fluids, and considering that

$$a_0 = \frac{b_0}{v}, \qquad a_1 = \frac{b_1}{v}, \qquad a_2 = \frac{b_2}{v}, \text{ \&c.,}$$

in which b_0, b_1, b_2 are constants to be determined by experiment, we have

$$p\,v = R\,\tau - \frac{b_0}{v} - \frac{b_1}{\tau\,v} - \frac{b_2}{\tau^2\,v} - \&c. \tag{142}$$

$$\therefore \left(\frac{d\,p}{d\,\tau}\right) = \frac{R}{v} + \frac{b_1}{\tau^2\,v^2} + \frac{2\,b_2}{\tau^3\,v^2} + \&c. = -M \text{ (say)},$$

$$\left(\frac{d\,v}{d\,\tau}\right) = \frac{R\,v + \frac{b_1}{\tau^2} + \frac{2\,b_2}{\tau^3} + \&c.}{2\,p\,v - R\,\tau} = N,$$

and (141) becomes

$$\left.\begin{aligned} C_v\,d\,\tau &= \tau\,M\,d\,v, \\ C_p\,d\,\tau &= \tau\,N\,d\,p, \\ \gamma\,d\,v &= \frac{N}{M}\,d\,p, \end{aligned}\right\} \tag{143}$$

which are the differential equations to the adiabatics for imperfect gases. From $(143)_2$ v can be eliminated by means of equation (142), resulting in an equation involving τ and p only; and τ from $(143)_3$ by means of the same equation; but the resulting equations will be too complex to admit of integration, and therefore the general finite equation to adiabatics is unknown.

It is customary to assume that the equation of the adiabatics for such superheated vapors as are used for engineering purposes, as steam, is of the same *form* as that for the sensibly perfect gases, at least, within the limits used in practice; and hence may be represented by the equation

$$p\,v^\gamma = c, \tag{144}$$

in which γ must be found for the particular substance, and the particular state of that substance.

To find γ for steam considered as a perfect gas, we found in Article 78 the volume of a pound of steam at 212° F. under the pressure of one atmosphere to be 26.5 cubic

feet; hence, if it followed the gaseous law down to 32°, the volume at the latter temperature would be

$$v_0 = 26.50 \div 1.366 = 19.39 \text{ cu. ft.};$$
$$\therefore p_0 v_0 = 19.39 \times 2116.2 = 41033;$$
$$\therefore \frac{p_0 v_0}{\tau_0} = 83.28.$$
$$K_p = 0.48 \times 778 = 373.44,$$
$$K_v = 373.44 - 83.28 = 290.16,$$
$$\therefore \gamma = \frac{K_p}{K_v} = 1.3, \text{ nearly}; \tag{145}$$

hence, the equation of the adiabatic for steam, considered as a sensibly perfect gas, will be

$$p\, v^{1.3} = p_1\, v_1^{1.3} \tag{146}$$

This value of γ is used for superheated steam at all temperatures.

But steam as used in the steam-engine is generally more or less saturated, for which case Rankine used $\frac{10}{9}$ for the *approximate* value of γ, so that for such cases the equation of the adiabatic will be

$$p\, v^{\frac{10}{9}} = p_1\, v_1^{\frac{10}{9}} \tag{147}$$

Rankine was the first writer to give even an approximate equation to the adiabatic of saturated steam. M. G. Schmidt, in his *Théorie des Machines à Vapeur*, 1861, assumed that steam comported like a perfect gas, and so assumed $\gamma = 1.4$, a value entirely without foundation, as shown by equations (145) and (147), and which that author later abandoned.

In 1863 Grashof reviewed the question, and found the mean value of $\gamma = 1.1354$.

Still later, Professor Zeuner, by a series of experiments in which the initial pressures varied from 1 to 4 atmospheres, final pressures from $\frac{1}{2}$ to 2 atmospheres, and in which the specific quantity of initial vapor (or the per cent of the fluid

in the cylinder that was vapor before cut-off) was 0.70, 0.80, 0.90; found results from which he concluded that:

The value of γ is dependent chiefly upon the initial specific volume of the vapor.

That it is nearly constant for the same initial state of the vapor for all the pressures observed from one to four atmospheres.

That the value of γ may be represented by the empirical formula

$$\gamma = 1.035 + 0.100\, x_1, \tag{148}$$

in which x_1 is the initial specific quantity of the vapor. This formula is limited to values of x_1 between 0.7 and 1 (*Théorie Mécanique de la Chaleur* (1869), (329–335). In this formula, if $x_1 = 0.76$, that is, if 24 parts in 100 of the fluid is initially water, it gives $\gamma = 1.111$, which is the constant value proposed by Rankine. If equation (148) can be extended to values of x_1 much less than 0.7, it appears that the adiabatic for saturated steam approximates more and more nearly to the isothermal of the perfect gas in which $\gamma = 1$; and for values of x_1 less than 0.50, the two curves will nearly coincide within the ranges of expansions used in ordinary practice. Hence the curve of adiabatic expansion of wet saturated steam approximates to that of the equilateral hyperbola.

But when we consider the complex nature of the problem —the temperature of the surrounding walls being modified by the nature of the metal, its thickness, its exposure externally; the time of the exposure internally depending upon the piston speed; rendering it practically impossible to realize exact adiabatic expansion—it is too much to expect any empirical formula to cover all the cases of approximate adiabatic expansion which might arise; and we conclude, as did Zeuner, that the empirical formula of Rankine, equation (147), is sufficiently exact for theoretical or practical purposes when the initial steam contains but little water.

To find the equation to the adiabatic for saturated vapor when liquid is present, make $d\,H = 0$ in equation (136), and it reduces to

$$-\frac{C\,d\,\tau}{\tau} = d\left(\frac{x\,H_e}{\tau}\right). \qquad (148a)$$

Integrating between initial and general limits gives

$$-\,C\,log\,\frac{\tau_1}{\tau} = \frac{x_1\,H_{e1}}{\tau} - \frac{x\,H_e}{\tau}$$

$$= x_1\,v_1\frac{d\,p_1}{d\,\tau_1} - x\,v\,\frac{d\,p}{d\,\tau}.$$

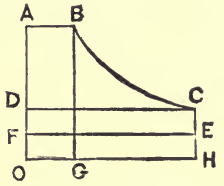


FIG. 43.

$$\therefore x\,v = \left(x_1\,v_1\frac{d\,p_1}{d\,\tau_1} + C\,log\,\frac{\tau_1}{\tau}\right)\frac{d\,\tau}{d\,p}. \qquad (149)$$

For steam C becomes J. By means of equation (86), if c and h_e be ordinary thermal units,

$$v\,\frac{d\,p}{d\,\tau} = \frac{H_e}{\tau} = J\,\frac{h_e}{\tau},$$

and (149) may be written

$$\frac{x\,h_e}{\tau} = \frac{x_1\,h_{e1}}{\tau_1} + J\,log\,\frac{\tau_1}{\tau}. \qquad (149a)$$

In this solution the specific volume of the liquid is neglected, since the volume of both the liquid and the vapor is essentially that of the vapor. In equation (149) $x_1\,v_1 = O\,G$, Fig. 43, will be the volume of one pound of the saturated steam and water at the beginning of expansion, and $x\,v$ the volume of the steam and water at any point of the expansion $B\,C$, corresponding to the temperature τ or pressure p. Eliminating $\frac{d\,\tau}{d\,p}$ by means of equation (82), and then τ by means of (81), the result will be the equation of the adiabatic $B\,C$; but the second member will be too

complex to be of practical value; and the approximate equation of Rankine (147) will be used instead. An important theoretical deduction may be made from the equation in its present form; thus, if the steam be dry at B, the point of cut off, x_1 will be unity, and making $x\,v = u$, we have with the aid of (86),

$$u = \frac{\tau\, v}{H_e}\left(J \log \frac{\tau_1}{\tau} + v_1 \frac{d\,p_1}{d\,\tau_1}\right); \tag{150}$$

$$\therefore \frac{v - u}{v} = 1 - \frac{\tau}{H_e}\left(J \log \frac{\tau_1}{\tau} + v_1 \frac{d\,p_1}{d\,\tau_1}\right), \tag{151}$$

which is positive for values of τ_1 less than 1436° F., the same limit that makes equation (140) negative. This shows that the volume of steam and water will be less than the specific volume, v, of steam only at the temperature τ. This is due to condensation, as stated in Article 99.

Let the initial volume of steam be *one cubic foot*, its weight will be $\frac{1}{v_1} = w_1$, and let $r = \frac{u}{v_1} =$ the *variable ratio of expansion*, then will equations (150) and (87) reduce to

$$r = \frac{\tau}{L}\left(w_1\, J \log \frac{\tau_1}{\tau} + \frac{L_1}{\tau_1}\right), \tag{152}$$

by means of which the ratio of expansion may be computed.

101. Condensers. A condenser consists of a vessel kept at a comparatively low temperature by means of cold water, for the purpose of condensing steam. In the jet condenser a liquid spray is forced *into* the vessel, and for the surface condenser the cold liquid circulates *about* the vessel or through tubes in the vessel, producing a cold surface. When the piston of the engine is very near the end of its stroke, a communication being made between the steam end of the cylinder and the condenser through the exhaust passage, the steam rushes into the condenser, and the greater

part of it is suddenly condensed—the pressure falling to two pounds per square inch, more or less. Using the subscript 2 to indicate the conditions at the end of the stroke, and 3 for those in the condenser at the end of the operation, and discarding the effect of molecular changes under varying pressures, thus assuming that the heat abstracted will be the difference in the heats in the initial and terminal strokes (which will be approximately correct), equation (148) will give,

$$H = J(T_2 - T_3) + x_2 v_2 \frac{H_{e2}}{v_2} - x_3 v_3 \frac{H_{e3}}{v_3}, \qquad (153)$$

for water and for the Fahrenheit scale, and is the heat abstracted from a pound of steam in reducing its temperature from T_2 to T_3 degrees.

The steam end of the cylinder will remain practically at constant volume during this change, and neglecting, as before, the specific volume of the water from which the steam is generated, and assuming that the volume of the space within which the change of temperature takes place is constant during the change, we have

$$x_2 v_2 = x_3 v_3 = u_2, \qquad (154)$$

and the preceding equation becomes

$$H = J(T_2 - T_3) + u_2 \left(\frac{H_{e2}}{v_2} - \frac{H_{e3}}{v_3} \right). \qquad (155)$$

In a continuously working engine a constant mass of vapor remains in the condenser at the end of each stroke, the amount condensed being equal to that exhausted, and H_{e3} may be neglected in (155), for which case we have

$$H = J(T_2 - T_3) + u_2 \frac{H_{e2}}{v_2} \qquad (155a)$$

The pressure of the vapor in the condenser determines its temperature, and that will be the inferior limit of temperature at which the steam will be worked.

EXERCISE.

Determine, approximately, the amount of water that must pass through the condenser of a steam-engine per pound of steam exhausted, having given $T_2 = 300°$ F. the temperature of the steam in the cylinder as it exhausts into the condenser, $x_2 = 0.90$ the fractional part of the steam and water in the cylinder that may be considered as pure saturated steam, the pressure in the condenser two pounds per square inch absolute, the water entering the condenser at 60° F., and leaving it at 100° F.

By means of a table of the properties of saturated steam, or by eqs. (78), (85) and (95), we find, using approximations to the larger numbers,

Temperature of the condenser for 2 lbs. per sq. in., $T_3 = 126$
From the problem, $T_2 = 300$

$$T_2 - T_3 = 174;$$

$$\therefore J(T_2 - T_3) = 174 \times 778 = 135400 \text{ ft.-lbs.}$$

Total heat of the steam at 300° will be $778 \times 1173 = 912600$
Heat in the water above 32°, $778 \times 270 = 210000$

Difference, $H_{e2} = 702600.$

The table gives, for the specific volume of the steam at 300°,

$$v_2 = 6.2 \text{ cu. ft.};$$

$$\therefore \frac{H_{e2}}{v_2} = 113000.$$

Total heat of steam at 126°, $778 \times 1120 = 871400$
Heat in the water above 32°, $778 \times 94 = 73100$

Difference, $H_{e3} = 798300$

The tabular value for v_3 is

$$v_3 = 172 \text{ cu. ft.};$$

$$\therefore \frac{H_{e3}}{v_3} = 4640.$$

These values in equation (155) give

$$H = 135400 + 604600 = 740000 \text{ ft.-lbs.}$$

The water supplied to the condenser being raised through $100 - 60 = 40$ degrees, the quantity required will be

$$q = \frac{740000}{778 \times 40} = 24 \text{ pounds, nearly.}$$

Equation (155*a*) gives 24.7 lbs.; that is, a condensing engine running with steam at 52 pounds gauge pressure will require about 25 pounds of water for the condenser for every pound of steam condensed if the temperature of the water be raised 40 degrees. If a greater difference of temperature of the water at arriving and leaving be allowed, it would require less water, or if the gauge pressure be higher, it will require more water for the same difference of temperature.

The numerical computation of (155) will be facilitated by a table of the latent heat of evaporation per cubic foot, since $H_{e_2} \div v_2 = L$.

102. Isodiabatic Lines. Let $C\,N$ and $B\,M$, Fig. 44, be any two isothermals cut by an arbitrary path $A\,D$. In passing from A to D a certain amount of heat will be absorbed, represented by the area between $D\,A$ and two adiabatics drawn from A and D respectively, as shown in Article 34. It is possible to find another path, $C\,B$, in working along which the same amount of heat will be emitted as was absorbed along $A\,D$. To prove this, conceive an indefinite number of isothermals between $C\,N$ and $B\,M$, and at the points of division with $A\,D$ draw adiabatics; then find a point near C, which call z, on the isothermal next below $C\,D$, such that when joined with C the area included between $z\,C$ and two adiabatics through z and C, respectively, will equal that between the corresponding pair at D. Proceed in this manner with the next isothermal, and so on to B; then will the area between $B\,C$ and the adiabatics through B and C respectively equal the area between $A\,D$ and the adiabatics through A and D respectively, which was to be proved. The lines $D\,A$ and $B\,C$ are called *isodiabatics* in reference to each other (Rankine's *Misc. Sc. Papers*, p. 345; *Steam-Engine*, p. 345).

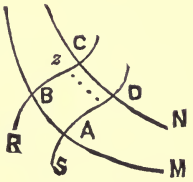


FIG. 44.

To find the analytical condition, conceive $C\,N$ and $B\,M$ to be consecutive isothermals, then the heat absorbed in passing from A to D will be, from equation (A),

$$d\,H = C_v\,d\,\tau + \tau\left(\frac{d\,p}{d\,\tau}\right)d\,v,$$

and that emitted along $C\,B$,

$$d\,H = C_v\,d\,\tau + \tau\left(\frac{d\,p_1}{d\,\tau_1}\right)d\,v_1;$$

which, according to the conditions of the problem, are to be equal, giving

$$\left(\frac{d\,p}{d\,\tau}\right)d\,v = \left(\frac{d\,p_1}{d\,\tau_1}\right)d\,v_1, \tag{156}$$

which relation is independent of the specific heat of the substance. For sensibly perfect gases we have

$$\left(\frac{d\,p}{d\,\tau}\right) = \frac{p}{\tau} = \frac{R}{v},$$

$$\left(\frac{d\,p_1}{d\,\tau_1}\right) = \frac{p_1}{\tau_1} = \frac{R}{v_1},$$

and by substituting above and integrating, we have

$$v = B\,v_1,$$

or

$$\frac{p}{p_1} = A, \textit{ a constant;} \tag{157}$$

that is, *the ratio of the pressures, or of the volumes, at the respective points where the successive isothermals cut the curves $A\,D$ and $B\,C$ must be constant.*

CHAPTER IV.

HEAT ENGINES—GENERAL PRINCIPLES.

103. Efficiency.—Heat engines, in practice, work in cycles, and when running under uniform conditions, the successive cycles will be identical, in which case the total effect will be that produced in one cycle multiplied by the *number* of cycles. It is, therefore, important to investigate the properties of one cycle.

The efficiency of a plant is the ratio of the work which the plant can produce to that of the energy supplied. Thus, if the plant consist of a furnace and engine, it is the ratio of the work it can do to the theoretical energy of the fuel supplied to the furnace.

The efficiency of an engine is the ratio of the work it can do to the energy of the heat absorbed.

In case of an hydraulic machine, it is the ratio of the work it can do to the theoretical energy of the waterfall.

The measure of the efficiency does not involve the magnitude of the machine, and, hence, is only an incidental element in proportioning the engine. If one pound of air when worked in a cycle will produce a given amount of work, two pounds will produce twice as much when worked in a similar cycle. The proportions of an engine having a given efficiency depend upon the amount of work to be done in one cycle.

104. Perfect elementary heat engine.—An engine receiving all its heat at one temperature and rejecting heat at one lower temperature, must pass through its series

of changes of pressure and volume according to Carnot's cycle. Such an engine is reversible. No such engine can be constructed or operated, but as it would give the highest theoretical efficiency of any engine working between the temperatures of the source and refrigerator, it serves as a theoretical standard of comparison, and is referred to as a *Perfect Elementary Heat Engine.*

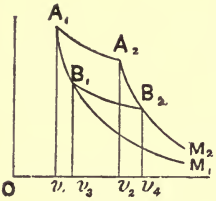


FIG. 45.

Let $A_1\ A_2\ B_2\ B_1$, Fig. 45, represent a Carnot's cycle, according to which the engine receives all its heat at the temperature τ_1, being the temperature of the isothermal $A_1\ A_2$; and rejects heat only at the temperature τ_2, being the temperature of the isothermal $B_1\ B_2$. Then will the heat absorbed in expanding from state A_1 to A_2 at the constant temperature τ_1 be, according to equation $(A)_1$, page 48, since $d\,\tau$ will be zero,

$$H_1 = 0 + \tau_1 \int_{v_1}^{v_2} \left(\frac{d\,p}{d\,\tau}\right) d\,v,$$

and the heat absorbed along the adiabatic $A_2\ B_2$ will be

$$0 = \int_{\tau_1}^{\tau_2} K_v\, d\,\tau + \int_{v_2}^{v_4} \tau \left(\frac{d\,p}{d\,\tau}\right) d\,v,$$

and the heat rejected along the isothermal $B_2\ B_1$,

$$-H_2 = -0 - \tau_2 \int_{v_1}^{v_2} \left(\frac{d\,p}{d\,\tau}\right) d\,v,$$

and along the adiabatic $B_1\ A_1$,

$$0 = -\int_{\tau_1}^{\tau_2} K_v\, d\,\tau - \int_{v_1}^{v_3} \tau \left(\frac{d\,p}{d\,\tau}\right) d\,v;$$

and the sum of these will give the heat transmuted into external work, since the cycle is complete; hence,

$$H_1 - H_2 = (\tau_1 - \tau_2) \int_{v_1}^{v_2} \left(\frac{d\,p}{d\,\tau}\right) d\,v. \qquad (158)$$

The efficiency, according to the preceding article, will be

$$E = \frac{H_1 - H_2}{H_1} = \frac{\tau_1 - \tau_2}{\tau_1} = \frac{T_1 - T_2}{T_1 + 460{\cdot}66}. \qquad (159)$$

Since equations (A) are general, and applicable to all substances, the result must be equally general; hence, *the efficiency of the perfect elementary engine depends only upon the highest and lowest temperatures between which it is worked, and is independent of the nature of the working substance.*

If iron, or any other solid, could be worked between the temperatures τ_1 and τ_2, according to Carnot's cycle, it would be just as *efficient* as if the substance were the most perfect gas. The range of volumes through which solids expand and contract is small, so that the work done in a cycle would be comparatively small, and the changes of temperature are so slow as to preclude the use of such substances in the construction of heat engines. But this fact does not affect the efficiency of the cycle.

The highest temperature at which the engine works cannot exceed that of the source, for it is an axiom that *heat cannot of itself flow from a hot body to one still hotter*, a principle stated by Clausius (*Theory of Heat*, p. 78).

Neither can it be worked at a lower temperature than that of the refrigerator, for it is held as an axiom that *a heat engine cannot be worked at a lower temperature than that of the coldest of surrounding bodies*, a principle stated by Thomson (*Math. and Phys. Papers*, p. 181). These axioms are the same in substance, and originally were stated independently by the respective authors.

If any of the heat absorbed is at a lower temperature than τ_1, while all is rejected at τ_2, the efficiency will be less

than if it were all absorbed at the higher temperature. To show this, let τ_3 be the constant temperature of the second source, then we would have

$$H_1 = \tau_1 \int_{v_1}^{v_2} \left(\frac{d\,p}{d\,\tau}\right) d\,v,$$

$$H_3' = \tau_3 \int_{v_2}^{v_3} \left(\frac{d\,p}{d\,\tau}\right) d\,v,$$

$$H_2 = \tau_2 \int_{v_1}^{v_3} \left(\frac{d\,p}{d\,\tau}\right) d\,v,$$

and the efficiency would be

$$\frac{H_1 + H_3' - H_2}{H_1 + H_3'},$$

which is less than the value of equation (159) so long as H_3' is less than H_1. A reversible engine has the highest efficiency for the heat utilized, and the *perfect elementary heat engine* has the highest efficiency of any engine working between the same limits of temperature.

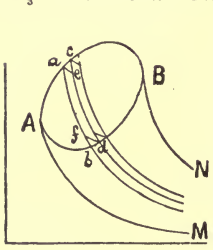


FIG. 46.

The principle of efficiency is applied in the same manner, whatever be the path of the fluid. Thus, if the cycle be $A\ a\ B\ d\ A$, Fig. 46, $A\ M$ and $B\ N$ adiabatics indefinitely extended, then, according to Article 34, we have

$$H_1 = MAaBN,$$

$$H_2 = MAdBN;$$

$$\therefore E = \frac{MAaBN - MAdBN}{MAaBN} = \frac{AaBdA}{MAaBN} = \frac{H_1 - H_2}{H_1}. \quad (160)$$

If the indicator card of the steam-engine were $A\ B\ C$ [illegible], Fig. 47, in which $A\ B$ is the steam line of constant tem-

perature, τ_1, $B\ C$ the expansion line of no transmission of heat extended until the pressure falls to that of the back pressure, $C\ J$ the back pressure line of constant temperature, τ_2, and $J\ A$ the compression line of no transmission of heat being made to pass through the initial state, A, then will the efficiency be

$$\frac{\tau_1 - \tau_2}{\tau_1} = \frac{T_1 - T_2}{\tau_1},$$

as before.

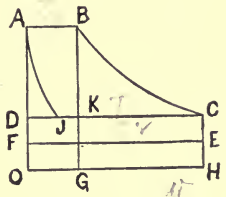


FIG. 47.

In Fig. 45 a constant quantity of air is supposed to remain in the cylinder of the engine during the changes forming the cycle, but in the steam-engine the heat is carried into the cylinder with the steam, so that the mass of steam increases with the stroke from A to B, Fig. 47; from B to C the mass remains constant; at C the exhaust is open, communicating with the refrigerator, and remains open until the piston reaches J, at which point the exhaust is closed, and the mass of steam remaining in the cylinder at J remains constant throughout the compression $J\ A$. At the completion of the cycle the fluid in the cylinder at the state A will have the initial pressure and volume, but since the changes of state are not effected with a constant mass of fluid the operation will not be that of a Carnot's cycle, and the above expression for efficiency will not be applicable.

The only theoretical mode of improving the efficiency of the elementary engine is to increase the range of temperatures between which it is worked.

It does not follow from this principle that different substances worked between the same limits of *pressure* will be equally efficient, for pressures are not proportional to the absolute temperatures, except for the sensibly perfect gases. If, however, the operation be in a Carnot's cycle, the temperatures corresponding to the pressures being found, equation (159) will be applicable.

It might be urged that some work would be expended in forcing the mass of steam into and out of the cylinder, thereby producing less external work than the same heat would do in case the changes were produced with a constant mass of fluid in the engine. In regard to this point, it is sufficient to observe that, if the argument be valid, the energy so absorbed is too insignificant compared with the heat energy of the fluid, to be considered.

Actual engines do not produce the indicator diagrams here assumed, and, hence, must be made the subject of special investigation.

EXERCISES.

1. In an ideal *elementary* engine working one pound of air, if the lowest pressure be that of one atmosphere, 2116.2 lbs. per square foot at B_2, Fig. 45, the absolute temperature of the refrigerator $\tau_2 = 550°$ ($T_2 = 89.34°$ F.), that of the source $\tau_1 = 950°$ ($T_1 = 489.34°$ F.), and the volume swept through by the piston during each single stroke 12 cubic feet; find the greatest and least volumes of the air in the cylinder, the power developed in one end of the cylinder during one cycle—or double stroke of the piston—the heat absorbed, and the efficiency.

To find the largest volume, v_4, we have, equation (3),

$$v_4 = \frac{53.21\ \tau_2}{p_4} = \frac{53.21 \times 550}{2116.2} = 13.83 \text{ cu. ft.}$$

To find p_2 and v_2, the adiabatic $A_2\,B_2$, equation (41), gives

$$p_2 = p_4 \left(\frac{\tau_1}{\tau_2}\right)^{\frac{\gamma}{\gamma-1}} = 2116.2 \left(\frac{19}{11}\right)^{3.463} = 14045 \text{ lbs.}$$

$$v_2 = v_4 \left(\frac{\tau_2}{\tau_1}\right)^{\frac{1}{\gamma-1}} = 13.83 \left(\frac{11}{19}\right)^{2.463} = 3.60 \text{ cu. ft.}$$

To find the least volume, v_1, the problem gives

$$v_4 - v_1 = 12;$$

$$\therefore v_1 = 13.83 - 12 = 1.83 \text{ cu. ft.}$$

And the isothermal $A_1 A_2$ gives, equation (3),

$$p_1 v_1 = 53.21 \times 950 = 50550 \text{ ft.-lbs.}$$

$$\therefore p_1 = 27630 \text{ lbs. per sq. ft.}$$

$$= 191.9 \text{ lbs. per sq. in.}$$

Similarly,

$$\frac{p_3}{p_4} = \frac{v_4}{v_3} = \frac{p_1}{p_2} = \frac{v_2}{v_1} = 1.97;$$

$$\therefore p_3 = 4162 \text{ lbs.}$$

$$v_2 = 7.03 \text{ cu. ft.}$$

The heat absorbed will be, equation (36),

$$p_1 v_1 \log \frac{v_2}{v_1} = 34207 \text{ ft.-lbs.}$$

The heat rejected will be

$$\frac{11}{19} \times 34207 = 19804 \text{ ft.-lbs.};$$

and, hence, the work done in one cycle will be

$$34207 - 19804 = 14404 \text{ foot-pounds,}$$

independent of the time.

The efficiency will be

$$\frac{14404}{34207} = 0.42,$$

according to which more than half the energy of the heat is rejected by the engine. The ratio of the greatest to the least volumes is

$$\frac{v_4}{v_1} = 7\tfrac{1}{2}, \text{ nearly,}$$

and of pressures,

$$\frac{p_1}{p_4} = 13.$$

2. In the preceding exercise, what must be the area of the piston in order to operate one pound of air between the limits assigned, the stroke of the piston being six feet.

3. In Exercise 1, if the engine make 20 revolutions per minute, what will be the horse-power developed on one side of the piston?

4. If, in Exercise 1, two pounds of air had been used, and the lowest pressure that of one atmosphere, the temperatures being the same as those given in the exercise, what would have been the greatest and least volumes of air, the volume swept through by the piston being 24 cu. ft.? Would the efficiency be the same? Would the work have been the same for the same volume swept through by the piston?

5. If in an *elementary* air engine the highest pressure be 150 pounds per square inch, the highest temperature 450° F., the lowest pressure 14.7 pounds per square inch, and lowest temperature 60° F., what will be the volume swept through by the piston per pound per stroke?

105. Regenerators consist of a chamber well filled with thin plates of metal so arranged as to present a large surface to the fluid and offer as little resistance to its passage as possible. The fluid, after escaping from the engine by passing through this chamber to the refrigerator, gives up a portion of its heat to the metal plates, the refrigerator finally absorbing the heat which is permanently rejected; after which, by passing back through the chamber and being at a lower temperature than during its former passage, it absorbs heat from the plates, thus requiring a less amount from the source in order to raise it to the required temperature. During the flow of the air from the cylinder the plates act as a refrigerator, by abstracting heat from the gas; but during the return of the gas they act in the opposite sense, and hence become *regenerators*.

If the temperature changed by insensible degrees in the regenerator, the efficiency would be unaffected, but such not being the case, they cause a loss of 5 or 10 per cent, even when well proportioned. Their great advantage consists in reducing the size of the cylinder, as will appear in the following exercise.

Assume that heat is absorbed at one temperature and rejected at another, as in the preceding case, but that the change of pressure from one isothermal to the other is effected at constant volume by passing the air through a regenerator, in which case the indicator diagram will be represented by Fig. 48. If τ_1 be the temperature of the isothermal $B\ C$, τ_2 of $A\ D$, C_v the specific heat at constant volume, the heat absorbed in passing from A to B to C, if the operation were reversible, would be, equation $(B)_1$, page 50,

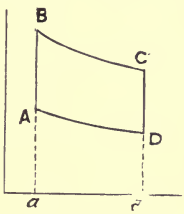


FIG. 48.

$$H_1 = C_v (\tau_1 - \tau_2) + R \tau_1 \int_{v_1}^{v_2} \frac{d v}{v},$$

heat rejected,

$$-H_2 = -C_v (\tau_1 - \tau_2) - R \tau_2 \int_{v_1}^{v_2} \frac{d v}{v};$$

$$\therefore H_1 - H_2 = R (\tau_1 - \tau_2) \int_{v_1}^{v_2} \frac{d v}{v} = U \text{ (say)},$$

which is the *mechanical energy expended*. But in determining the efficiency, the loss of heat in passing to and fro through the regenerator must be added to H_1; since that amount of heat must be drawn from the source and is not accounted for in the preceding value of H_1, and, representing this by the expression

$$n C_v (\tau_1 - \tau_2),$$

in which n is a fraction, $C_v = 0.169 \times 778 = 131$, we have for

$$\textit{the efficiency} = \frac{U}{H_1 + 131\ n\ (\tau_1 - \tau_2)},$$

in which n will be $\frac{1}{20}$, or $\frac{1}{10}$, or whatever fraction represents the heat lost by the regenerator.

EXERCISE.

In an air engine with a regenerator producing changes at constant volume, let p_1, v_1; p_2, v_2; τ_1, τ_2, be the same as in the first of the preceding exercises; determine the ratio of the pressures and volumes.

Considering the engine as perfect, the work done will be the same as in Exercise 1, page 164, for the expansion during the absorption of heat must be the same.

We will have,

$$p_1 = 27630, \qquad p_2 = 14045;$$
$$v_1 = 1.83 = v_3,\ v_2 = 3.6. = v_4, \text{ Fig. 48};$$

then

$$p_4 = p_2 \frac{\tau_2}{\tau_1} = \frac{11}{19} \times 14045 = 8132 \text{ lbs.},$$

$$p_3 = p_1 \frac{\tau_2}{\tau_1} = \frac{11}{19} \times 27630 = 15996 \text{ lbs.};$$

$$\therefore \frac{v_4}{v_1} = 1.97,$$

$$\frac{p_1}{p_4} = 3.40.$$

Comparing these results with the exercise referred to, it appears that the greatest volume in that case was nearly 4 times the greatest volume in this; hence, the volume of the cylinder with the regenerator, under the conditions imposed, need be only about one-fourth as large as without

106. Air engines have been made in which the changes of temperature have been effected at nearly constant pressure, and others in which the change takes place at nearly constant volume. These conditions require special forms of mechanism, which will be considered further on; but the work performed in a cycle may be computed from the indicator card, as in Articles 104 and 105.

107. Heat engines, whether of air, or steam, or other vapor, are assumed to transform a certain amount of heat energy into work independent of the mechanism involved. That is, aside from the friction of the engine wastes due to leaks and clearance, it is immaterial whether the engine be single-acting, double-acting, reciprocating, oscillating, rotary, disk, trunk, compound, or any other of the many forms of engines used; the work done will be the same in all the engines by the same fluid worked between the same limits of temperature.

Therefore, considering the engine as a *heat engine* only, we have only to consider the thermal changes produced in the working fluid during a complete cycle, involving the temperature of the feed water, and the initial and final temperatures in the cylinder. But as *a piece of mechanism*, the several forms have their mechanical advantages, which must be considered in the light of practical mechanism. All the details of the engine, such as the valve mechanism, the size of the bearings, the strength of the parts, compactness, etc., belong to constructive mechanism, and are treated of in works which consider these engines as machines.

In order to analyze a heat engine it is necessary to know the law according to which it receives and rejects heat; and since, in actual engines, all these laws are not known, assumptions in regard to them are made which are supposed to be approximately correct.

108. Steam-engine.—Steam in the cylinder works under such a variety of conditions that a complete analysis

requires the consideration of several hypotheses. Thus, steam may be superheated, in which case it will expand, approximately, like a perfect gas; or it may be saturated, in which case, by expanding without transmission of heat, it may remain constantly at the point of saturation; or by means of a steam jacket, the steam, by being constantly supplied with heat, may be considered as dry saturated steam. The curve of expansion may be too complex to be analyzed with great exactness. When steam enters the cylinder it may, and generally will, be hotter than the walls of the cylinder, and give up heat to the walls, thus reducing the pressure, even if it does not actually condense any of the steam; and as the steam becomes cooler by expansion, the walls of the cylinder will give up heat to the steam, thus raising its pressure at the latter part of the stroke. The water in the cylinder, if any, may also be re-evaporated. In either case the restored heat taking place near the end of the stroke does not compensate for the loss at the beginning, for the former can act through only a small part of the stroke, and as soon as the exhaust opens the restored heat escapes with the steam and is lost. Water in the cylinder may result from condensation of the saturated steam, as shown in Article 99, or it may be carried over from the boiler with the steam in the form of very small drops, as a spray. If the cylinder be jacketed the walls will be kept at a more nearly uniform temperature, and thus condensation in the cylinder be prevented, which is a great gain in the working of the engine. Condensation in the steam jacket does not affect the working of the engine. The refinements resulting from these numerous conditions are beyond the reach of analysis, because the laws governing their actions are unknown. This fact, however, is not seriously prejudicial to analysis, for the hypotheses assumed agree so nearly with actual cases as to give results, not only approximately correct, but so nearly correct as to be reliable in ordinary

practice. If, however, it becomes necessary to investigate these refinements, or so-called exceptional conditions, the problem of the steam-engine in this respect ceases to be *analytical*, and is essentially *empirical*. It must not be inferred that theory, even in this case, is useless, or is to be ignored, for it is only by *theory* that exceptions are known. Theory gives the first grand approximation to the truth, when, by comparing the results with actual cases, the defects in the theory become known, and thus, in turn, furnish the means of correcting or amending the original theory; after which a second and nearer approximation may be made, and so on, bringing the results of theory and of practice more nearly to an agreement. A consideration of these many conditions demands a special treatise; we will consider only a few special cases.

109. Ideal steam diagram.—Let $A\,B\,C\,E\,F$ be an *ideal* diagram of a steam-engine, $A\,B$ being the steam line at constant temperature and pressure, $B\,C$ the expansion line, $C\,E$ the fall in pressure at the end of the stroke, due to the sudden opening of the exhaust passage, $E\,F$ the back pressure line, $O\,H$ the line of absolute zero of pressures; then $O\,A = G\,B$ will be the total forward pressure up to the point of cut-off, $C\,H$ the forward pressure at the end of the stroke, $H\,E = O\,F$ the back pressure.

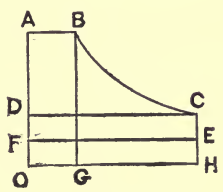


FIG. 49.

The admission line $A\,B$ is an isothermal of constant pressure, and in this respect resembles the case described in Articles 74 and 77, in which a liquid was evaporated under constant pressure *at* a constant temperature. In that case more and more liquid was evaporated, producing more and more steam, as the volume increased, while here more and more steam enters from the source as the volume increases. We might proceed, as with the perfect engine, to

find the heat absorbed and rejected throughout the cycle, and take the sum; but it is customary to find the results directly in terms of pressure and volume.

The *ideal* diagram is one freed from all irregular and disturbing causes, such as late opening for admission, initial expansion, wire drawing at the point of cut-off, slow closing of the port, irregularities in the expansion line $B\ C$, too early opening of the exhaust near C, a want of sufficient opening at E, and of compression near F; but such a diagram represents the greater part of the work done, and by applying *theory* to it a result approximately correct will be obtained.

110. Isothermal expansion.—Assume that the steam is superheated and the cylinder steam jacketed, then will the expansion line be nearly isothermal. Assume it to be exactly so, and let

$p_1 = O\ A = G\ B$, Fig. 49, be the initial pressure,

$p_2 = H\ C$, the terminal pressure,

$p_3 = H\ E$, the back pressure,

p = any ordinate to $B\ C$,

$v_1 = O\ G$ = the volume occupied by one pound of steam in the cylinder up to the point of cut-off,

$v_2 = O\ H$, the volume of one pound at full stroke,

$r = v_2 \div v_1$ = ratio of expansion,

v = any volume between G and H,

p_m = the mean *absolute* pressure, being such an ideal pressure as would if exerted throughout the stroke produce the same work as that of the variable pressures,

p_e = the mean *effective* pressure.

The equation of $B\ C$ will be, page 103,

$$p\,v = p_1\,v_1 = p\,r_1 = p_1\,r, \tag{162}$$

Saturated steam after being generated in a boiler is conceived to be superheated in a separate vessel.

We have

$$\text{area } G\ B\ C\ H = \int_{v_1}^{v_2} p\ d\ v = p_1\ v_1\ log_e \frac{v_2}{v_1}, \qquad (163)$$

and

$$O\ A\ B\ C\ H = p_1\ v_1\ (1 + log_e\ r);$$

also,

$$p_m\ v_2 = O\ A\ B\ C\ H;$$

$$\therefore p_m = \frac{p_1}{r}\ (1 + log_e\ r;) \qquad (164)$$

$$\therefore \frac{p_m}{p_1} = \frac{1 + log_e\ r}{r}, \qquad (165)$$

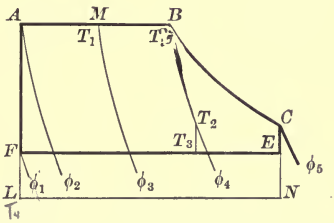
$$p_e = p_m - p_2. \qquad (166)$$

The effective energy exerted by one pound of steam against the piston

$$= A\ B\ C\ E\ F\ A = U = (p_m - p_2)\ v_2. \qquad (167)$$

To find the heat absorbed per pound of steam, let **H** = the heat absorbed.

It may be represented by a diagram thus: Let L be the initial state of the feed water as to pressure and temperature, T_4; A the state at the temperature of boiling, T_1; M the state of dry saturated vapor at the pressure p_1 and temperature T_1; B the state of the superheated steam at the pressure p_1 and temperature T_5; $B\ C$ the isothermal expansion curve, then will the temperature at C be T_5, the same as at B. (The temperatures marked T_2 and T_3 are for use in the two following exercises.)



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Through $L\ A\ M\ B$ and C draw adiabatics, then $H_1 = \varphi_1\ L\ A\ \varphi_2 = C\ (T_1 - T_4)$ = the heat absorbed by the liquid before boiling = $778\ (T_1 - T_4)$; $H_2 = H_e = \varphi_2\ A\ M\ \varphi_3 = 1117830 - 544.6\ \tau_1$, (Eq. [78]); $H_3 = \varphi_3\ M\ B\ \varphi_4 = K_p\ (T_5 - T_1)$ = the heat absorbed in producing superheating = $0.48 \times 778\ (T_5 - T_1) = 373\ \tau_5 - 373\ T_1 - 171953$; $H_4 = \varphi_4\ B\ C\ \varphi_5 =$ *the latent heat of expansion*, which equals the work done during isothermal expansion of a gas considered perfect = $p_1\ v_1\ log\ r$, Equations (36), (163), which by (164) becomes

$$p_1\ v_1 \left(\frac{r\ p_m}{p_1} - 1\right).$$

From equation (162)

$$373\ \tau_5 = 4\tfrac{1}{2}\ p_1\ v_1 \text{ nearly.}$$

$$\therefore \mathbf{H} = H_1 + H_2 + H_3 + H_4 = \varphi_1\ L\ A\ B\ C\ \varphi_5,$$

$$= 694816{\cdot}4 + p_1\ v_1\left(\frac{r\ p_m}{p_1} + 3\tfrac{1}{2}\right) - 140\ T_1 - 778\ T_4. \quad (171)$$

EXERCISE.

1. Let $p_1 = 100 \times 144 = 14400$, $T_5 = 450°$ $r = 10$, $p_3 = 2\tfrac{1}{2} \times 144 = 360$ lbs.; $T_4 = 110°$.

Find

$$T_1 = 327.6°,$$
$$\tau_5 = 910.66°,$$
$$p_1\ v_1 = 75922;$$
$$v_1 = 5.27;$$
$$r\ v_1 = 52.7;$$
$$p_m = 4756;$$
$$p_e = 4396;$$
$$U = 231757;$$

which is the effective work done by one pound of the steam against the piston; then, (171),

$$\mathbf{H} = 1132730 \text{ ft.-lbs.},$$

which is the heat expended per pound of steam in the cylinder.

Pressure equivalent to that heat—

$$p_h = \frac{\mathbf{H}}{v_2} = 21364 \text{ lbs.},$$

which is such an ideal pressure that if it worked against the piston while it swept through the same volume as when driven by the one pound of steam, it would do an amount of work equal to the entire energy of the heat expended.

Efficiency of the steam—

$$\frac{U}{\mathbf{H}} = \frac{p_m - p_3}{p_h} = \frac{231517}{1113350} = \frac{4756 - 360}{21364} = 0.205. \quad (171a)$$

111. Adiabatic expansion *of dry saturated steam.* First, assume the approximate law

$$p\ v^{\frac{10}{9}} = p_1 v_1^{\frac{10}{9}} = \textit{constant}. \quad (172)$$

The work during expansion will be, Fig. 49,

$$G\ B\ C\ H = \int_{v_1}^{v_2} p\ d\ v = p_1\ v_1 \left(9 - 9\ r^{-\frac{1}{9}}\right), \quad (173)$$

and the total work per pound,

$$O\ A\ B\ C\ H = p_1\ v_1 \left(10 - 9\ r^{-\frac{1}{9}}\right).$$

Terminal pressure, equation (172)—

$$p_2 = \frac{p_1}{r^{\frac{10}{9}}}. \quad (174)$$

Mean total forward pressure—

$$p_m = \frac{O\ A\ B\ C\ H}{v_2} = p_1 \left(\frac{10}{r} - \frac{9}{r^{\frac{10}{9}}}\right). \quad (175)$$

Mean effective pressure—

$$p_e = p_m - p_3 = p_1 \left(\frac{10}{r} - \frac{9}{r^{\frac{10}{9}}}\right) - p_3. \quad (176)$$

Work done per POUND *of steam—*

$$U = p_e v_2 = p_1 v_1 \left(10 - 9\ r^{-\frac{1}{9}}\right) - p_3 v_2. \quad (177)$$

Work done per CUBIC FOOT *of steam admitted—*

$$\frac{U}{v_1} = r\,p_e = p_1\left(10 - 9\,r^{-\frac{1}{9}}\right) - r\,p_3. \quad (178)$$

Heat expended per pound of steam admitted—

This will be the heat supplied to the water per pound above the temperature of the feed water *plus* the latent heat of evaporation, and is given by equation (93), which in the present notation becomes

$$\mathbf{H} = J\,(\tau_1 - \tau_4) + H_{e1}.$$

Eq. (78), $$= 778(T_1 - T_4) + 867003 - 544.6\,T_1. \quad (179)$$

Heat expended per *cubic foot* of steam admitted—

$$\frac{\mathbf{H}}{v_1} = 778\,w_1\,(T_1 - T_4) + \mathrm{L}, \text{ (Art. 78).} \quad (180)$$

Efficiency of the steam—

$$\frac{U}{\mathbf{H}}. \quad (181)$$

EXERCISE.

Let $p_1 = 14400$ lbs.; $r = 10$; $p_3 = 360$ lbs.; feed water, 110° F., as in the preceding exercise.

Then, omitting fractions of temperature after τ_1,

τ_1,	equation	(80),	$= 788.26°$, using p_1; $\therefore T_1 = 327.60°$.
p_2,	"	(174),	$= 1115$ lbs.
τ_2,	"	(80),	$= 640°$, using p_2; $\therefore T_2 = 180°$.
τ_3,	"	(80),	$= 590°$, " p_3; $\therefore T_3 = 134°$.
T_4			$= 110°$.
v_1,	"	(89),	$= 4.36$ cu. ft.
v_1,	"	(86),	$= 4.37$ " "
v_2,	"	(172),	$= 43.7$, or $10\,v_1$.
p_m,	"	(175),	$= 4363$ lbs. per sq. ft.
p_e,	"	(176),	$= 4003$ " " " "
U,	"	(177),	$= 174931$ ft.-lbs.
$\mathbf{H}$,	"	(179),	$= 857706$ " "

Efficiency—

$$\frac{U}{\mathbf{H}} = 0.204. \qquad (181a)$$

112. Adiabatic expansion of saturated steam *according to the theoretical law.* If the steam initially have x_1 part of moisture, use equation (k) or (m), p. 192. If initially dry, Fig. 49 and equations (150) and (86) give

$$A\,B\,C\,D = \int_{p_2}^{p_1} u\,d\,p = \int_{\tau_2}^{\tau_1} \left(J\,log_e \frac{\tau_1}{\tau} + \frac{H_{e1}}{\tau_1} \right) d\,\tau$$

$$= J\left[\tau_1 - \tau_2 \left(1 + log_e \frac{\tau_1}{\tau_2} \right) \right] + \frac{\tau_1 - \tau_2}{\tau_1} H_{e1}. \qquad (182)$$

For the work *per pound* of steam working full cycle,

$$U = A\,B\,C\,E\,F = J\left[\tau_1 - \tau_2 \left(1 + log_e \frac{\tau_1}{\tau_2} \right) \right] +$$

$$\frac{\tau_1 - \tau_2}{\tau_1} H_{e1} + (p_2 - p_3)\, v_2. \qquad (183)$$

The heat expended per pound of steam admitted to the cylinder will be the same as in the preceding Article, or

$$\mathbf{H} = J\,(T_1 - T_4) + H_{e1}. \qquad (184)$$

The efficiency will be

$$\frac{U}{\mathbf{H}}. \qquad (185)$$

[Messrs. Gantt and Maury determined the *Efficiency of Fluid Vapor Engines* according to this hypothesis—using these equations—for Water, Alcohol, Ether, Bisulphide of Carbon, and Chloroform (*Thesis*, Stevens Institute of Technology, 1884; *Van Nostrand's Engineering Magazine*, 1884 (2), pp. 413–432)].

EXERCISE.

Let $p_1 = 14400$ lbs.; $p_3 = 360$ lbs.; $T_4 = 110°$ F., as in the preceding exercise, and $p_2 = 1115$ lbs., as found in that exercise.

If the ratio of expansion were given, p_2 could be found only by a tedious approximation; therefore, we have assigned the final pressure.

We have,

$\tau_1 = 788.26°$; $\therefore T_1 = 327.66°$, as before.
$\tau_2 = 640°$; $\therefore T_2 = 180°$, " "
$\tau_3 = 590°$; $\therefore T_3 = 134°$, " "
$\tau_4 = 570°$; $\therefore T_4 = 110°$, " "
$p_1 = 14400$ lbs. " "
$p_2 = 1115$ lbs. " "
$v_1 = 4.375$ cu. ft. " "

$$v_2 = \frac{H_{e2}}{\tau_2 \left(\frac{dp}{d\tau}\right)_2} = 48.40, \text{ Eq. (86)},$$

$$r = \frac{v_2 \tau_2}{v_1 H_{e2}} \left(778 \log_e \frac{\tau_1}{\tau_2} + \frac{H_{e1}}{\tau_1}\right) = 9.55, \text{ Eq. (152)},$$

$$u_2 = \frac{v_2 \tau_2}{H_{e2}} \left(778 \log_e \frac{\tau_1}{\tau_2} + \frac{H_{e1}}{\tau_1}\right), \text{ Eq. (150)},$$

$$= r v_1 = 41.76,$$

$$v_2 - u_2 = 6.64.$$

$U = 171507$ ft.-lbs., Eq. (183).

The preceding exercise gives,

$$\mathbf{H} = 857706 \text{ ft.-lbs.}$$

Efficiency of fluid—

$$\frac{U}{\mathbf{H}} = \frac{171507}{857706} = 0.200. \qquad (185a)$$

Steam condensed due to expansion only—

$$\frac{v_2 - u_2}{v_2} = 0.137,$$

or nearly 14 per cent.

Mean effective pressure—

$$p_e = \frac{U}{u_2} = \frac{171507}{41.76} = 4170 \text{ lbs.}$$

Mean total forward pressure—

$$p_m = 4170 + 360 = 4530 \text{ lbs.}$$

It will be seen that there is little or no advantage in using the exact, but more complex, formulas of this Article over the approximate ones of the preceding Article.

The efficiencies found in the three preceding cases are :—

For superheated steam, expanding isothermally (171*a*) 0.205
For saturated steam, expanding adiabatically, approximate law (181*a*) 0.204
" " " " " " theoretical law (185*a*) 0.200

The effect on the efficiency by superheating is too small to be of practical importance. As this fact appears to be contrary to the popular opinion, it is well to observe that the superheated steam in Article 109 is not used in the most economical manner ; for a much larger amount of heat is thrown away at the end of the stroke than in the example of saturated steam, so that if it were utilized in heating feed water, or worked in another engine, or used for any other useful purpose, the efficiency of the plant would be increased. Or if it had been expanded down to that of the terminal pressure of the other cases, $p_2 = 1115$ lbs., it would have shown a greater efficiency ; but to accomplish this result the ratio of expansion must be greater, other data being the same. These considerations have reference to the *efficiency of the fluid* only, but in considering the *efficiency of the plant*, the size and cost of the engine enter as elements of the problem. Thus, to do the respective works, 231757 and 174931, deduced in two of the preceding exercises, with two engines making the same number of revolutions in the same time, according to the conditions assumed, the volume of the cylinder of the one supplied with superheated steam must be larger than that supplied with saturated steam in the ratio of the volume of a pound

of superheated steam at admission to that per pound of saturated steam, or, as

$$\frac{5.27}{4.375} = 1.20;$$

but the ratio of the works done will be

$$\frac{231757}{174931} = 1.32;$$

hence, per cubic foot of the cylinder capacities the former engine will do

$$\frac{1.32}{1.20} = 1.10 \text{ times}$$

the work of the latter.

The engine using isothermal expansion and doing 231757 foot-pounds of work per pound of steam, if it uses the pound per minute, will do

$$\frac{231757}{33000} = 7.02$$

horse-powers per pound of steam; and, per hour, it will require

$$\frac{1980000}{231757} = 8.54 \text{ pounds}$$

per horse-power. The engine which expands adiabatically, doing 174931 foot-pounds of work, would require

$$\frac{1980000}{174931} = 11.32 \text{ pounds}$$

per horse-power per hour. These results are for perfect conditions, no allowance having been made for wastes, clearance, or initial condensation of steam. It is a very good plant that does not consume more than seventeen pounds of feed water per indicated horse-power per hour, although reliable records of some good tests show less than this amount. Some multiple expansion engines have been reported as

consuming about thirteen pounds, as determined from the indicator card, but that mode of determining the weight of steam does not allow for the condensation of steam. The only reliable way is to *weigh* the water used. Thirty to forty pounds is more common in practice.

The heat of combustion of a pound of pure carbon is 14500 B.T.U., and if it could all be utilized for the purpose it would evaporate $14500 \div 966 = 15$ pounds of water at and from 212°; hence, if the feed water be at 110° F. and boiling point at 327° F., as in the two preceding exercises, it would, according to the table on page 112, evaporate $15 \div 1.14 = 13.15$ pounds; and to develop one I.H.P. per hour it would require

$$11.32 \div 13.15 = 0.861 \text{ pounds}$$

of coal. This does not allow for waste in producing steam. If the efficiency of the furnace be 0.70, it would require $0.861 \div 0.70 = 1.31$ pounds of coal.

Case of no expansion. In many simple direct-acting steam pumps, the full pressure of steam is maintained throughout the stroke. For this case $r = 1$ in equation (177), and the indicated work will be

$$U = (p_1 - p_3)\, v_1, \tag{185b}$$

when v_1 is the volume of a pound of the vapor at the pressure p_1. The work done during the forward pressure will be the external work performed during evaporation at the pressure p_1, and is sometimes called *the external latent heat of vaporization.* That part of the apparent latent heat which performs disgregation work will be lost at the exhaust.

The volume of the cylinder, the piston making n single strokes per minute for m horse-powers, will be

$$V = \frac{33000\, v_1\, m}{n\, U} \text{ cu. ft.} \tag{185c}$$

Water consumed per indicated horse-power per hour,

$$W = \frac{33000 \times 60}{U} \text{ pounds.} \qquad (185d)$$

EXERCISES.

1. In a direct-acting steam pump, let the uniform gauge pressure be 70 pounds and back pressure 16 pounds, feed water 60° F.; required the work done per pound of steam, efficiency, volume of the cylinder for one horse-power if there be 50 double strokes per minute, and the water consumed.

$p_1 = 70 + 14.7 = 84.7$,
$p_1 - p_2 = 68.7$,
$r_1 = 775.6$, Eq. (81); or, $T_1 = 315°$ F.;
$v_1 = 5.14$, Eq. (86) or (89);
$U = 68.7 \times 144 \times 5.14 = 50849$ ft. lbs., Eq. (185*b*);
$W = 38.9$ lbs. per hour, Eq. (185*d*);
$V = \frac{1}{33}$ cu. ft. $= 52.4$ cu. in., Eq. (185*c*);
$\mathbf{H} = 893844$, Eq. (179);
$E = 0.057$, Eq. (185),

or, the theoretical efficiency of the fluid will be about 5.7 per cent.

In actual practice the loss from condensation and radiation in these pumps is considerable, and the clearance is not only relatively large but somewhat irregular—especially in the smaller sizes, and it is found that the water consumption ranges from 75 to 125 pounds per horse-power per hour, with the possibility of being outside these limits in either direction. The mean average efficiency for the *fluid* admitted (steam and water) will be for small pumps of this class about $\frac{1}{3}$ of the theoretical, or,

$E = 0.019$, approximately,
and, $U = 17000$ ft. lbs., approximately,
also, $W = 120$ lbs. of water per horse-power per hour.

But the size of the cylinder need not be correspondingly increased, for the condensed steam will occupy but little volume.

The efficiency of the furnace, boiler and connections may be taken at 50 per cent, giving for the entire plant

$$E' = 0.0095,$$

or about 1 per cent of the theoretical heat of the fuel burned in the furnace.

It has been found by actual measurements that the average duty (or the work which 100 pounds of coal can do) in direct-acting pumps feeding 75 to 100 horse-power boilers, with coal of good quality, may, in the absence of direct experiment, be taken as 10000000 foot-pounds. This is 100000 foot-pounds per pound of coal, or $100000 \div 778 = 128.5$ thermal units, which is about $\frac{1}{100}$ of the heat of combustion of the average of commerical coal. The efficiency of such a plant, then, is actually about 1 per cent of the heat in coal of good quality. Such a plant will require from 9 to 15 pounds of coal per indicated horse-power per hour.

2. In the preceding Exercise, if the stroke be five inches, what will be the diameter of the cylinder?

3. If, in a direct-acting steam pump, the gauge pressure be 100 pounds, back pressure 16 pounds, feed water 90° F., find the efficiency of the fluid, and compare the result with that in Exercise 1.

4. If, in Exercise 3, the gauge pressure be 40 pounds, required the efficiency of the fluid.

5. Explain the several causes of the loss of the 99 per cent (more or less) of the heat of combustion as found in these Exercises. What effect has the temperature of the feed water upon the efficiency?

112a. General equations of vapor engines. —Consider only one pound of fluid in the cylinder, and let $B\ C$ be the curve of saturation, and $E\ F$ any adiabatic in which there is only a fraction of the pound of vapor throughout the expansion.

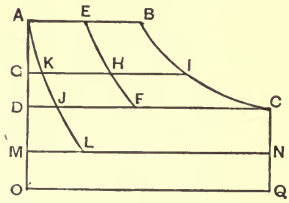

FIG. 50*a*.

$A\ B$, Fig. 50*a*, will represent the volume of a pound of vapor at the absolute pressure $O\ A = p_1$ and absolute temperature τ_1, $G\ I$ the volume at the absolute temperature τ and pressure $O\ G = p$.

Let $x_1 = A\ E \div A\ B$ = the fractional part of the fluid at the state E that is vaporized,

$v_1 = A\ B$, $x_1\ v_1 = A\ E$,

$x = G\ H \div G\ I$,

$v = G\ I$, volume corresponding to the pressure $O\ G$,

$x\ v = G\ H$,

c, the specific heat of the liquid,

h_e, the latent heat of evaporation at temperature τ in ordinary heat units, which will be

h_{e1} at temperature τ_1.

Then will the equation of the adiabatic $E\ F$ be, Eqs. (86) and (149),

$$G\ H = x\ v = \left(c\ log_e \frac{\tau_1}{\tau} + \frac{x_1\ h_{e1}}{\tau_1}\right) \frac{\tau\ v}{h_e}, \qquad (a)$$

which may be put under the more symmetrical form

$$\frac{x h_e}{\tau} + c\ log_e \frac{\tau}{\tau_0} = \frac{x_1\ h_{e1}}{\tau_1} + c\ log_e \frac{\tau_1}{\tau_0} = a\ constant, \qquad (b)$$

in which τ_0 is any arbitrary temperature. Since the vapor is to be continually saturated, this equation is limited to the conditions that x_1 must not be negative, and must be less than unity, and at the same time x, for any amount of expansion, must be less than unity.

Let subscript $_2$ be used for the terminal state F, then

$$\frac{x_2 h_{e2}}{\tau_2} + c \log_e \frac{\tau_2}{\tau_0} = \frac{x_1 h_{e1}}{\tau_1} + c \log_e \frac{\tau_1}{\tau_0}.$$

The difference between the initial and terminal weights of vapor will be

$$x_1 - x_2 = x_1 - \left(c \log_e \frac{\tau_1}{\tau_2} + \frac{x_1 h_{e1}}{\tau_1} \right) \frac{\tau_2}{h_{e2}}, \qquad (c)$$

and this may be negative, zero, or positive. We will designate those vapors whose specific heats are negative as "steam-like vapors," and those which are positive as "ether-like vapors," *steam* and *ether* being typical of their respective classes.

If the fluid be water, then $c = 1$, and let $x_1 = 0.436$ at $\tau = 800°$ F. (absolute), $h_e = 1436.8 - 0.7\,\tau$. Then equation (a) gives

for $\tau = 900°$,	$x = 0.404$,	$\tau = 600$,	$x = 0.450$,
$\tau = 800°$,	$x = 0.436$,	$\tau = 500$,	$x = 0.436$,
$\tau = 700°$,	$x = 0.450$,	$\tau = 400$,	$x = 0.407$,
$\tau = 650$,	$x = 0.453$,	$\tau = 200$,	$x = 0.277$;

from which it appears that steam *increased* with the expansion as the temperature fell from 900° to 650°, or from 340° to 190° on the Fahrenheit scale; and after that it decreased continually with the temperature. This change of the weight of steam can take place only by the evaporation of water initially in the presence of the vapor, and by condensation later in the expansion. The converse is also true, that if, in the initial state, only a fraction of the fluid be vapor, the liquid may at first be evaporated by adiabatic compression, but it may reach a state beyond which *it will be condensed by adiabatic compression.* Thus, in the example above given, if at 600° F. (absolute) 45 per cent. of the fluid be vapor, it will increase to 45.3 per cent., after which it will *condense indefinitely with adiabatic compression.*

If at 650° there be 45.3 per cent. of steam, the vapor will condense both by adiabatic compression and expansion from that state.

This may be illustrated by the annexed diagram, Fig. 50*b*, in which the relations are greatly exaggerated. Let *D E F* represent successive states of constant steam weight, and *A B C* an adiabatic of part liquid and vapor. These curves may intersect each other at two points *a* and *b*; above *a* the weight of vapor in the adiabatic will be less than at *a*, and the adiabatic will lie to the left of *D E F*, and below *b* it will lie below the curve of constant steam weight. The adiabatic is less curved than the curve of constant steam weight.

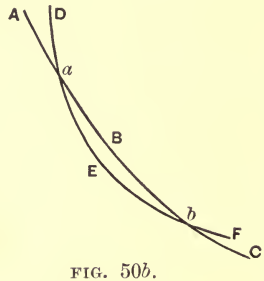

FIG. 50*b*.

To find the minimum weight of vapor such that, by continued compression of steam-like vapor, the liquid will be continually evaporated.

In equation (*b*) first find the value of τ that will make the left member a minimum when $x = 1$. Neglecting all powers of τ above the first in the latent heat of evaporation, Regnault's experiments give

$$\frac{h_e}{\tau} = \frac{a}{\tau} - b,$$

where a and b are constants depending upon the particular fluid. Using this value, it will be found that the required function is a minimum for

$$\tau = \frac{a}{c};$$

that is, τ will be near the "temperature of inversion," which, in the case of steam, is about 1436° F. (absolute), or 976° F. actual. Since the law of the latent heat of evapo-

ration here given is not exact, and, even if it were, Regnault's experiments would not warrant the extension to such high temperatures, we will discard fractions, and treat the entire number, 1436, as if it were exact. Since the adiabatic law is not applicable above this state, the maximum condensation by adiabatic expansion will be found by beginning at this state and expanding down to the required temperature. In equation (*c*), letting $x_1 = 1$, $\tau_1 = 1436$, $h_{e1} = 1436 - 0.7\ \tau$, $c = 1$, then

$$1 - x = 1 - \frac{2.3026\ log_{10}\frac{1436}{\tau} + 0.3}{\frac{1436}{\tau} - 0.7.}$$

	Abso. Temp.	Per cent. of Steam.	Per cent. of Water.	Temp. Deg. F.
If	$\tau = 800$,	$x = 0.808$,	$1 - x = 0.192$,	340.
	$= 700$,	$x = 0.753$,	$1 - x = 0.247$,	240.
	$= 672$,	$x = 0.725$,	$1 - x = 0.265$,	212.
	$= 600$,	$x = 0.692$,	$1 - x = 0.308$,	140.

It thus appears that if 72½ per cent. of the fluid be saturated steam, or 26½ per cent. of it be water at 212° F., the steam will condense continually by adiabatic expansion, or the water be continually evaporated by adiabatic compression. If there be less than twenty-six per cent. of water at 212°, the water will all become evaporated before the temperature reaches the critical temperature, and, after passing that state, compression will produce superheating. Every adiabatic having more than 72½ per cent. of steam at 212° is tangent to some curve of constant steam weight; and hence, with the exception of the adiabatic tangent to the curve of saturation, will have a state of maximum steam weight, at which point the curves of constant steam weight and the adiabatic will have a common tangent. From this state condensation of steam will result from compression as well as from expansion. The adiabatic which is tangent to

the curve of saturation passes through the state of the temperature of inversion.

According to the preceding table, at $\tau = 672°$, if $72\frac{1}{2}$ per cent. is steam, compression will produce evaporation up to 1436°. If at $\tau = 672°$ we assume 70 per cent. of steam, we find the following results:

$\tau =$ 672,	$x = 0.70$.	$\tau =$ 1100,	$x = 0.849$.
$\tau =$ 700,	$x = 0.73$.	$\tau =$ 1200,	$x = 0.855$.
$\tau =$ 800,	$x = 0.76$.	$\tau =$ 1250,	$x = 0.860$.
$\tau =$ 900,	$x = 0.794$.	$\tau =$ 1300,	$x = 0.859$.
$\tau =$ 1000,	$x = 0\ 825$.	$\tau =$ 1400,	$x = 0.84$.

It will be seen that the amount of steam will be a maximum at 1250° F. absolute.

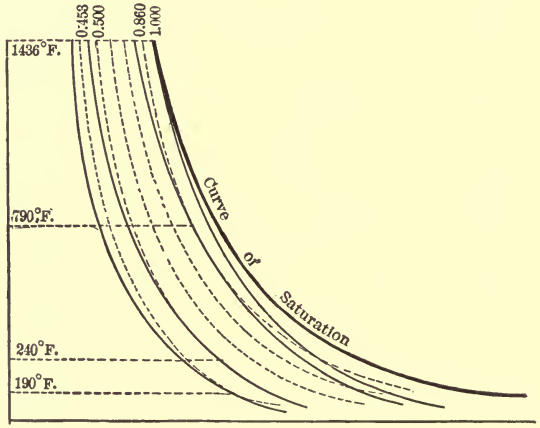

FIG. 50*c*.

In Fig. 50*c* the dotted lines are curves of equal steam weights, and the full lines—except the curve of saturation—are adiabatics, one of which is tangent to the curve of saturation; another tangent to the curve whose constant steam weight is 86 per cent., the point of tangency being at the

temperature of 790° F.; another is tangent to the curve of 50 per cent. of steam at 240° F.; and the fourth tangent to the curve of 45.3 per cent. of steam at 190° F. absolute.

In order to show the properties on a small scale, it is necessary to exaggerate the relations, thus distorting what would be the correct figure.

An examination of ether will show that the results here deduced for steam are not necessarily applicable to other vapors. In "ether-like vapors" the temperature of inversion is below ordinary temperatures; and for such if $x_1 = 1$, condensation will result from adiabatic compression for temperatures above that of inversion. Thus, for ether, omitting terms above the first power of τ, we have from Regnault's experiments,

$$h_e = 93.3214 + 0.3870\ \tau.$$
$$c = 0.517.$$

Hence, from equation (139), page 147,

$$s = 0.517 - \frac{93.32}{\tau} = \text{specific heat of the saturated vapor.}$$

If $s = 0$, then $\tau = 180°$ (absolute), or $-280°$ F.; and this is the temperature of inversion. Assuming any temperature above this, as $\tau_1 = 520°$, and $x_1 = 1$ in equation (a), then

$$x = \frac{0.5664 - 2.3026\ log_{10}\frac{\tau}{520}}{\frac{93.3214}{\tau} + 0.3870}.$$

From this it appears that x will diminish as τ increases, and finally become zero for $\tau = 915°$, nearly.

There appears to be no proportion of vapor to liquid such that they will be the same at two different states on an adiabatic, as has been found for steam. It may be shown that for any value of x_1, x will decrease as τ increases, showing

that reëvaporation does not take place during adiabatic compression.

If the fluid be initially all liquid, then $x_1 = 0$, which in equation (a) gives for the equation of $A\,J$, Fig. 50a,

$$x\,v = c\,\frac{\tau v}{h_e}\,log_e\,\frac{\tau_1}{\tau}, \qquad (e)$$

or,

$$x = c\,\frac{\tau}{h_e}\,log_e\,\frac{\tau_1}{\tau}, \qquad (f)$$

This expression may in some cases have a maximum, from which it appears that if the fluid be initially all liquid, under adiabatic expansion the liquid may be evaporated until the temperature is so reduced as to produce the maximum weight of vapor, after which the vapor will condense.

Thus, for steam $c = 1$, and if $\tau_1 = 800$, x will be a maximum for $\tau = 350°$ (absolute), nearly, at which state x will be 0.24; or 24 per cent. of the liquid will have become vapor. At 300°, $x = 0.239$; for $\tau = 200°$, $x = 0.21$. All these latter temperatures are, however, so much below any used in practice, that it is not probable that the formula for evaporation will be applicable; and we may assert, that, within practical limits, steam will be continually generated under adiabatic expansion, if in the initial state the fluid be entirely water.

With ether, if initially liquid, evaporation will increase with adiabatic expansion until it all becomes saturated vapor, after which it will superheat; provided that the liquid becomes vapor before the temperature of inversion is reached.

The numerical values of these results will be modified in some cases considerably—if higher powers of the temperature be included in the analysis.

The ratio of expansion will be

$$r = \frac{D\,F}{A\,E} = \frac{x_2\,v_2}{x_1\,v_1}. \qquad (g)$$

If, at the cut-off, B, the fluid be all vapor, as it may be for steam-like vapors, then $x_1 = 1$, and reducing by means of equation (a) we have

$$r = \left(c \log_e \frac{\tau_1}{\tau_2} + \frac{h_{e1}}{\tau_1}\right) \frac{\tau_2}{h_{e2}} \cdot \frac{v_2}{v_1}, \qquad (h)$$

which is the equivalent of equation (152), page 154.

For ether-like vapors, if the final state is that of vapor only, then $x_2 = 1$, and substituting x_1 from equation (a) gives

$$r = \frac{v_2 \, h_{e1}}{\left(\frac{h_{e2}}{\tau_2} - c \log_e \frac{\tau_1}{\tau_2}\right) \tau_1 \, v_1}. \qquad (i)$$

The weight of ether vapor at B, the beginning of expansion, in order that the pound of fluid shall be all vapor at C, the end of the expansion, will be x_1 in equation (a) when $x = 1$, or

$$x_1 = \left(\frac{h_{e2}}{\tau_2} - c \log_e \frac{\tau_1}{\tau_2}\right) \frac{\tau_1}{h_{e1}}. \qquad (j)$$

In practice, the adiabatic expansion of steam-like vapors may be approximately realized, but there is well-nigh an insuperable difficulty in securing the adiabatic expansion of saturated ether-like vapors; for, in the former case, if steam be in the state of saturation at the instant of the cut-off, it will continue to be saturated during expansion; but, with the latter, if no ether liquid be present at the instant of cut-off, the vapor will superheat during expansion, and instead of realizing equation (a), the curve of expansion will be of the form

$$p \, v^n = a \; constant,$$

in which n will be the ratio of the specific heat at constant pressure to that at constant volume. We will continue to consider the vapor as saturated.

To find the work A E F D, Fig. 50*a*, p. 184, the expansion *E F* being adiabatic, the vapor being saturated throughout expansion, we have

$$U_1 = A\,E\,F\,D = \int G\,H.\,dp = \int \left[c\,log_e \frac{\tau_1}{\tau} + \frac{x_1 h_{e1}}{\tau_1} \right] \frac{\tau v}{h_e} dp$$

$$= J \int_{\tau_2}^{\tau_1} \left[c\,log_e \frac{\tau_1}{\tau} + \frac{x_1 h_{e1}}{\tau_1} \right] d\tau$$

$$= J \left[c \left(\tau_1 - \tau_2 - \tau_2\,log_e \frac{\tau_1}{\tau_2} \right) + \frac{\tau_1 - \tau_2}{\tau_1} x_1 h_{e1} \right]; \quad (k)$$

which becomes equation (182) if $x_1 = 1$.

If, in this expression, the value of x_1 from equation (*a*) be substituted, and subscript $_2$ be attached to those variables which are without subscripts, we will have

$$U_1 = J \left[c \left(\tau_1 - \tau_2 - \tau_1\,log_e \frac{\tau_1}{\tau_2} \right) + \frac{\tau_1 - \tau_2}{\tau_2} x_2 h_{e2} \right] \quad (l)$$

Equation (*k*) is better adapted for the discussion of steam-like vapors, and equation (*l*) for ether-like vapors; for in the former x_1 may be unity, and in the latter x_2 may be unity.

Eliminating $log_e \frac{\tau_1}{\tau}$ from these equations by means of equation (*a*) gives

$$U_1 = J \left[c\,(\tau_1 - \tau_2) + x_1 h_{e1} - x_2 h_{e2} \right], \quad (m)$$

in which x_1 and x_2 are limited as before.

If, during the return stroke, the fluid be refrigerated so as to maintain the constant temperature τ_2, the pressure will be uniform and equal *C D*; and if at some point, as *J*, adiabatic compression begins and is continued until the temperature is raised to τ_1 at *A*, let x_3 be the weight of vapor at

state A, then will the work done by compression be found by simply changing x_1 to x_3, since all the other quantities remain as before:

$$\therefore U_2 = J\left[c\left(\tau_1 - \tau_2 - \tau_2 \log_e \frac{\tau_1}{\tau_2}\right) + \frac{\tau_1 - \tau_2}{\tau_1} x_3 h_{e_1}\right]; \quad (n)$$

hence the work done in the cycle $A\ E\ F\ J\ A$ will be

$$U_1 - U_2 = J \frac{\tau_1 - \tau_2}{\tau_1} h_{e_1}\left(x_1 - x_3\right).$$

The heat absorbed will be

$$J h_{e_1}\left(x_1 - x_3\right);$$

hence, the efficiency will be

$$E = \frac{U_1 - U_2}{J h_{e_1}(x_1 - x_3)} = \frac{\tau_1 - \tau_2}{\tau_1},$$

which is the same as that of the perfect elementary engine.

Neglecting compression and clearance, we have

$$U = A\ E\ F\ D + (p_2 - p_3)\, x_2\, v_2,$$

where $p_2 = O\ D$, $p_3 = O\ M$, absolute pressures. If τ_4 be the temperature of the feed water, the heat expended will be

$$\mathbf{H} = Jc\,(\tau_1 - \tau_4) + x_1\, H_{e_1},$$

where $H_{e_1} = J h_{e_1}$. Hence the efficiency will be

$$E = \frac{J\left[c\left(\tau_1 - \tau_2 - \tau_2 \log_e \frac{\tau_1}{\tau_2}\right) + \frac{\tau_1 - \tau_2}{\tau_1} x_1 h_{e_1}\right] + (p_2 - p_3)\, x_2\, v_2}{J\left[c\,(\tau_1 - \tau_4) + x_1 h_{e_1}\right]} \quad (o)$$

From this result it appears that in the case of actual engines, the specific heat of the working fluid and the latent heat of evaporation both affect the efficiency. If the feed

water be at the temperature of the exhaust, then $\tau_4 = \tau_2$, and the preceding expression may be reduced to

$$E = \frac{\tau_1 - \tau_2}{\tau_1} - \frac{c\,\tau_2\left(log_e\frac{\tau_1}{\tau_2} - \frac{\tau_1 - \tau_2}{\tau_1}\right) + \frac{1}{J}\left(p_2 - p_3\right) x_2\, v_2}{c\,(\tau_1 - \tau_2) + x_1\, h_{e1}}. \qquad (p)$$

By retaining x_1 and x_2, equation (o) is applicable both to "steam-like" and "ether-like" vapors, only observing that neither x_1 nor x_2 can exceed unity, and that they are related to each other through equation (a).

To find the work done during adiabatic expansion when the initial state A is that of liquid only, make $x_3 = 0$ in the value of U_2, or $x_1 = 0$ in equation (k), giving

$$U_3 = A\,D\,J = J\,c\left[\tau_1 - \tau_2\left(1 + log_e\frac{\tau_1}{\tau_2}\right)\right], \qquad (q)$$

and if the temperature at L be τ_3 then will $A\,M\,L$ be found by substituting τ_3 for τ_2 in the preceding equation.

Actual engines do not expand down to the back pressure, neither is the pound of fluid retained in the cylinder; but at the end of the expansion the exhaust is opened, and the vapor escapes until the exhaust is closed at the point L in the back stroke. The adiabatic $A\,L$ will then be for only a fraction of a pound of fluid. To find its equation let z be the fraction of the pound of fluid, including both liquid and vapor, then equation (a) gives

$$G\,K = z\,x\,v = z\left(c\,log_e\frac{\tau_1}{\tau} + \frac{x_1\,h_{e1}}{\tau_1}\right)\frac{\tau\,v}{h_e}. \qquad (r)$$

If the fluid be all liquid at A, then $x_1 = 0$, and

$$G\,K = z\,c\,\frac{\tau\,v}{h_e}\,log_e\frac{\tau_1}{\tau}, \qquad (s)$$

which reduces to equation (f), if $z = 1$ as it should.

But in practice there is clearance and the fluid will not be reduced to a liquid at state A. Representing the clearance by $P\,A$, Fig. 50*d*; then will $z = P\,A \div A\,B$, where $A\,B$ is the volume of a pound; and equation (r) will be the equation of the adiabatic. It will, however, be more convenient to use the approximate equation

$$p\,v^{1.0} = p_1\,v_1^{1.0},$$

as has been done in the following equation (u).

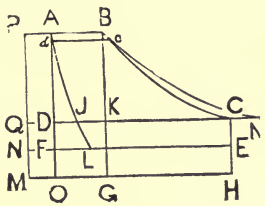

FIG. 50*d*.

Some practical considerations. *A steam or hot-air jacket* is sometimes used to prevent liquefaction of steam in the cylinder by keeping the walls of the cylinder hot. Liquefaction of steam in the jacket produces no bad effect, although it represents cost for fuel. The entire action is too complex to admit of definite computation.

The velocity of the steam through the steam pipe, if not more than 100 feet per second, does not, according to D. K. Clark, produce any appreciable loss by frictional resistance. The loss of pressure in passing through the ports into the cylinder is, in practice, from 3 to 10 pounds, and in exceptional cases even more.

Wire-drawing of steam is the reduction of pressure due to friction. This does not represent a corresponding waste of energy, for it produces heat, thus superheating it—that is, produces a temperature higher than the boiling point corresponding to its pressure, the pressure being lower than at the boiler; but the entire energy is never restored in this way. It is better to cut-off earlier with throttle open than to throttle and cut-off later, to produce the same work.

Superheating may be produced by wire-drawing, by a steam-jacket, by circulating a hot fluid through flues in the steam-chest, by heating the pipes conducting the steam to the

engine, by heating in the cylinder by hot pipes, or by injecting some superheated vapor into a body of saturated vapor. The object of superheating is to prevent condensation, to diminish the back pressure by producing steam of less density, and to increase the efficiency of the fluid.

When steam is superheated to such an extent that it may, without material error in practice, be treated as perfectly gaseous, it is sometimes called *steam gas.* Experiments of Hirn and others show that a very moderate amount of superheating produces steam gas; from which it is inferred that the formulas for steam gas will be practically correct for ordinary superheated steam.

EXERCISE.

Find the work per pound of ether working in an engine without clearance or compression, expansion complete, between the absolute pressures of 100 and 14.7 pounds per square inch; the ratio of expansion and the efficiency, the fluid being entirely saturated vapor at the end of the expansion, and the temperature of the liquid ether 60°.

Since expansion is complete, the final pressure, 14.7 pounds, will equal the back pressure.

The specific heat of liquid ether is $c = 0.517$. To find the initial and terminal temperatures we have, equation (81),

$$\tau = \frac{1}{\sqrt{\frac{A - log_{10} p}{C} + \frac{B^2}{4\,C^2}} - \frac{B}{2\,C}},$$

in which for ether, $A = 7.5641$.

$$B = 2057.8,\ log\ B = 3.313425.$$

$$C = 164950,\ log\ C = 5.217355.$$

Hence, $\tau_1 = 676$, $\tau_2 = 558$; $\therefore\ T_1 = 216°$ F., $T_2 = 98°$ F.

Latent heat of evaporation as determined by Regnault,

$$h_e = 171.24 - 0.0487\ T - 0.000473\ T^2;$$

hence, for terminal state, since $T_2 = 98°$....$h_{e_2} =$ 162
for initial state, since $T_1 = 216°$....$h_{e_1} =$ 139
Work, $ABCD$, Fig. 50a, $x_2 = 1$ in Eq.(l),ft.-lbs. $U =$ 22200
Initial weight of vapor, Eq. (j), lbs.....$x_1 =$ 0.92
Efficiency, Eq. (o), making $x_1 = 0.92$, $p_2 = p_3$. $E =$ 0.137
Volume of a lb. of liquid ether, cu. ft..... $v =$ 0.0223
Vol.of lb.of vapor at $p_1 = 100$ lbs.,Eq.(84),cu. ft. $v_1 =$ 2.98
" " " " " " $p_2 = 14.7$ " " " " " " $v_2 =$ 12.52
Ratio of expansion, Eq. (i)..... $r =$ 4.6

If there be a clearance, and sufficient fluid be retained to just fill the clearance by compression, as indicated by $L\,A$, Fig. 50d, this fluid will act as a cushion, and the energy in it will be stored and restored with each stroke, and will not form any part of the working fluid. In this case, freeing the diagram of the effects of the cushion fluid, as in Stirling's engine, page 224, u_2 will be represented by a line equal to $J\,C$, in which case equation (n) becomes applicable by subtracting from it a trilinear area, of which the hypothenuse is the curved line $J\,L$, and the base the projection of $L\,J$ on $F\,E$. But the *mean effective pressure* will be diminished because the effective work per revolution will be less, the back pressure being greater.

Ratio of expansion with clearance. $A\,B\,C\,E\,L$, Fig. 50d, being the diagram described by the indicator, the cut-off being at B, $A\,B$ will be the *apparent* steam line, and $P\,B$ the *real* steam line.

Let

$$r' = \frac{F\,E}{A\,B} = \text{the } \textit{apparent} \text{ ratio of expansion,}$$

$$r = \frac{Q\,C}{P\,B} = \text{the } \textit{real} \text{ ratio of expansion,}$$

$$c = \frac{P\,A}{F\,E} = \text{the ratio of the volume of clearance to the piston displacement,}$$

$$s = F\,E = \text{the stroke of the piston;}$$

then

$$r = \frac{Q\,C}{P\,B} = \frac{F\,E + P\,A}{A\,B + P\,A} = \frac{1 + \dfrac{P\,A}{F\,E}}{\dfrac{A\,B}{F\,E} + \dfrac{P\,A}{F\,E}} = \frac{1+c}{\dfrac{1}{r'}+c} = \frac{r' + c\,r'}{1 + c\,r'}, \qquad (t)$$

$$\therefore r < r'.$$

$$P\,A = c\,s; \quad A\,B = \frac{s}{r'}; \quad \frac{A\,P}{A\,B} = c\,r'.$$

The adiabatic $A\,L$ will be for a fractional part of one pound of the vapor, and if $A\,B$ be proportional to the volume of one pound of the vapor, and $A\,J$ terminates at the end of the clearance, then will the weight of vapor required to produce $A\,L$ be

$$\frac{A\,P}{A\,B} \text{ of one pound} = c\,r';$$

and the volumes will be in the same ratio at equal pressures. Hence,

$$\frac{N\,L}{P\,A} = \frac{c\,r'\,v_3}{c\,r'\,v_1} = \left(\frac{p_1}{p_3}\right)^{\frac{9}{10}};$$

$$\therefore N\,L = c\,s\left(\frac{p_1}{p_3}\right)^{\frac{9}{10}}, \qquad (u)$$

which determines the point where compression must begin.

The mean effective pressure will be diminished, but, like many other elements, the exact amount cannot be determined theoretically, except by a full solution of the problem; still a sufficiently near approximation may be found by means of equation (176), page 175, which is

$$p_e = p_1\left[10 - \frac{9}{r^{\frac{1}{3}}}\right]\frac{1}{r} - p_3;$$

in which $r^{\frac{1}{3}}$ will be between 1 and 2 for all ratios of expansion used in practice, and when p_1 is large compared with p_3, p_e will vary, approximately, inversely as r. Hence, if p_e' be the mean effective pressure with clearance, and p_e

without, we have, approximately,

$$p_e' = \frac{r}{r'}\, p_e. \qquad (v)$$

The *piston displacement per minute* in doing the same work as without clearance will be *increased* in the ratio

$$r' \div r.$$

If the steam is completely exhausted during each return stroke, the *real* volume of steam will be

$$P\ B = (1 + c\ r')\ A\ B, \qquad (w)$$

or $1 + c\ r'$ times the *apparent* volume.

The mean *absolute* pressure will also be *diminished*, to find an approximate expression for which, conceive that the piston displacement equalled the volume of the cylinder, including the clearance; then would the work done be

$$p_m\ (1 + c)\ u_2.$$

But the work done in passing over the clearance would be

$$p_1\ c\ u_2;$$

and if p_m' be the *mean absolute* pressure, we have

$$p_m'\ u_2 = p_m\ (1 + c)\ u_2 - p_1\ c\ u_2;$$

$$\therefore\ p_m - p_m' = c\ (p_1 - p_m). \qquad (x)$$

The expenditure of heat per pound of steam per stroke without clearance, or with cushion space just filled by the compression of vapor, being

$$\mathbf{H} = H_{e_1} + C\ (\tau_1 - \tau_2), \qquad (y)$$

with clearance and complete exhaustion with each return stroke will be, equations (w) and (y),

$$\mathbf{H}\ (1 + c\ r') = [H_{e_1} + C\ (\tau_1 - \tau_2)]\ (1 + c\ r'). \qquad (z)$$

The efficiency of the fluid will also be *diminished;* for

$$\frac{\textit{effective work with clearance}}{\textit{effective work without clearance}} = \frac{p_m' - p_3}{p_m - p_3};$$

and the expenditure of steam, and consequently of heat, is greater with clearance, as shown by equation (z), and hence the *diminution* of the efficiency will be

$$E - E' = \frac{p_m' - p_3}{(1 + c\,r')(p_m - p_3)}, \text{ nearly.}$$

Although the effects due to these and other practical considerations cannot be thoroughly analyzed, yet their discussion shows that they do not oppose, or revolutionize, the general theory of the vapor engine—they simply modify its results. A more complete knowledge of vapor engines requires special experiments and a study of the engine itself under varied conditions. Theory teaches much, and we are thankful that we know so much, and regretful that we know so little.

113. Cut-off. With a given plant, if the cut-off be early more work may be done with a given amount of fuel than if the cut-off be late in the stroke; and it is proposed to find the cut-off which shall give the most work per pound of steam admitted to the cylinder. This problem may be called *the point of cut-off that will produce the greatest Efficiency of Fluid.*

With a given plant, it may be proposed to find the point of cut-off such that the owner may realize the greatest profit by selling the power produced. This condition will involve the first cost of the plant, attendance, repairs and deterioration. The deterioration may be such that the cost of the entire plant will be absorbed in the course of a few years, or if sold during this time, it will be the difference between the original cost and the amount received by the sale. In this case, if the cut-off be early fuel may be saved, but the other changes may make the cost of the power delivered more per dollar expended than if the cut-off were later, thus making it a problem of maxima and minima. This may be called the *Owner's Problem.*

Again, in the plant of the preceding case, the parts may be improperly proportioned: but if a definite amount of

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work is to be produced, the designer may be required to proportion the plant so that the boilers shall be of the proper size for working most economically for producing the required amount of steam, and the engine so proportioned that by cutting off properly the power produced shall cost the least per dollar expended. If cut-off be too late in this case, more steam will be required, requiring larger boilers and more fuel, while the engine may be smaller, thus costing less; or if cut-off be too early, requiring less steam and smaller boilers, the cylinder and every part of the engine must be larger, costing more, so that this is a problem of maxima and minima, and may be designated as the *Designer's Problem.*

These and similar problems have received the general title, *The Most Economical Point of Cut-off.*

A general solution of the *owner's* problem was made by Rankine, and is made the basis of the solution of the other two. It is substantially as follows:—

Let p_1 = the initial *absolute* pressure in the cylinder per square foot,

p_m = the *mean absolute* pressure,

F = the resistance of the engine other than the useful load, including friction and back pressure,

h = the cost of producing unity of weight of steam in unity of time (one hour), which consists of the cost of fuel, repairs, wages of firemen, interest on cost of boilers, and depreciation;

k = interest on the cost of the engine, *plus* engineer's wages, *plus* cost of repairs, *plus* depreciation of value of engine, *plus* cost of waste and oil, reduced to cost per square foot per hour;

A = area of the piston in square feet,

l = length of stroke in feet.

n = the number of times the cylinder takes steam in unity of time (one hour), once per revolution for single-acting engines, and twice for double-acting;

$l\,n$ = the number of linear feet swept through by the piston in unity of time—one hour;

v = the volume of unity of weight of steam at the pressure p_1, taken from a table of "saturated steam" or from equation (86), or (89);

W = weight of steam used by the engine in unity of time—one hour,

v_2 = the volume swept through by the piston in one stroke,

v_1 = the volume swept through to the point of cut-off,

r = the ratio of expansion $= v_2 \div v_1$

Z = an auxiliary quantity $= p_m\, v_2 \div p_1\, v_1 =$ the ratio of the work done at full stroke, working expansively, to the work done up to the point of cut-off.

Then

$$Z = \frac{p_m\, v_2}{p_1\, v_1} = \frac{p_m}{p_1}\, r; \tag{186}$$

$$\therefore p_m = \frac{p_1}{r}\, Z. \tag{187}$$

$k\,A$ = the interest on the cost of the engine per hour for the steam used, and

$\frac{k\,A}{W}$ = the interest for each pound of steam used per hour; hence, the total cost per pound per hour will be

$$h + k\,\frac{A}{W}.$$

The volume of W pounds of steam will be $v\ W$, and at full stroke, $r\ v\ W$;

$$\therefore A\ l\ n = r\ v\ W; \tag{188}$$

and the preceding expression becomes

$$h + k\ \frac{r\ v}{l\ n}.$$

The useful work per stroke will be

$$U = (p_m - F)\ v_2,$$

which per pound of steam per hour becomes

$$\frac{n\ v_2\ (p_m - F)}{W} = \frac{A\ l\ n}{W}\ (p_m - F), \tag{189}$$

which by means of Equations (187) and (188) becomes

$$v\ (p_1\ Z - F\ r);$$

hence, the work done per unit of cost (one dollar) of steam will be

$$\frac{v\ (p_1\ Z - F\ r)}{h + k\ \frac{r\ v}{l\ n}} = \frac{Z - \frac{F}{p_1}\ r}{\frac{h\ l\ n}{k\ v} + r}\ .\ \frac{p_1\ l\ n}{k}, \tag{190}$$

which is to be a maximum in reference to r as a variable, and will be a maximum when the factor

$$\frac{Z - \frac{F}{p_1}\ r}{\frac{h\ l\ n}{k\ v} + r} \tag{191}$$

is a maximum. Equation (186) shows that Z is a function of p_m and r; but the form of the function p_m is not definitely known, depending, as has been previously stated, upon the behavior of the steam in the cylinder—whether it be dry, saturated, superheated, wire-drawn, &c. If the

mean forward pressure be found in any manner as a function of r, a graphical solution may be made as follows: Draw two axes, $O\,X$ and $O\,Y$, and construct the locus $A\,B$, Fig. 50, representing Equation (186), by laying off spaces $O\,Q$ on $O\,X$ to represent r and corresponding ordinates $Q\,P$ to represent Z. Through the origin O draw a line $O\,N$, such that

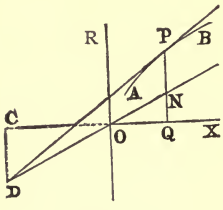

FIG. 50.

$$tan\ NOX = \frac{F}{p_1},$$

then will any ordinate $Q\,N$ be

$$Q\,N = \frac{F}{p_1}\,r;$$

$$\therefore N\,P = Z - \frac{F}{p_1}\,r,$$

which is the numerator of expression (191).

On the negative axis of x lay off a distance

$$O\,C = \frac{h\,l\,n}{k\,v},$$

and at C erect a perpendicular intersecting $N\,O$ prolonged, at D. From D draw a tangent to the locus $A\,B$, the point of tangency being P, then will the corresponding abscissa, $O\,Q$, be that value of r, which will make (191) a maximum. For, the part $P\,N$ of any ordinate between $D\,N$ and $D\,P$ will be proportional to $C\,Q$, its distance from C; but $C\,Q$ is the denominator of (191), and if the line $D\,P$ be above the tangent, nothing will be determined by it, and if below, assume that it passes through some point, as A, on the curve. For the ordinates between the diverging lines we have

$$\frac{P\,N}{C\,Q} = \frac{ordinate\ through\ A}{abscissa\ of\ A}.$$

But the part of the ordinate below A will be less than that represented by the numerator of the second member of this equation, thus making the ratio less.

114. Special cases.

1st. *Let the expansion be isothermal;* then will

$$\frac{p_m}{p_1} = \frac{1}{r}(1 + log_e\, r), \tag{192}$$

as given in equation (165); and

$$Z = \frac{p_m}{p_1}\, r. \tag{193}$$

From these equations the following table has been computed, which is applicable to *perfect gases*, and *superheated steam* working expansively at constant temperature.

r.	$\frac{p_m}{p_1}$. Eq. (192).	Z. Eq. (193)
1	1.000	1.00
$1\frac{1}{4}$	.978	1.22
$1\frac{1}{2}$	.937	1.41
$1\frac{3}{4}$	.891	1.56
2	.846	1.69
$2\frac{1}{2}$	.766	1.92
3	.700	2.10
$3\frac{1}{2}$	.644	2.25
4	.596	2.39
$4\frac{1}{2}$	.556	2.50
5	.522	2.61
$5\frac{1}{2}$	.492	2.70
6	.465	2.79
8	.385	3.08
10	.330	3.30
20	.200	4.00

With these values of r and Z the locus $A\ B$ may be

constructed, and the value of r found therefrom will give the required maximum for sensibly perfect gases.

2*d*. *Let the expansion be adiabatic*, according to the approximate law.

Then

$$\frac{p_m}{p_1} = \frac{10}{r} - \frac{9}{r^{\frac{10}{9}}}, \text{ Eq. (175),}$$

and

$$Z = 10 - 9\, r^{-\frac{1}{9}},$$

from which the following table is computed.

r.	$\frac{p_m}{p_1}$.	Z.
1	1.000	1.00
$1\frac{1}{4}$	.976	1.22
$1\frac{1}{2}$	.931	1.35
$1\frac{3}{4}$	.844	1.54
2	.834	1.66
$2\frac{1}{2}$	.784	1.88
3	.678	2.03
$3\frac{1}{2}$	.620	2.17
4	.571	2.29
$4\frac{1}{2}$	.530	2.39
5	.495	2.47
$5\frac{1}{2}$	.464	2.55
6	.438	2.63
8	.357	2.86
10	.303	3.03
20	.177	3.55

Results found by constructing the locus $A\ B$ from this table will be applicable to saturated steam expanding adiabatically. Other hypotheses might be assumed, and corresponding results obtained, but as extremely accurate results will not be expected in practice, the hypotheses of adiabatic expansion will answer for ordinary cases.

In regard to the *Efficiency of Fluid*, we have in equation (191) k and h both zero, rendering that term indeterminate, and equation (190) reduces to infinity, as it should, since the cost of the steam is not included in the latter problem, which condition only requires the most work per pound of steam entering the cylinder. This requires that

$$U = (p_m - F)\, r\, v_1 \tag{194}$$

shall be a maximum, and this is equivalent to making the numerator of the left member of (190) a maximum, and this is a maximum when the ordinate $P\,N$ is a maximum, giving $O\,Q$ for the corresponding value of r. Equation (194) is easily reduced for a maximum for the isothermal expansion of gases. For we have from equation (164)

$$U = \left[\frac{p_1}{r}(1 + log_e r) - F\right] r\, v_1$$
$$= [p_1 + p_1\, log_e r - F\, r]\, v_1,$$

which will be a maximum, when

$$r = \frac{p_1}{F}, \tag{195}$$

that is, *the ratio of expansion must be such as to reduce the terminal pressure to that of the back resistance.*

If frictional resistances be neglected, F will represent the back *pressure*, which will be that of the exhaust steam; in which case we have

$$p_3\, r\, v_1 = p_1\, v_1;$$

or

$$p_3\, v_2 = p_1\, v_1;$$

or *the work on the back stroke will equal the work done before cut-off;* hence, *the useful work will equal the latent heat of expansion.*

EXERCISE.

(The following exercise is an abstract of a paper by Messrs. Wolff and Denton, *Transaction of the American Society of Mechanical Engineers* (1881), 147, 281, except that

we assume that the engine was used 10 out of 24 hours, while they assumed that it was run continuously. It was an example of a Buckeye non-condensing engine.)

1. Cylinders { 26″ diameter, 48″ stroke, } pressure { 75.3 lbs. gauge, 14.7 " air, }

90.0 " absolute.

10 revolutions per minute, working 10 hours daily.

Assume clearance, $c = 2\frac{1}{2}$ per cent,

Condensation = 30 per cent above that indicated,

Back pressure = 15.7 lbs.

Friction = 2.0 "

$\therefore F = 17.7$ lbs.

Coal, 5 dollars per 2000 lbs.

Evaporation, 9 lbs. of water per pound of coal.

We have—

CHARGEABLE TO THE BOILER:

If the engine work full stroke, and no allowance be made for clearance and condensation, the cost of the coal per hour will be

$$\frac{0.7854 \times (26)^2 \times 48}{1728} \times 2 \times 600 \times 0.21185 \times \frac{1}{9} \times \frac{5}{2000} = 10.415$$

(Vol. 1 stroke feet. Double stroke. Wt. 1 cu. ft. Lbs. of coal per lb. of water. Cost of coal dollars per lb. Dollars per hour.)

Add $2\frac{1}{2}$ per cent for clearance..... .260

Sum = 10.675

Add 30 per cent of \$10.675 for condensation.... = 3.202

Add wages of fireman (\$2.25), laborer (\$1.25).. = 0.146

Interest on cost of boilers, $8500 \times 0.06 \div (365 \times 24)$ = 0.058

Depreciation, say $\frac{1}{12} \times \frac{8500}{365 \times 24}$ ..... = 0.082

Repairs, if \$190 per year ..... = 0.022

Sum = 14.185

CHARGED TO THE ENGINE.

Assume cost of engine when set to be \$9000.

Interest, \$9000 × 0.06 ÷ (365 × 24) ..... = 0.063
Wages of engine runner..... = 0.200
Depreciation, say $\frac{1}{25}$ of 9000 ÷ (365 × 24) .. = 0.042
Repairs, say \$150 per year..... = 0.017
Oil and waste, say ..... = 0.030
Sum = 0.352.

Then,

$$\frac{F}{p_1} = \frac{17.7}{90.0} = \frac{1}{5.09}$$

$$\frac{h}{k} \times \frac{l\,n}{v} = \frac{\dfrac{14.185 \times 1728}{0.7854 \times (26)^2 \times 48 \times 2 \times 600 \times 0{\cdot}21185}}{\dfrac{0.352 \times 144}{0.7854 \times (26)^2}} \times \frac{4 \times 2 \times 60}{4.7}$$

$$= 40.47 = O\ C.$$

$$O\ C \times \frac{F}{p_1} = \frac{40.47}{5.09} = 7.95 = C\ D.$$

In Fig. 50, lay off $O\ C = 40.47$ and $C\ D = 7.95$, and from the point D thus found draw the tangent $D\ P$, and from P let fall the perpendicular $P\ Q$, then will $O\ Q$ be the ratio of expansion for this case, which, for adiabatic expansion, will be

$$O\ Q = 3.4.$$

In a class of forty students working independently and with different scales, the results differed only by two or three tenths, using the same law of expansion, a result near enough for practice in the present state of the science. The exact value may be found by trial and error, by substituting in expression (191). (Addenda.)

(Literature. General solution by the late Professor Rankine, *Phil. Mag.* (1854), 21, 176; *Trans. Roy. Soc. Edinburgh*, Vol. XX., Part II.; *Ship Building; Miscellaneous Scientific Papers*, pp. 288-299. *The Engineer*, 1866, April 2d, p. 248, gives a modification of his graphical

method. Wolff and Denton, *Transaction of the American Society of Mechanical Engineers* (1881), pp. 147–281 (and discussion by others in the same *Transactions*); *Trans. Inst. Civ. Eng.*, London, 1881–82, Vol. IX., Part II., 75, Part III., 44; *American Engineer*, 1881, June, July, Aug., Nov. Emery, *Jour. Frank. Inst.*, May, 1875. Marks, *ibid.*, 1880, '83, '84. Thurston, *ibid.*, 1880, '81, '83, '84. Wood, *ibid.*, May, 1884.)

2. Let the dimensions of the engines and cost of plant be as in the preceding exercise, the gauge pressure 70.3 lbs., 10 revolutions per minute, working 24 hours daily, condensation 25 per cent, back pressure F. = 18 lbs., coal, evaporation, oil, waste, and interest as in the preceding exercise; also wages, except that they are for 12 hours instead of 10; life of boiler 10 years, repairs $225; life of engine 20 years, repairs of engine $175 per year; engine to run 300 days of the year; find the most economical point of cut-off.

3. If the cost of producing the steam be neglected, or $h = 0$, in Exercise 1, find the proper point of cut-off.

4. In Exercise 1 if the "cost of the engine" be neglected, or $k = 0$, find the proper value for r. (Use the left member of (190).)

115. Multiple expansions.—Engines are made with two or more cylinders, so arranged that after steam has done some work in one cylinder it may be exhausted into another, and from the second into the third, and so on, and the expansions continued in the successive cylinders. If two cylinders are employed, the combination is called a *compound engine*; if three cylinders, *triple expansion*; if four, *quadruple expansion*. Since multiple expansions have come into practice, it might be well to drop the term *compound*, and substitute *double expansion*. The cylinders may be arranged in any desirable manner. If placed end to end, having a common piston rod, they are called *tandem*. They may be placed side by side, close to each other, or separated many feet and connected by a large pipe. The cylinder first receiving steam is called the *high* pressure cylinder, and

has the smallest piston displacement per stroke, and the others the *low* pressure cylinders, each increasing in size as they are more remote in the grade of expansion from the high pressure. In triple expansions, the smaller cylinder is called the *high pressure*, the next, the *intermediate*, and the third or largest, the *low pressure* cylinder. The high-pressure cylinder may exhaust directly into the next one, or into a receiver, and in the same manner from the next cylinder, and so on. In some cases double expansion is accomplished in three cylinders, the high-pressure cylinder being between the two low-pressure cylinders.

If the fluid retained its state of aggregation, there would be no theoretical gain in expanding in two or more cylinders over that of expanding in one between the same limits of temperature; but, on the other hand, there would be a loss, for the spaces between the cylinders serve as clearances which must be filled with steam. Steam cards from multiple-expansion engines clearly show this loss; yet experience proves that there is a gain of efficiency. This is chiefly due to the fact that liquefaction of the steam is less when expanded in several cylinders, for the walls of the cylinders are kept at a more nearly uniform temperature, being more nearly that at which the steam enters the cylinder.

There is also a mechanical advantage, since the initial stress on the crank pins will not be so excessive. With triple expansions, the initial stress on the crank pin may be about one-half or one-third of what it would be if expansion were made in one cylinder only.

This arrangement also produces a more uniform rotation of the shaft, which in the case of vessels driven by propellers is favorable to greater efficiency of speed. So that if a single expansion and a triple expansion should show the same economy of fuel per horse-power, the triple expansion in the same vessel ought to show greater economy of fuel for a given mileage.

116. Condensation.—The laws which govern the liquefaction of steam in the cylinder are not well known. Theory recognizes three sources for the appearance of water in the cylinder: *first*, water carried from the boiler to the cylinder in the form of a spray, in which particles of liquid water are mingled with the steam; *second*, liquefaction produced by the expansion of saturated steam; and, *third*, liquefaction produced by the walls of the cylinder. The first pertains chiefly to the construction and management of the boiler; the second has been discussed from a theoretical standpoint by Rankine and Clausius, as stated in Article 98. Rankine, in an example with assumed data, in which the ratio of expansion was $32\frac{1}{4}$, found that nearly 18 per cent of the steam entering the cylinder was liquefied during expansion from this cause (*Misc. Sc. Papers*, p. 399). Theory shows that superheated steam loses nothing from this cause so long as it remains above the condition of saturation, and actual engines confirm this result. Calorimeter tests of steam-jacketed engines have shown a total loss from liquefaction of from 10 to 20 per cent.

The third has been discussed by Professor Cotterell in his work on *The Steam-Engine*, pp. 246–269, in which he shows that this may be the principal cause of the loss of *efficiency of the fluid*. The laws of conduction and radiation are not sufficiently well known to enable one to establish a complete theory of liquefaction in this regard; and if they were, the variations of temperature due to expansion, as well as the varying temperature of the walls from the beginning to the end of the stroke, would greatly complicate the problem.

If the liquefied water be deposited upon the inner surface of the cylinder, as it will be in the third case, it will facilitate the conduction of heat, and the result will be very different from the condition in which the water remains

distributed throughout the steam, as it is supposed to be in the first and second cases named above.

The reduction of temperature, due to the action of the walls, would also have an influence upon the theory involved in the second case, in a manner which has not yet been considered.

Initial condensation is that which takes place during the admission of steam, and is due chiefly to exposure to surfaces colder than the steam, and is independent of the case investigated by Rankine. It can be reduced by keeping the walls at nearly the temperature of the entering steam, and hence may be nearly prevented by a steam-jacket, and in other cases may be reduced by late cut-off and high speed. The economy of high expansion is so well established by theory and confirmed by experience, when condensation is avoided, that other means than that of a late cut-off will be sought for preventing liquefaction.

Theory does not enable us to compute the amount of condensation for any particular case; it must, therefore, be determined by direct experiment. A few examples are given in the following notes.

NOTES.

117. Experiments on steam-engines:

(*a.*) *Hirn's experiments.*—By far the most complete set of experiments scientifically conduced were those under the direction of M. Hirn, by MM. O. Hallauer, W. Grosseteste, and Dwelshauverse Déry, begun in 1873, and extending over several years. The results of the experiments are published in the *Bulletin Spécial of the Société Industrielle de Mulhouse*, 1876. Smith on *Steam Using* contains a summary of these experiments, pp. 188–285.

(*b.*) *Navy experiments.*—Mr. B. F. Isherwood, while chief

engineer of the United States Navy, made extensive experiments upon the boilers and engines of several steamships of the navy, under the general direction of the Department of the Navy, which were published in two large volumes, entitled *Experimental Researches in Steam Engineering*, Philadelphia, 1863, 1865.

Mr. Isherwood made the first attempt, so far as known, to determine the amount of liquefaction of steam in unjacketed cylinders for various ratios of expansion. The experiments were made on the engines of the steamer *Michigan*, in 1861, and showed a large amount of liquefaction, increasing at low speeds and high expansions. These were discussed by Rankine, and published in the *Proceedings of the Institution of Engineers*, Scotland, 1861–62.

(*c*.) *Navy experiments—continued.*—Messrs. Emery and Loring, in 1874, experimented on the engines of the United States revenue steamers *Bache*, *Rush*, *Dexter*, and *Gallatin*, the reports for which were published by the Government, and also in *Engineering*, Vols. XIX. and XXI.

From these and other experiments, some have concluded that, for unjacketed cylinders, it will be sufficient to allow for the liquefaction of steam in the cylinder, due to all causes:

For full stroke,	12	per cent of the total feed water,
2 expansions,	23	" " " " " " "
4 "	36	" " " " " " "
6 "	54	" " " " " " "
8 "	67	" " " " " " "
10 "	72	" " " " " " "
12 "	75	" " " " " " "

(*American Engineer*, 1884, May 23, p. 207.)

For dry steam at high temperatures these allowances are probably much too large, although they may sometimes be realized with saturated vapor.

(*d*) *Experiments at the Stevens Institute.* Messrs. Gately

and Kletzsch experimented upon a Harris-Corliss engine having an 18-inch cylinder, 42-inch stroke, for the purpose of determining the laws of condensation under different conditions. The engine was not jacketed, but was covered with lagging and a non-conducting substance. They found that if

y	= cut-off	= 0.13,	then,	cylinder	condensation	= x	= 0.50
"	"	= 0.225	"	"	"	"	= 0.41
"	"	= 0.33	"	"	"	"	= 0.34
"	"	= 0.45	"	"	"	"	= 0.27
"	"	= 0.59	"	"	"	"	= 0.225

which values are well represented by the equation

$$(x + 0.12)\,(y + 0.44) = 0.3543.$$

If z = the area of the surface exposed in square feet and x the per cent of condensation, as before, the experiments gave

$$x\,z - 1.026\,x - 4.77\,z = 221.36.$$

Varying boiler pressures gave

p	= pressure	= 80.00	pounds,	x	= per cent	cyl.	condensation	= 35.24
"	"	= 66.85	"	"	"	"	"	= 37.83 - 37.85
"	"	= 52.33	"	"	"	"	"	= 36.84
"	"	= 37.00	"	"	"	"	"	= 41.43
"	"	= 22.30	"	"	"	"	"	= 41.19

which may be represented by the equation

$$x = 45 - 0.1266\,p.$$

(*Graduation Thesis*, 1884; *Jour. Frank. Inst.*, 1885, Oct., Nov. and Dec.)

Messrs. Blauvelt and Haynes, by calorimeter tests upon the engines of the steamship *Hudson* of the Cromwell Line, found in some cases only 10 per cent of liquefaction when the cut-off was $\frac{1}{10}$. The pistons were 48 inches in diameter, stroke 6 feet; steam-jacketed cylinder; 800 horse-power engine. (*Thesis*, 1886.)

The experiments of Mr. James S. Merritt upon a small direct-acting steam pump, at various piston speeds, the di-

ameter of the steam cylinder being eight inches, and the stroke about $9\frac{1}{4}$ inches, gave the following results:

Double strokes per minute.	Indicated horse-power.	Water consumed per I. H. P. per hour.		Steam condensed in cylinder; per cent of the feed water.
		Calculated pounds.	Actual pounds.	
10	1.25	46	114	60
20	2.36	50	89	43
30	3.63	48	100	52
40	4.92	48	75	36
50	5.34	51	86	40

(*Thesis*, 1886.)

These results are not as uniform as is desirable in such an experiment. The one for 20 strokes appears to be exceptional.

Messrs. McElroy and Parsons, in their test of the boilers and compound engines of the tug *Blue Bonnet* found that 37 per cent of the feed water was unaccounted for by the indicator cards. The cylinders were, respectively, 19″ × 22″ and 22″ × 22″. The cards indicated that twenty pounds of water per horse-power per hour were consumed. Boiler pressure, 70 lbs. (*Thesis*, 1887.)

(*e*) *Cylinder condensation.* A series of experiments to determine the amount of cylinder condensation were made by Major English, England, and published in *Engineering*, 1887. It is, however, questionable whether they are of much practical value, since the conditions do not conform to actual practice.

118. Miscellaneous:

(*f*) The weight of steam required by an engine is indicative of its efficiency. Eighteen pounds per horse-power per hour is very good practice. Thirty-five to fifty pounds is common in ordinary practice.

(*g*) At Calumet, Mich., an engine was tested working at

about 600 horse-power, for over ten days, which ran with 16.3 pounds of feed water. (*Am. Soc. Mechanical Engineers*, Discussion, Hartford meeting, p. 14.)

(*h*) In a three days' test of the steamship *Para*, having triple-expansion engines, the actual weight of steam consumed per I. H. P. per hour was 13.4 lbs. Adding 15 per cent for initial condensation gives ~~15.5~~ 15.4 lbs.; coal, 1.54 lbs. per indicated horse-power; evaporative power of the coal, 12.0 lbs. from and at 212° F. The *Stella* consumed 13.7 lbs. of steam per I. H. P., with 1.36 lbs. of coal whose evaporative power was 13.9 lbs. of water from and at 212° F. (*Proc. Institution Mech. Eng.*, 1886–87, pp. 492–506.)

(*i*) *Large Ocean Steamers.*

	City of Rome.	*Umbria* and *Etruria.*	*Servia.*
Length, feet	542.5	500	515
Breadth, "	52.0	57	52
Displacement, tons	11230	9860	10,960
Indicated H. P.	11890	14321	10,300
Speed, miles per hour	18.2	20.2	17
Coal, per day, tons	185	315	205
Coal per I. H. P.	2.2	2.1	2
Cylinders, Diam., ins.	3 @ 46	1 @ 71	1 @ 72
Cylinders, Diam., ins.	3 @ 86	2 @ 105	2 @ 100
Cylinders, Stroke, "	72	72	78
Steam pressure, lbs.	90	110	

(*j*) In the year 1840, the time of crossing the Atlantic Ocean in a steamship was about 13 days.

The recent short passages have been:

City of Rome	6 d.	18 h.	0 m.
Oregon	6 d.	10 h.	35 m.
Etruria	6 d.	5 h.	31 m.

(*Scribner's Magazine*, 1887, p. 315.)

In 1887, beginning May 28th, the time of the *Umbria* was 6 d., 4 h., 12 m. In 1888, beginning May 24th at Queenstown, the time of the *Etruria* was 6 d., 1 h., 55 m.

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Distance, 3028 miles; average speed, 20.7 miles per hour. On June 1st the average speed was 24.08 miles per hour.

(*k*) Count de Pambour represented that the friction of an engine increased with the load, the expression for the resistance being of the form

$$R = R_0 + x\,R,$$

in which x is a coefficient, greater than unity, R the variable load, and R_0 the resistance of the unloaded engine.

But Mr. Charles T. Porter, about 1871, wrote that "experiments with a friction brake have shown no appreciable difference between the losses of power in friction when very small and very large loads were driven by the same engine. (*American Machinist*, Dec. 25, 1886, p. 8.)

The result of Porter's experiments has more recently been confirmed by several other experimenters. (*Trans. Am. Soc. Mech. Eng.*, Vol. VII., pp. 86–113.)

(*l*) The *Stiletto* is a torpedo boat 95 feet long, weighs 28 tons, develops 450 H. P., having 16 H. P. per ton of displacement, works with 150 pounds pressure, ran 30 miles in 77 minutes, or an average of 23.7 miles per hour, passing the *Mary Powell*, hitherto the fastest boat on the North River. The highest speed attained is not given. (*The Mech. Eng.*, June 27, 1885.)

(*m*) The compound locomotives tried on the Boston and Albany Railroad failed as economizers of fuel, and were changed to the ordinary form.

(*n*) *Steam pressure* used in marine engines has gradually increased at an average rate of more than two pounds per year for many years. The following values are given in *Scribner's Magazine* for May, 1887.

1825	2 to 3 pounds.
1830	5 "
1835	8 "
1840	10 "
1845	14 "
1850	21 "
1855	25 "
1860	30 "
1865	40 "
1870	50 "
1875	60 "
1880	70 "
1882	80 "
1886	150 to 160 "

The writer does not clearly state why these particular values are given. Some, though relatively few, steamships carry 160 pounds pressure, and if this be the highest, then, on the same plan, the highest for preceding years ought to have been given. Many locomotive boilers carry 160 pounds pressure, and have done so for several years.

(*o*) *Ideal efficiency.* It is sometimes convenient for reference to have an ideal maximum efficiency for steam powers. Assume, then, that the steam pressure is 200 pounds per square inch by the gauge, and that the back pressure is one pound per square inch absolute, and that the forward pressure decreases to one pound absolute; then will the temperature corresponding to the higher pressure be 388° F. and to the lower 102° F.; and the maximum efficiency when working between these temperatures will be

$$\frac{388 - 102}{388 + 460} = \frac{286}{848} = 0.325,$$

which is somewhat less than $\frac{1}{3}$. To realize this result would, according to the approximate adiabatic law, require about 118 expansions, which fact alone shows that such an efficiency is far beyond existing possibilities in a working engine. The waste of heat in the furnace of, say, 25 per cent, loss of pressure between the boiler and the engine, loss of heat at the exhaust, and other losses reduce the

efficiency of steam plants below 15 per cent of the heat energy of the fuel.

(*p*) *Quadruple-expansion engines.* In the *American Machinist* of December 3d, 1887, is an article taken from London *Engineering*, describing a system of quadruple-expansion marine engines, which have been placed on several new steamers. The boilers for these engines are designed to carry 180 lbs. pressure. It is claimed that these are 6 to 8 per cent more efficient than triple-expansion engines.

(*q*) *Efficiency of plant.* Take the case of the steamship *Ohio*, of the International Steamship Company, which has triple-expansion engines of 2100 I. H. P., the gross tonnage of the vessel being 3325 tons. The engines were guaranteed to consume not more than 1.25 lbs. of coal per I. H. P. per hour. The cylinders were 31 in., 46 in., 72 in., and 51 in. stroke.

The trial trip developed an I. H. P. for 1.23 lbs. of coal.*

The calorific capacity of this coal is not known to us, but it is quite certain that on such a trial the best of coal would be used.

If the heat of combustion was 15000 thermal units—which is a very high value—there would have been expended

$$1.23 \times 15000 = 18450 \text{ thermal units}$$

per horse-power per hour, or

$$18450 \times 778 = 14354100 \text{ foot-pounds}$$

of energy to produce one horse-power, or

$$33000 \times 60 = 1980000 \text{ foot-pounds}$$

of work per hour, in which case the

$$\textit{efficiency of plant} = \frac{1980000}{14354100} = 0.138,$$

or nearly 14 per cent.

* *The Mechanical Engineer*, Sept. 10th, 1887, pp. 49, 50, taken from *Industries*

If the coal contained 14000 thermal units—a fair value for very good coal—the

$$\textit{efficiency of plant} = \frac{1980000}{13397100} = 0.148,$$

or nearly 15 per cent.

The boilers would naturally be in excellent condition for such a trial, and if they, including the steam connections up to the steam-chest, gave an efficiency of 75 per cent, then we would have for

$$\textit{efficiency of the engines only}, \frac{13.8}{75} = 0.184 \text{ in former case}$$

$$\frac{14.8}{75} = 0.197 \text{ “ latter “}$$

If the efficiency of the boiler and connections were 70 per cent—a fair value—the efficiency of the engines would be 0.197 in the former case and 0.211 in the latter.

It is a remarkably good plant that will produce an indicated horse-power with 1.23 pounds of the best coal While it is well known that such a trial may be so conducted as to give a result too favorable to the contractors—by not giving proper credit to the heat generated just before starting, or by letting the fires run too low at the close, or by not standardizing the indicator, &c.—yet, on the other hand, the proprietors of the vessel would naturally check all the conditions so as to determine for themselves if the terms of the contract were fulfilled. We therefore feel some confidence that marine steam plants have been made that have developed an actual efficiency of some 14 or 15 per cent; and certainly the conventional 10 per cent efficiency used by popular writers is exceeded in some cases.

Tests of commercial coal taken at random show that the heat of combustion frequently falls below 12000 thermal units per pound. Ocean steamships have been reported as

developing an I. H. P. per hour with 2.1 pounds of coal, and in some cases with less than that amount; hence if this coal contained 12000 thermal units, the efficiency would be

$$\frac{19800000}{196000000} = 0.101,$$

or over 10 per cent.

If the initial pressure in the cylinder were 160 pounds to the square inch absolute (about 145 gauge pressure) and back pressure 3 pounds, and the steam saturated, then would the initial temperature be 363° F. and the lower 142° F., and if this heat were used in a perfect elementary engine between these limits, the efficiency would be

$$E = \frac{363 - 142}{363 + 460} = 0.268,$$

and 70 per cent of this is 0.1876, which is nearly one of the values found above.

(*r*) The torpedo boat *Ariete*, built for the Spanish Government, has twin screws and compound engines, $14\frac{1}{2}''$, $24\frac{1}{4}''$, by 15″ stroke. Average boiler pressure, 152 lbs., revolutions, 395 per minute, developing 155 H. P., and steaming 24.9 knots (28.8 miles) for two hours continuously. One measured mile was run at the rate of 26 knots (30.1 miles) per hour.

(*s*) Steamer *Anthracite*, in 1880, had triple expansion engines.

Pounds of water per I. H. P. per hour	21.68
Evaporation per lb. of coal at and from 212°	9.27
" " " " combustible " "	11.25
Steam-pressure in the boiler, gauge, lbs.	216.5
" " " 1st cylinder, gauge, lbs.	201.6
Terminal pressure in the 3d cylinder, gauge, lbs.	9.55
Expansion, times	25.7
Coal per I. H. P. per hour, lbs.	2.61

HOT-AIR ENGINES.

119. Stirling's (or Laubereau's) Hot-air Engine. This engine was invented by Dr. Robert Stirling about the year 1816, and improved by his son, Mr. James Stirling; for the details of which see *Proceedings of the Institution of Civil Engineers*, 1845. It was further improved by M. Laubereau, the form of which is shown in Fig. 51. It consists of two cylinders of different diameters, having a free communication between them. The smaller piston, *B*, Fig. 52, is the working piston, and drives the engine. The larger piston or plunger, *A*, is made chiefly of plaster of Paris or other non-conductor of heat, and is somewhat smaller than the bore of the cylinder, so that the air may pass freely past it. Also an annular space about the cylinder is filled with thin plates or small wires which heat quickly as the hot air passes among them, and as quickly give up their heat to the cold air on its return. This device is the *regenerator* referred to in Article 105. The top of the cylinder at *C* may be made double to admit of the passage of water; or, what is better, the upper end of the cylinder may be filled with an extensive coil of small copper tubes through which water is made to flow by means of a force-pump worked by the engine, the object being to maintain a low temperature in that end of the cylinder, and thus cool the air at that end, and hence is called the *refrigerator*. The object of the plunger is to transfer a mass of air from one end of the cylinder to the other and back again, and so on alternately, which is accomplished by the reciprocating motion of the plunger. The plunger is sometimes called *the displacing piston*.

FIG. 51.

The plunger, A, is operated by a cam so constructed and arranged that when the piston, B, is near its upper dead point, the plunger will be driven very quickly to the lower end of its stroke and remain there until the piston descends to near its lower dead point, when the plunger will be driven quickly to the upper end of its stroke.

The cylinder containing the plunger is called *the receiver*. The mass of air in the entire engine is constant, and is so maintained by a small air-pump worked by the engine, forcing air into the passage D. The mass of air in the lower part of the receiver, and which is referred to as being below the plunger, when the plunger and piston are at the upper ends of their strokes, is called the *working air*, and all the other air is called *cushion air*. These masses of air are not separated, neither do they keep entirely separate from each other, but intermingle somewhat. When both pistons are at the upper ends of their strokes, the entire mass of air will have its greatest volume, and that which we consider working air will have its greatest volume; but when the plunger is up and piston down, some of that which we call *cushion air* will be forced into the lower part of the receiver, and thus become virtually working air. This latter quantity will be variable, and will not be considered as working air.

The cushion air will be at nearly uniform temperature, and will be considered the same as that of the refrigerator.

To make this engine double acting requires two receivers having plungers working in opposite directions, one receiver being connected with the upper end of the working cylinder and the other with the lower.

120. Theory of Stirling's Engine, or of a hot-air engine in which changes of temperature of the working fluid are made at sensibly constant volume. In this analysis we avoid the refinements which would result from following exact conditions

by assuming *ideal* conditions, which will represent approximately the real ones. For this purpose we make the following assumptions:

(*a*) That the working air is that under the plunger when both the piston and plunger are at the upper ends of their strokes, and the working air is at its highest temperature.

(*b*) That the cushion air remains at constant temperature—thus neglecting the fact that a part of it enters the receiver and virtually becomes working air.

(*c*) That the mass of working air is transferred instantly, and that the changes of its temperature are also instantaneous.

(*d*) The air in the clearance at the lower end of the receiver is discarded, but might be included in the cushion air.

Other assumptions will be made as the subject is developed.

In Fig. 53, let *A* represent the state of the working fluid when at its least volume and greatest pressure; in which condition the temperature will be greatest and the working piston will be at the bottom of its stroke, the

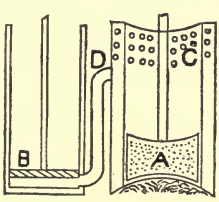

FIG. 52.

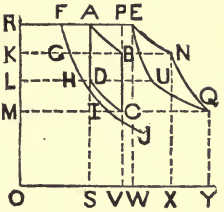

FIG. 53.

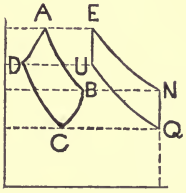

FIG. 54.

volume in the receiver and clearances being *R E*. As the working piston rises, the plunger remaining at the top, the air will expand at a constant temperature to the state *B*, the path of the fluid being the isothermal *A B*. Now let the plunger be suddenly depressed—the working air will at once be transferred to the upper end of the re-

ceiver and there cooled by the refrigerator. If the piston were stationary, the volume of the working air would diminish and its *path* would be some line inclined to the left of $B\,C$; but in order to secure the ideal condition of *a change of temperature at constant volume*, conceive that the motion of the piston is so regulated as to relieve the pressure on the working air in such a way as to make the path of the fluid a straight line, $B\,C$, perpendicular to the axis of volumes.

To accomplish this, the working piston must sweep over the volume $A\,V$ while the pressure is falling from B to C. When the piston descends, sweeping over the volume $V\,W$, the working fluid will be compressed at the constant temperature of the refrigerator to the state D, the path being the isothermal $C\,D$. At the state D, let the plunger be suddenly raised to the top of the receiver, while the piston moves according to such a law that the change of temperature will be made at constant volume. $A\,B\,C\,D$ will be an ideal diagram of the working fluid under the conditions imposed. The motion here imposed upon the piston is only a rough approximation to its actual motion.

Prolong $C\,B$ to P; then will $R\,P$ be the greatest volume of the working air, which, according to the hypothesis, occupies the lower part of the receiver when the piston is near the upper end of its stroke. Between the plunger, when at the upper end of its stroke, and the piston, when at its lower end, will be the space at the upper end of the receiver, the space in the ports and passage D, Fig. 52, and the clearance below the piston B. Take $P\,E$ to represent the volume of these spaces. The diagram shows that with this arrangement none of the so-called working air will enter the working cylinder, and the latter may be kept cool to facilitate lubrication. The volume $P\,E$ should, in practice, be small compared with the volume of the lower part of the receiver, say from 5 to 15 per cent of the volume $K\,B$.

To find the path of the cushion air, take $R\ F = A\ E$; then, according to supposition *b*, the equilateral hyperbola $F\ J$ will be the isothermal representing the changes of pressure and volume of cushion air. If the weight of cushion air was the same as that of the working air, $F\ J$ would fall upon $D\ C$; but as it is generally less, it is placed below.

To construct the *ideal* indicator diagram, make $A\ E = R\ F$, $B\ N = K\ G$, $C\ Q = M\ I$, $D\ U = L\ H$; then will E, N, Q, U be corners in the *ideal* diagram that should be described by the working engine under the conditions imposed. In an actual diagram the corners are rounded.

To find the efficiency, let

τ_1 = the highest absolute temperature of the working air,

τ_2 = the lowest " " " " " "

r = the ratio of expansion.

Since the gas is sensibly perfect, and the expansion along $A\ B$, Fig. 53, is isothermal, we have, for the heat absorbed from the furnace, equation (36),

$$H_1 = R\ \tau_1\ log_e \frac{v_b}{v_a} = R\ \tau_1\ log_e r;$$

and for the heat rejected along $C\ D$,

$$H_2 = R\ \tau_2\ log \frac{v_c}{v_d} = R\ \tau_2\ log_e r;$$

and for that absorbed along $D\ A$ from the regenerator, equation (37),

$$H_3 = C_v\ (\tau_1 - \tau_2),$$

and rejected along $B\ C$,

$$H_4 = C_v\ (\tau_1 - \tau_2).$$

The heats H_3 and H_4 cancel each other. In practice it is found that a certain amount of energy is lost in the regenerator, as stated on page 167, which we represent by

$$n\ C_v\ (\tau_1 - \tau_2).$$

The heat transmuted into work per pound of working air per revolution will be

$$U = H_1 + H_3 - H_2 - H_4 = 122.5\ (\tau_1 - \tau_2)\ log_{10} r, \quad (201)$$

foot-pounds.

If the regenerator were perfect, the efficiency would be

$$E = \frac{U}{H_1} = \frac{\tau_1 - \tau_2}{\tau_1}, \quad (202)$$

which is the same as that of the perfect elementary engine. Allowing for the imperfection of the regenerator, and letting U' be the work actually performed by the motor per pound of air, we have

$$E = \frac{U'}{H_1 + 131\ n\ (\tau_1 - \tau_2)}.$$

The dimensions of an engine must be found in designing it.

Let, in Fig. 53,

p, with the subscript of the letter at a corner of the diagram, represent the pressure at that corner;

v, with the same subscript, represent the volume;

$r = K\,B \div R\,A$ = ratio of expansion of the working air;

$q = R\,E \div R\,A$ = ratio of the total volumes in the cylinders, passages, and clearances, when the working piston is on its lower dead centre to the least volume of working air.

All the volumes have reference to one pound of working air.

We have $K\,B = r\,.\,R\,A,$

or, $$v_b = r\,v_a, \quad (203)$$

$$v_e = q\,v_a,\ q > r;$$

$$P\,E = (q - r)\,v_a;$$

$$A\,E = (q - 1)\,v_a; \quad (204)$$

$$R\,\tau_1 = p_a\,v_a = p_b\,v_b, \quad (2), \text{p. } 11, \quad (205)$$

$$R\,\tau_2 = p_c\,v_c = p_d\,v_d.$$

If s be the ratio of the mass of working air to that of the cushion air, and τ_2 the absolute temperature of the isothermal $F J$, we have

$$R\,\tau_2 = p_f \,.\, s\,v_f = p_g \,.\, s\,v_g = p_i \,.\, s\,v_i, \text{ \&c.} \quad (206)$$

$$v_b = v_c\,; \qquad v_d = v_a\,; \qquad p_f = p_a\,; \qquad p_i = p_c. \quad (207)$$

From these we find—

$$\frac{v_b}{v_a} = \frac{v_c}{v_d} = \frac{p_a}{p_b} = \frac{p_d}{p_c} = r = \frac{v_g}{v_f} = \frac{v_i}{v_h}. \quad (208)$$

Pressures,

$$p_b = \frac{p_a}{r}\,; \quad p_c = \frac{\tau_2}{\tau_1}\cdot\frac{p_a}{r}\,; \quad p_d = \frac{\tau_2}{\tau_1}\,p_a. \quad (209)$$

Volumes per pound *of the working air*—

$$\left.\begin{aligned} v_a = v_d &= 53.21\,\frac{\tau_1}{p_a}\,; \\ v_b = v_c &= r\,v_a = 53.21\frac{r\,\tau_1}{p_a}. \end{aligned}\right\} \quad (210)$$

Volumes of cushion air, per pound of working air,

$$v_f = A\,E = (q-1)\,v_a = \frac{q-1}{r}\,v_b, \quad (204), (210)_2. \quad (211)$$

$$v_g = r\,v_f = (q-1)\,v_b \quad (212)$$

$$v_i = \frac{p_f}{p_i}\,v_f = \frac{p_a}{p_c}\,v_f = (q-1)\,\frac{\tau_1}{\tau_2}\,v_b, \quad (206), (207). \quad (213)$$

$$v_h = \frac{v_i}{r} = \frac{q-1}{r}\cdot\frac{\tau_1}{\tau_2}\,v_b. \quad (214)$$

Total volumes,—

$$\left.\begin{aligned} v_e &= v_a + v_f = \frac{q}{r}\,v_b. \\ v_n &= v_b + v_g = q\,v_b. \\ v_q &= v_c + v_i = \left(1 + (q-1)\,\frac{\tau_1}{\tau_2}\right) v_b. \\ v_u &= v_d + v_h = \left(1 + (q-1)\,\frac{\tau_1}{\tau_2}\right)\frac{v_b}{r}. \end{aligned}\right\} \quad (215)$$

Also

$$\frac{\textit{Mass of cushion air}}{\textit{Mass of working air}} = \frac{v_1}{v_e} = (q-1)\,\frac{\tau_1}{\tau_2}. \qquad (216)$$

Volume swept through by the piston per pound of working air per stroke—

$$v_q - v_e = \left[1 + (q-1)\frac{\tau_1}{\tau_2} - \frac{q}{r}\right] v_b. \qquad (217)$$

If there be given, instead of the ratio of expansion, the ratio of the volume swept through by the working piston to that swept through by the plunger, or $\frac{v_q - v_e}{v_b}$, then r may be found from equation (217), giving

$$r = \frac{q}{1 + (q-1)\frac{\tau_1}{\tau_2} - \frac{v_q - v_e}{v_b}}. \qquad (218)$$

If q is not given, find an approximate value of r by making $q = r$, giving

$$r = \frac{v_q - v_e}{v_b} \cdot \frac{\tau_2}{\tau_1} + 1\,; \qquad (219)$$

the correct value of which will be somewhat less than the value thus found, and q will be somewhat greater. Assume q about 2 to 5 per cent more than the value of r found from (219), and find the correct value of r from equation (218).

If the piston remained on its dead points while the plunger was moving, and the plunger on its dead point while the piston was moving, the indicator diagram, and the diagram representing the changes in the working fluid, would be as shown in Fig. 54. The approximate analysis of this case presents no serious difficulty.

121. In designing an engine of the Stirling type, the horse-power to be delivered and the number of revolu-

tions per minute must be known, in addition to the data already assumed. The number of revolutions will be limited by the piston speed and the length of stroke. The average piston speed may be between 100 and 200 feet per minute. One of Stirling's engines, having a four-foot stroke, was run, in actual practice, at about 28 revolutions per minute, giving an average piston speed of about 224 feet per minute.

An air engine, reported upon by M. Tresca, had a stroke of 0.4 m. (1.3 ft.) and made about 90 revolutions per minute, giving a piston speed of about 120 feet per minute.

The large air engines in the steamer *Ericsson* had an average piston speed of 108 feet per minute.

Let N = the number of revolutions per minute,
S = the average piston speed,
l = the length of stroke of the piston,
h = number of horse-power required of the engine,
W = the work required of the engine per minute;
then,

$$S = 2\,N\,l \tag{220}$$

$$W = 33000\,h. \tag{221}$$

Let w = the number of pounds of working air required; then, since the work done by one pound per revolution will be theoretically, the value of U in equation (201), we have:

$$w = \frac{W}{N\,U}. \tag{222}$$

But the actual work U will be less than the theoretical, and we will assume it to be 0.7, the theoretical. (In designing it is better to assume too small a fraction rather than too large.) Then

$$w = \frac{33000\,h}{0.7 \times 122.5\,(\tau_1 - \tau_2)\,log_{10} r \times N}. \tag{222a}$$

If r be assumed, the weight of air in one cubic foot will be, (205), $(210)_2$,

$$\frac{1}{v_b} = \frac{p_a}{R\, r\, \tau_1}. \tag{223}$$

The initial pressure, p_a, may be assumed, since it can be produced and maintained by the air pump in connection with the heat derived from the furnace.

The volume of the lower part of the receiver will be, (223), (222),

$$\frac{R\, r\, \tau_1}{p_a}\, w \text{ cu. ft.} \tag{223a}$$

Assume the stroke of the plunger to be $y\, l$, in which y is a fraction, say $\frac{3}{4}$, $\frac{2}{3}$, or $\frac{1}{2}$; and A its area; then

$$y\, l\, .\, A = \frac{R\, r\, \tau_1}{p_a}\, w;$$

$$\therefore A = 53.21\, \frac{r\, \tau_1\, w}{y\, l\, p_a}. \tag{224}$$

The volume swept through by the piston per pound of working air per stroke being given by equation (217) and the stroke, l, having been assumed, we have for

the section, B, of the working cylinder, $= \frac{v_q - v_e}{l}\, w$,

or

$$B = \left[1 + (q-1)\, \frac{\tau_1}{\tau_2} - \frac{q}{r}\right] \frac{v_b\, w}{l}, \text{ (217)}, \tag{225}$$

in which q will exceed r, and will be assumed.

If $\frac{v_q - v_e}{v_b}$ be given, instead of r, first find r approximately by equation (219); then assume q greater than r, and find r by (218), after which proceed as before.

The mean effective pressure for the single-acting engine is such an uniform pressure as would, if acting throughout the upward stroke, do the same work as is done by the fluid during one revolution of the engine. Since w pounds of air

does the work $w\ U$ during this time, we have

$$p_e\ B\ l = w\ U;$$

$$\therefore p_e = \frac{w\ U}{B\ l} = \frac{U}{v_q - v_e}, \text{ (225).} \qquad (226)$$

EXERCISES.

1. Let $T_1 = 600°$ F.; $T_2 = 120°$ F.; $p_a = 120$ lbs. per sq. in.; stroke of working piston, $2\frac{1}{2}$ feet; 30 revolutions per minute; $\frac{v_q - v_e}{v_b} = \frac{1}{2}$ = the ratio of piston displacement to plunger displacement; and 5 horse-power be developed. Find

$\tau_1 = 1060°$; $\tau_2 = 580°$, omitting decimals.

$\frac{\tau_1}{\tau_2} = 1.83$; $\frac{\tau_2}{\tau_1} = 0.55$, nearly.

$r = 1.275$, approximately, (219).

Assume $q = 1.30$;

then $r = 1.25$, (218).

$U = 5693$ ft.-lbs., (201).

$E = 0.453$, (202).

$E' = 0.317$, if $0.7\ E$.

$p_a = 17280$ lbs. per sq. ft. (given).

$v_b = 4.08$ cu. ft., $(210)_2$.

$v_q - v_e = 2.04$ feet.

Pounds of air, $w = 1.0$, nearly, (222), theoretical,

or, $w = 1.43$ lbs., $(222a)$, practical.

Section of plunger, $A = \frac{2.336}{y}$ sq. ft., (224); and if $y = \frac{1}{2}$, then

Diameter of plunger = 2.46 feet.

Section of piston B = 1.17 sq. ft., (225).

Diameter of piston B = 1.23 feet.

Mean effective pressure, $p_e = 2755$ lbs. per sq. ft., (226).

Find also the numerical values of v_e, v_q, v_a, v_f, v_g, p_b, and p_c.

2. In a double-acting engine made by Stirling, having a piston 16 inches diameter and a stroke of 4 feet, making 28 revolutions per minute, it was found by calculation, and also by means of a friction brake, that the work done per minute on the piston was 1670000 foot-pounds. There were passed through the refrigerator 250 lbs. of water per minute, and its temperature was increased 18° F. while passing. Assuming that the heat, except that doing mechanical work, was absorbed by the water passing through the refrigerator; find the foot-pounds of heat abstracted by the water, the efficiency of the fluid, provided seven tenths of the heat were abstracted by the refrigerator; the horse-power of the engine; and the foot-pounds of heat unaccounted for by the absorption of heat by the water in the refrigerator.

3. In the engine described in the preceding Exercise, 83 pounds of coal were used per hour, possessing an estimated thermal capacity of 11580 B. T. U.; find the efficiency of the plant; also of the furnace, if that of the engine be 0.3.

Ans. *Efficiency of plant*, 0.133.

Efficiency of *furnace*, 0.44.

From this it will be seen that the efficiency of the furnace is considerably less than that of the steam boiler, which, in good condition, may be assumed to be between 0.60 and 0.75. The efficiency of the plant is nearly equal to that of the most efficient steam plants of the present day. See page 201. But exact comparisons cannot be made, for the thermal capacity of the coal is not known with sufficient exactness, nor the comparative physical properties of air and steam in regard to conductivity.

122. Ericsson's Engine, *in which the changes of temperature are made at constant pressure.* About the year 1833 John Ericsson constructed in London a so-called

"caloric engine," which attracted much attention, especially from scientific men; but it was not a commercial success. His efforts at producing large engines of this class culminated in making in New York, in 1853, a vessel of 2200 tons, called the *Ericsson*, in which the motors consisted of four immense caloric engines.* (For dimensions, see Exercise 1, following.) After experimenting with these weak giants—giants in size, but weak in power—they were abandoned; but he produced another hot-air engine,† which was extensively introduced in various parts of the world; still, after a few years, many of them were removed and replaced by steam engines. Their great bulk, the noise attendant upon their working, and the rapid destruction of their furnaces, were prejudicial to their general use. More recently Captain Ericsson has designed a small hot-air pumping engine, which is being extensively used, the principles of which we will consider.

123. Description. Fig. 55 is an external view of a small hot-air, Ericsson pumping engine, and Fig. 56 is a sectional view of the same. Within a cylinder of uniform bore are two pistons, A and B, of which B is the driving piston and operates the mechanism in a manner so clearly shown as not to need explanation. The lower piston, A, which we will generally call the *plunger*, is made of some substance which is practically a non-conductor of heat. Its office is to transfer a body of air from the space below it to the space above, and back again, and so on alternately, and for this reason is known as the displacing, or transferring, piston. In the position shown, the plunger A is at the upper end of its stroke, and the piston B, being governed in its speed by the crank l, is moving most rapidly, and is driven by the expansion of the air in the lower part of

* *Journal of Arts and Science*, Sept., 1883.

† *Contribution to the Centennial Exhibition*, 1875, by John Ericsson, pp. 425–38.

the receiver *d*. The plunger remains nearly stationary, descending but little while the piston completes its upward stroke. The air in the upper part of the receiver is cooled by the water which has been raised by the pump *r* circulating in the annular space *x x*, so that the water raised for other useful purposes acts as a refrigerator of the engine. During the earlier part of the return stroke of the piston, the plunger descends a little faster than the piston, main-

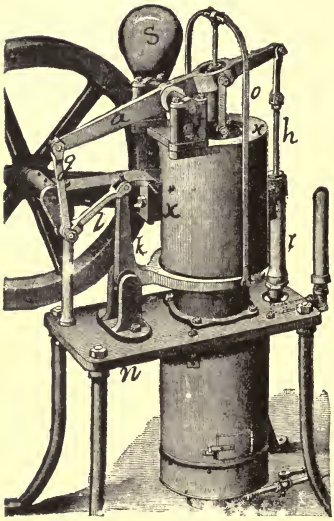

FIG. 55.

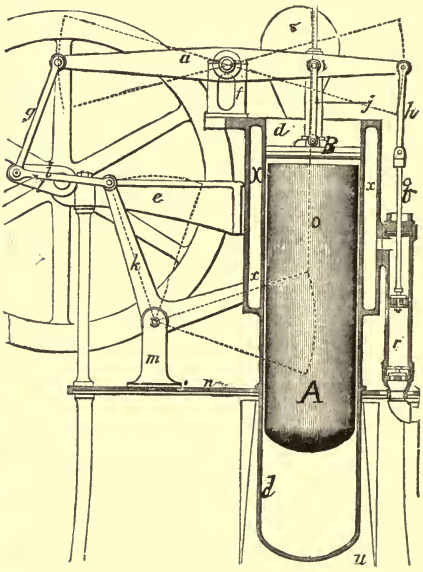

FIG. 56.

taining a nearly uniform pressure upon the piston while air is transferred from below the plunger to the space above, the volume and temperature both decreasing. When the piston has reached about the position shown in Fig. 56 on its downward stroke, the plunger will have reached the lower end of its stroke and all the working air will have been transferred above and its temperature maintained at its inferior limit while it is compressed by the completion

of the downward stroke of the piston B; after which the plunger will rise to the position assumed at the beginning of this description, during which the working air will be transferred to the lower part of the receiver, and its temperature and volume both increased at nearly constant pressure. The mass of air in the engine is constant.

124. Analysis. Fig. 57 is a copy of an indicator diagram taken from a small engine of this class in Stevens Institute of Technology. It will be seen that the changes of temperature at constant pressure are clearly indicated,

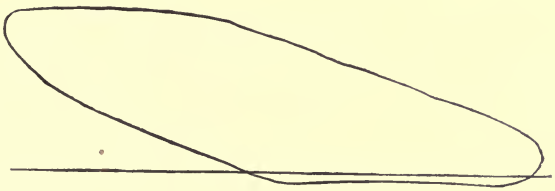

FIG. 57.

and the isothermals, being nearly straight lines, show that the variation of pressure is small compared with the change of volume.

Assuming that the change of state from that of constant pressure to that of constant temperature is instantaneous, the diagram of one pound of the working fluid may be represented by $D\,E\,F\,G$, Fig. 58. F will represent the state of the working fluid at its highest temperature, τ_1, greatest volume and least pressure; hence the plunger and piston will both be at the upper ends of their strokes in the ideal case; and the mass of air below the plunger will be considered as *working* air, and all the other air *cushion air*. $I\,F$ will represent the volume of one pound of working air at its highest temperature, and corresponds to the space below the plunger. Let $F\,F_1$ to the same scale correspond to all the space above the working air; then will

$I F_1$ correspond to the entire volume of the cylinder per pound of working air. The cushion air being supposed to remain at the inferior limit of temperature, take $I L = F F_1$ and construct the isothermal $L K$ for the temperature τ_2, to represent the *path* of the cushion air. Make $D D_1 =$

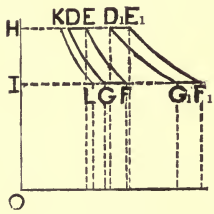

FIG. 58.

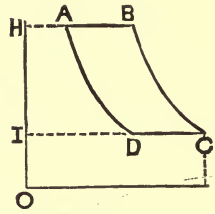

FIG. 59.

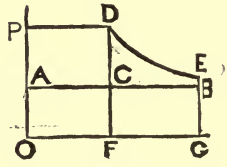

FIG. 60.

$H K = E E_1$, $G G_1 = I L$; then will $D_1 E_1 F_1 G_1$ be the real indicator diagram of the engine. If D_1 falls to the left of F, the piston at the lower end of its stroke will pass into the space occupied by the working air at its greatest volume.

Let τ_1 be the absolute temperature of the isothermal $E F$, and τ_2 that of $D G$, and C_p the dynamic specific heat of air at constant pressure; then will the heat absorbed per pound of air be, from state D to state E,

$$H_4 = C_p (\tau_1 - \tau_2);$$

along $E F$,

$$H_1 = \tau_1 \int \left(\frac{d p}{d r}\right) d v = R \tau_1 log \frac{v_f}{v_e}$$

$$= R \tau_1 log_e r, \text{ Eq. (36)};$$

along $F G$,

$$-H_3 = - C_p (\tau_1 - \tau_2);$$

along $G D$,

$$-H_2 = - R \tau_2 log_e r;$$

hence, the work done per pound of air per revolution, if

the conditions were perfect, would be the sum of these, or

$$U = 122.5\ (\tau_1 - \tau_2)\ log_{10} r. \tag{227}$$

Efficiency of fluid with perfect regenerator,

$$E = \frac{U}{H_1} = \frac{\tau_1 - \tau_2}{\tau_1}; \tag{228}$$

or the efficiency would be the same as that of the perfect elementary engine. There being no regenerator, the efficiency of fluid, if working perfectly without radiation, will be

$$E = \frac{U}{H_1 + H_4}. \tag{228a}$$

If all the losses due to radiation and the refrigerator be represented by $n\ C_p\ (\tau_1 - \tau_2)$, then the efficiency would be

$$E = \frac{U}{H_1 + 184\, n\, (\tau_1 - \tau_2)}. \tag{228b}$$

The value of n is not known, but will exceed unity in this class of engines, especially with very slow speed.

In this analysis, the pressure at state G will be assumed to equal that of the atmosphere, although it may be somewhat less, as shown in Fig. 57; then if

p_a be the pressure per square foot of the atmosphere, τ_a its absolute temperature, and v_a the volume of a pound; then

$$p_a\, v_a = R\, \tau_a,\ (2); \tag{229}$$

$$\therefore v_a = 53.21\, \frac{\tau_a}{p_a}$$

Let p and v, with subscripts, as in Article 120, represent respectively the pressures and volumes at the corresponding states; then

$$p_g = p_a;\quad p_g\, v_g = R\, \tau_2; \tag{230}$$

$$\therefore v_g = R\, \frac{\tau_2}{p_a} = \frac{\tau_2}{\tau_a}\, v_a,$$

from which it appears that if τ_2 exceeds τ_a, v_g will exceed v_a. v_a may be taken, roughly, at $12\frac{1}{2}$ cubic feet. If r be the ratio of expansion, we have, making $\tau = \tau_2$ in Eq. (2),

$$\frac{v_g}{v_d} = \frac{p_d}{p_g} = r = \frac{p_e}{p_f} = \frac{v_f}{v_e} = \frac{v_l}{v_k}. \qquad (231)$$

From the figure and equation (231), we have

Pressures—

$$p_f = p_g = p_{f_1} = p_{g_1}\ ;\quad p_d = p_e = p_{d_1} = p_{e_1} = r\,p_g. \qquad (232)$$

Volumes—working air,

$$v_f = \frac{\tau_1}{\tau_2}\,v_g = \frac{\tau_1}{\tau_a}\,v_a\ ;\qquad v_d = \frac{v_g}{r}\ ;\qquad v_e = \frac{\tau_1}{\tau_2}\,v_d = \frac{\tau_1}{\tau_2}\cdot\frac{v_g}{r}\ ; \qquad (233)$$

Cushion air; let q be the ratio of the entire *volume* of air in the cylinder to the greatest volume of working air, then

$$v_l = v_{f_1} - v_f = (q-1)\,v_f\ ;\qquad v_k = \frac{v_l}{r} = \frac{q-1}{r}\cdot\frac{\tau_1}{\tau_a}\,v_a\ ; \qquad (234)$$

Total volumes—

$$\left.\begin{aligned} v_{f_1} &= q\,v_f. \\ v_{g_1} &= v_g + v_l = \left(q - 1 + \frac{\tau_2}{\tau_1}\right) v_f. \\ v_{d_1} &= \frac{v_{g_1}}{r} = \left(q - 1 + \frac{\tau_2}{\tau_1}\right)\frac{v_f}{r}. \\ v_{e_1} &= \frac{v_{f_1}}{r} = \frac{q}{r}\,v_f. \end{aligned}\right\} \qquad (235)$$

Volume swept through by the piston per pound of working air per revolution—

$$2\,(v_{f_1} - v_{d_1}) = 2\left(q\,(r-1) + 1 - \frac{\tau_2}{\tau_1}\right)\frac{v_f}{r}. \qquad (236)$$

Mean effective pressure per unit area of the piston, or energy expended per foot of volume swept through by the piston, distributed over two strokes—

$$p_E = \frac{U}{2\,(v_{f_1} - v_{d_1})}. \qquad (237)$$

Mean total forward pressure—

$$p_{\text{M}} = \frac{v_{\text{e}} - v_{\text{d}} + v_{\text{e}} \log_{\text{e}} r}{v_{\text{f}_1} - v_{\text{d}_1}} \cdot p_{\text{e}}. \tag{238}$$

Mean back pressure—

$$p_{\text{B}} = p_{\text{M}} - p_{\text{E}}. \tag{239}$$

$$\frac{\textit{Greatest vol. working air}}{\textit{Piston displacement}} = \frac{v_{\text{f}}}{v_{\text{f}_1} - v_{\text{d}_1}}, \text{ nearly.} \tag{240}$$

Let

N be the number of revolutions per minute,
S, the average piston speed,
l, the length of stroke of the working piston,
W, the work in foot-pounds developed by the piston per minute,
HP, the horse-power developed per minute,
w, the pounds of working air per revolution,
A, the area of the working piston;

then

$$S = 2\,N\,l;$$
$$W = 33000\,HP; \tag{241}$$
$$W = w\,U\,N = 2\,p_{\text{E}}\,l\,A\,N. \tag{242}$$

If the isothermals are so nearly right lines that they may be considered as straight, the indicator diagram may be treated as a trapezoid; hence, its area, referring to Fig. 58, will be

$$G\,F \times H\,I = G_1\,F_1 \times H\,I;$$

or, $$(v_{\text{f}} - v_{\text{g}})(p_{\text{d}} - p_{\text{g}}) = 53.21\,(\tau_1 - \tau_2)(r - 1), \tag{243}$$

for the work done per pound of air per revolution.

EXERCISES.

1. In the steamer *Ericsson*, there were four single-acting working cylinders, producing an aggregate of 300 horse-power, as determined by an indicator. The pistons were 14 feet in diameter; length of stroke, 6 feet; revolu-

tions, 9 per minute; fuel, 1.87 pounds of coal per horse-power per hour, the total heat of combustion of each pound being estimated at 14000 thermal units.

Let

$$T_1 = 420° \text{ F.}, \quad T_2 = 120° \text{ F.}$$

From this data find—

Mean eff. pressure, lbs. per sq. in. per double stroke..	1.1.
Average piston speed, feet per minute.....	108.
Vol. swept through by piston per I. H. P., ft. per min.	222.
Heat of combustion of one lb. coal..... 14000×778 ft.-lbs. =	10892000.
Duty, 1 lb. coal..$33000 \times 60 \div 1.87$ ft.-lbs. =	1059000.
Efficiency of plant.... $\dfrac{33000 \times 60}{1.87 \times 14000 \times 778}$ =	0.0971.
Theoretical efficiency of fluid, (228)..... E =	0.340.
Actual efficiency if 0.8 of theoretical.....	0.272.
Probable efficiency of furnace. $0.0971 \div 0.272$ =	0.357.

Notwithstanding the good efficiency of the plant, the excessive size of the cylinders and other practical considerations prevented the general introduction of this class of engines for large powers.

2. Let the bore of the air cylinder of the pumping engine shown in Figs. 55 and 56 be 6 inches, stroke of the working piston, B, $2\frac{1}{2}$ inches, stroke of the plunger, O, $5\frac{3}{8}$ inches, diameter of the plunger, $5\frac{5}{8}$ inches (length of the plunger about 20 inches), being the dimensions of this engine in the Institute. The piston at the lower end of its stroke passes into the plunger space about one inch, and near the middle of the stroke, as shown in Fig. 56, there is about $\frac{1}{16}$ of an inch between the piston and plunger, thus reducing the cushion air to a minimum. The furnace extends upward about one half the length of the plunger, above which the cushion air surrounds the plunger when at the upper end of its stroke; and as only that is effective work-

ing air which is subjected *both* to the refrigerator and furnace, the working air will be less in volume than that of the plunger-displacement; but the relation cannot be determined with accuracy. As nearly as we can determine in this engine, we have

$$\frac{v_{f_1} - v_{d_1}}{v_f} = 0.75, \text{ Eq. (240).} \qquad (243a)$$

Assume $\tau_a = 520$; $T_2 = 130°$ F.; $T_1 = 720°$ F.; total air volume per pound of working air at its greatest volume, $q = 1.2$; and 50 revolutions per minute.

Find :—

$$\tau_2 = 590;\ \tau_1 = 1180;\ \tau_1 - \tau_2 = 590;\ \frac{\tau_2}{\tau_1} = 0.5.$$

Greatest vol. of a pound of working air, $(233)_1$ cu. ft.	$v_f = 29.67.$
Greatest total volume, $(235)_1$	$v_{f_1} = 35.60.$
Least total volume, $(235)_2$, or $(243a)$	$v_{d_1} = 11.35.$
Volume swept through by the piston per pound of working air per stroke, $(243a)$, cu. ft.	$v_{f_1} - v_{d_1} = 24.25.$
Ratio of expansion, $(235)_3$ or (236)	$r = 1.8.$
Work per lb. of air per revolution, (227), ft.-lbs.	$U = 18450.$
M. E. P., (237), $(243a)$, $(233)_1$, lbs. per sq. ft. double stroke	$p_E = 414.6.$
M. E. P. for the single working stroke of air, lbs. per sq. ft.	$p_E = 829.2.$
M. E. P., for the single working stroke of air, per sq. in.	$p_E = 5.76.$
Area of working piston, sq. in.	28.2744.
Work per revolution, ft.-lbs. $\frac{28.2744 \times 2\frac{1}{2}}{1728}$.	$829.7 = 33.92.$
Work per minute, ft.-lbs.	$50 \times 33.92 = 1696.$
Horse-power	$1696 \div 33000 = 0.051.$

Efficiency, if $n = 1$ in Eq. (228a)..... $E = 0.128.$
Efficiency of plant if eff. of furnace be 0.3 0.037.
Pounds of working air, (241)..... $w = 0.0018.$

The pump described in this Exercise is used by the students in their experimental course, and from one of those I make the following abstract;

At 50 rev. per m. indicated *M. E. P.* was, lbs. per sq. in. . 5.42.

I. work per m., ft.-lbs. . $\dfrac{5.42 \times 28.2744 \times 2\frac{1}{2} \times 50}{12} = 1597.$

Indicated horse-power .....1597 ÷ 33000 = 0.0484.
Weight of gas, cu. ft. per hour.....15.7.
Weight of one cubic foot of the gas..... ....0.04584.
Calorific power, B. T. U. per lb.....13650.
" " " " " " cu. ft.....625.7.

Ind. ef. of furnace and fluid, $\dfrac{1597 \times 60}{15.7 \times 625.7 \times 778} = 0.0125,$

or about $1\frac{1}{4}$ per cent.

This result shows the great loss of heat in this engine without a regenerator. Either the furnace has an efficiency of only about 0.1, or if its efficiency be 0.3, then n in equation (228a) should be between 3 and 4. If the efficiency of the furnace be about 0.2, then n must be 2 or more.

The *effective* power as determined by the water pumped was 640 foot-pounds per minute; hence the loss by leakage and friction was estimated to be

$$\frac{1597 - 640}{1597} = 0.599;$$

or nearly 60 per cent.

Efficiency of plant, including fuel, engine and pump

$$\frac{640 \times 60}{15.7 \times 625.7 \times 778} = 0.00502;$$

or only one half of one per cent of the theoretical heat of combustion of the fuel was utilized by the plant. This was one of the best of fifteen experiments.

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The manufacturers guarantee that this size of pump will raise 200 gallons of water per hour 50 feet high with 18 cubic feet of gas. This would give an *effectual* work of $200 \times \frac{231}{1728} \times \frac{50}{60} \times 62\frac{1}{2} = 1383$ foot-pounds. This is more than twice the amount found by the experiment above cited; but a part of the difference may be due to the fact that more gas is required than was consumed in the experiment, the quality of the gas, the condition of the engine, etc.; and the remainder—if any—to the art of advertising.

A very small power steam-engine with furnace and boiler may require from 8 to 12 pounds of coal per horse-power per hour, giving an indicated efficiency of some 2 per cent, more or less. The hot-air pumping engine is used not on account of its superior efficiency, but on account of its greater economy and safety—there being no danger of explosion, and requiring but little expense for attendance.

125. Ratio of expansion *to give a maximum mean effective pressure.* A general solution cannot be made. We will assume some elements, and thus illustrate the process for a particular case.

Let $q = 1.3$; $\frac{v_{f_1} - v_{d_1}}{v_f} = 0.8$; $m = \frac{\tau_2}{\tau_1}$; and let $12\frac{1}{2}$ cubic feet of air weigh a pound.

Then, (236)

$$m = 0.5\, r - 0.3.$$

With these conditions, and equations (227) and (237), we have

$$p_{\text{E}} = \frac{122.5}{20}\, \tau_{\text{a}}\, (1.3 - 0.5\, r)\, log_{\text{e}}\, r, \qquad (244)$$

which is a maximum for $r = 1.69$, as may be found by trial. The corresponding value of m will be

$$m = 0.545.$$

The value of p_z in (244) will be zero for

$$r = \frac{1.3}{0.5} = 2.6,$$

under which condition, the engine, if frictionless, would run without doing work, and would simply change the states of the working fluid. The mean effective pressure is unaffected by the initial temperature; simply the range of temperatures being involved in the value of m, which may finally be expressed as a function of r, as above. The work per minute will depend upon the number of revolutions, and heat must be supplied in sufficient quantity and with sufficient rapidity to maintain the assumed temperatures.

REMARK.—The application of the analysis in the two preceding articles is very delicate, for there are so many physical conditions that cannot be definitely determined, and a small change in any one of them may produce a large change in the results. Thus, it will be seen that changing the value of q from 1.2, as in the preceding Exercise, to 1.3 in this Article, changes r from 1.75 to 1.69. (The value used in the Exercise is 1.8, being the nearest entire tenth, but its value is exactly 1.75.) The "log r" will be changed in a greater ratio, thus affecting the final result in a corresponding manner. The solution given shows that the fraction $\frac{v_{f'} - v_{a'}}{v_f}$ is large when the engine is run at its best effect; and if it be assumed as 0.5 it would produce much less work per minute. The greater the difference of the temperatures the greater the work done per pound of working air, and the greater the ratio of the absolute temperatures the greater the efficiency, other things being equal.

The working of the engine is quite as delicate as the analysis, it being much affected by friction in the cylinder and stuffing boxes, and the condition of the furnace.

126. Heat received and rejected only at constant pressure. An engine involving this principle was proposed by Joule and Thomson (*Phil. Trans.*, 1885), but, so far as known, has not been constructed. In this engine the expansion and compression of the fluid would be adiabatic. $A\ B\ C\ D$, Fig. 59, page 238, would be the indicator diagram of such an engine, in which $A\ B$ and $D\ C$ are parallel to the axis $O\ v$, and $B\ C$ and $A\ D$ are adiabatics.

GAS ENGINES.

127. A gas engine is a hot-air engine in which the cylinder containing the working air is also the furnace, heat being produced by the rapid combustion of the fuel in the cylinder—so rapid as to be called *an explosion*. The fuel is an inflammable gas. When the piston is moving ~~back~~-forward in its stroke, air and gas are drawn into the cylinder, and, at the proper time, the gas is ignited, an explosion takes place, the air is suddenly heated and a high pressure produced; after which a part of the energy thus developed is imparted to the piston during the remainder of the stroke, and the other part is forced out of the cylinder at the exhaust.

The two most prominent systems which have been developed are: one in which the charge is fired with every revolution, when the cylinder is about half full of air and gas; the other at each alternate revolution, when the piston is near its remote dead point. In the former, the energy developed can act on the piston during only about one half of a single stroke; while in the latter it will act during nearly the whole stroke; so that the latter ought to be, as it is found to be in practice, much more efficient than the former. Fig. 60, page 238, illustrates an ideal diagram of the former engine.

In nearly all the more recent gas engines the piston draws in the charge of gas and air during a full ~~backward~~ forward stroke, then compresses it during the next ~~forward~~ backward stroke; and when just past the next dead point the gas is ignited and the piston is driven by the energy thus developed during the next ~~back~~forward stroke, and during the next ~~forward~~ backward stroke the products of combustion are forced out; thus requiring two revolutions to complete a cycle. These are trunk engines. Gas engines are made which take a charge at

both ends of the cylinder and thus resemble double-acting engines, although, in reality, there is only one explosion during each revolution. Others, like the Clerk engine, compress the charge in an auxiliary cylinder which is fired in one end of the working cylinder with every revolution. Thus, while the steam-engine has been improved by passing from single acting to double, quadruple, &c., acting during each revolution, the gas engine has been improved by passing from double to single acting during each revolution, and, finally, to one action during a bi-revolution.

128. History. The origin of the gas engine is not definitely known. It appears to be an outgrowth of an effort to use gunpowder as the fuel, which substance was suggested for this purpose as early as 1680 by the celebrated Huyghens. The gas engine proper was first patented in England more than a century later, 1794, and, although in the years following there were many improvements and many patents, yet it became of no practical value until about 1860, during which year M. Lenoir constructed in Paris the first gas engine that was actually introduced into public use; and during the five years immediately following several hundred were used in France. It was patented in England by J. H. Johnson. It was of the non-compression type, and in its external appearance resembled the ordinary double-acting steam-engine. The charge was fired at each end during each revolution. It contained no new principle, and its success was the result of the care and thoroughness with which the details were worked up. A section is shown in Fig. 61. Fig. 62 is an indicator diagram taken from a two horse-power engine of this class,

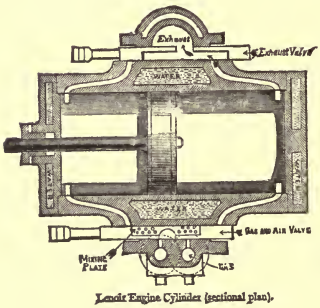

FIG. 61.

as shown in the *Journal of the Franklin Institute*, Vol. LI., 1866, Feb., p. 176.

The length of the line *A B*, Fig. 62, represents the length of stroke of the engine, while the line itself is the atmospheric line. Three lines are traced representing the action at one end of the cylinder during six revolutions. From *A* to *b* the charge was taken in at atmospheric pressure, but from *b* to *c*, the inlet valve being partly closed, the pressure fell, and at *c* the valve closed, the charge was fired and the pressure suddenly raised; and the energy thus developed drove the piston to the end of its stroke. During the return stroke the products of combustion are driven out at atmospheric pressure.

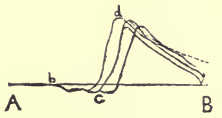

FIG. 62.

In 1867 Otto and Langen exhibited their free piston engine, of which Fig. 63 is an external view of one of this class in Stevens Institute. The principle was not new, but its details were so well worked up that it became a commercial success. It acts by drawing in a charge of air and gas during the first few inches of its stroke, then the valve is closed, the charge fired, and the piston, which is free, is shot upward, and a partial vacuum formed within the cylinder, while the pressure of the atmosphere on the piston gradually brings it to rest and then forces it downward. During the downward motion a pawl on the piston rod engages a ratchet on the main shaft, thus imparting to the latter a rotary motion, which is rendered nearly uniform by the fly-wheel.

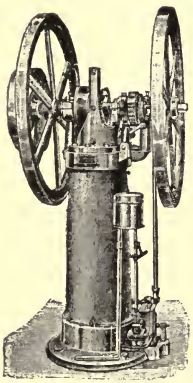
Otto and Langen Free Piston Engine.

FIG. 63.

The idea of compressing the charge before explosion was

mentioned as early as 1801, but the system now generally used was patented by Barnett, an Englishman, in 1838, and by Million, a Frenchman, in 1861, and further developed by M. Beau de Rochas in France and Sir C. W. Siemens in England, both in 1862. The advantages of compression became fully recognized by this time, and the principle has been incorporated into nearly all gas engines constructed since that date.

In 1876 M. Otto produced his "Otto Silent" engine, which, for smoothness and quietness of running, and the economy in the use of the gas fuel, far exceeded all pre-

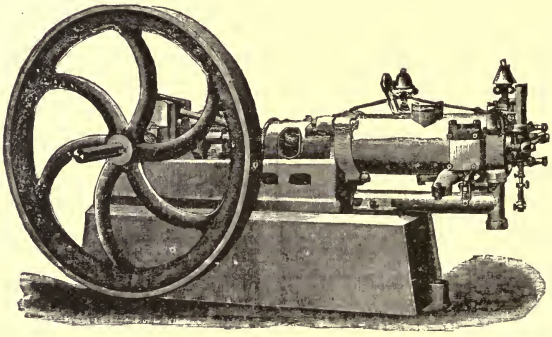

FIG. 64.

vious inventions of this class of engines, and in less than ten years after its invention it is claimed that 15,000 were sold. No new principle was incorporated, the success being entirely dependent upon the skilful use of the principles developed by others. Fig. 64 is an external view of an "Otto," used in making experiments in Stevens Institute.

Successful gas engines of many varieties are now used. At the American Institute Fair, in the fall of 1887, six different types were exhibited by as many different inventors; among which was an "Otto" containing the most recent improvements, some of which were exceedingly

ingenious, and a "Baldwin" of recent invention, which for silent running and uniformity of motion seemed to be all that could be desired. All these engines outwardly resembled the modern horizontal steam-engine. Some engines of this class are duplex, some vertical, but mostly horizontal.

The great improvement made in the gas engine is strikingly illustrated by the fact that the first successful ones, Lenoir's, consumed about 100 cubic feet of gas per indicated horse-power per hour, while an Otto has consumed less than 20 cubic feet for the same power. Some of the earlier engines consumed more than 100 cubic feet, and the latter ones more generally require about 24 cubic feet. The lowest figure given above was for a rich gas and an 8 H. P. engine.

129. Some details. Between the piston at its dead point and the end of the cylinder is a space not swept over by the piston, called the *combustion chamber;* the volume of which is 0.4, more or less, of the entire volume of the cylinder.

The fly-wheel is large compared with the power developed to insure more uniform running. The speed is also regulated in part by a governor, which operates differently in different engines. In some it cuts off a part of the supply of gas with each charge; in others it cuts off an entire charge until the speed is properly reduced; and in still others it closes the exhaust so as to retain a part, or all, of the products of combustion of the previous explosion, thus preventing a full charge of both air and gas being taken in.

The gas is ignited in various ways. A flame of gas, external to the cylinder, communicating with the interior by a small orifice covered by the piston until the charge is taken in and then uncovered during its regular stroke, has proved to be efficient. The orifice may be so small as to remain open during the explosion, but in the more recent engines

the orifice is covered by a valve. The flame may be carried a very short distance in a cavity of a valve. Incandescent metal, so rendered by a flame or by an electric current, has been successfully used. In some cases ignition has been produced by an electric spark; and in still others by chemical action. This is an exceedingly important detail, and has given inventors much trouble, as its action must not only be certain, but must act promptly at a definital part of the stroke, and, sometimes, more frequently than 90 times per minute.

A space is provided for the circulation of water about the cylinder, through the piston, and also through the cylinder heads. This is rendered necessary to prevent injury to the metal from the high temperatures due to the explosion. Mr. Dugald Clerk made experiments upon gas exploded in a closed vessel, determining the time of explosion and the pressures resulting, from which the temperature was computed by means of equation (1), page 11. He gives the following results:

MIXTURES OF AIR AND OLDHAM COAL GAS.

TEMPERATURE BEFORE EXPLOSION, 17° C.

Mixture.		Max. press. above atmos. in pounds per sq. in.	Temp. of explosion calculated from observed pressure.	Theoretical temp. of explosion if all heat were evolved.
Gas.	Air.			
1 vol.	14 vols.	40	806° C.	1786° C.
1 vol.	13 vols.	51.5	1033° C.	1912° C.
1 vol.	12 vols.	60	1202° C.	2058° C.
1 vol.	11 vols.	61	1220° C.	2228° C.
1 vol.	9 vols.	78	1557° C.	2670° C.
1 vol.	7 vols.	87	1733° C.	3334° C.
1 vol.	6 vols.	90	1792° C.	3808° C.
1 vol.	5 vols.	91	1812° C.	
1 vol.	4 vols.	80	1595° C.	

(*The Gas Engine*, by D. Clerk, p. 111.)

The computed temperatures may not have been the highest

at the instant of explosion, and, as Mr. Clerk says, are merely averages; and it may be taken, that coal gas mixtures with air give upon explosion temperatures ranging from 800° C. (1500° F. nearly) to nearly 2000° C. (3600° F.), depending upon the dilution of the mixture. Since cast iron will melt when subjected to a prolonged heat of about 2000° F. (p. 89), the heat of explosion would destroy the working surface if it were not cooled by some artificial means; but with the means employed, cylinders have been used for years, and a wearing surface maintained as perfect as in the steam-engine. The *glow* result-

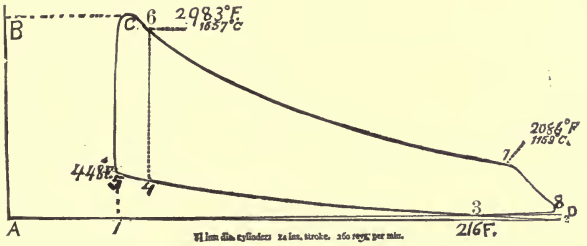

FIG. 65.

ing from the explosion has been observed by inserting in the cylinder a small tube containing a strong glass through which one could look.

130. Theory. We will consider the bi-revolution compression system. Fig. 65 is an actual indicator diagram taken from a 10 horse-power Otto engine during an experiment in the Institute, except that we have added the part *A* 1 *C B* to represent the combustion chamber, and reduced the linear dimensions one half. It is a fair sample of many others that were taken. *A D* is the atmospheric line, 1 *D* the stroke of the piston, *A* 1 the clearance, 2 5 the compression line, 5 *C* the explosion line, *C* 7 the expansion line. The explosion is nearly, but not quite, instantaneous, as

shown by the line 5 C—the piston moving a very short distance before the explosion is complete. During the taking in of the charge the pressure follows the line 12; during compression, the line 25; during explosion, the line 5 C; during expansion, the line C 7; during exhaust, 78; during the fourth stroke the products of combustion are forced out, following 81.

In subjecting these operations to analysis, we proceed, as before, to construct an ideal indicator diagram, in which we assume that the explosion takes place instantaneously at the remote dead point—that the expansion and compression lines are adiabatic—that the fall of pressure at the end of the stroke takes place without change of volume—and that the charge is taken in and expelled at atmospheric pressure; thus producing a diagram like 0, 1, 2, 3, Fig. 66, in which C 0 represents the pressure of the atmosphere.

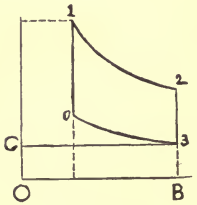

FIG. 66.

Letting the subscripts corresponding to the corners of the diagram represent the corresponding states; then will the heat energy developed by the explosion be the area 3012 indefinitely extended to the right between the adiabatics 03 and 12 prolonged; the value of which for each pound of the substance will be

$$H_1 = C_v (\tau_1 - \tau_0),$$

as in Exercise 3, page 64. The heat rejected in passing from state 2 to state 3 will be, similarly,

$$H_2 = C_v (\tau_2 - \tau_3);$$

hence the efficiency will be

$$E = \frac{H_1 - H_2}{H_1} = \frac{\tau_1 - \tau_0 - (\tau_2 - \tau_3)}{\tau_1 - \tau_0} = 1 - \frac{\tau_2 - \tau_3}{\tau_1 - \tau_0}. \qquad (245)$$

Since $v_0 = v_1$ and $v_3 = v_2$, we have from equation (42), if γ

be constant, and the same for the curve of expansion as for compression,

$$\frac{\tau_2}{\tau_1} = \frac{\tau_3}{\tau_0};$$

$$\therefore \frac{\tau_2 - \tau_3}{\tau_1 - \tau_0} = \frac{\tau_2}{\tau_1} = \frac{\tau_3}{\tau_0};$$

$$\therefore E = 1 - \frac{\tau_2}{\tau_1} = 1 - \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = \frac{\tau_0 - \tau_3}{\tau_0}; \qquad (246)$$

according to which it appears that the efficiency depends only upon the ratio of the temperatures just before and just after compression—or, generally, upon the temperatures at the extremities of either adiabatic, but otherwise is independent of the temperature of the explosion.

The work per pound will be

$$U_{\text{lb.}} = H_1 - H_2 = C_v [(\tau_1 - \tau_0) - (\tau_2 - \tau_3)]. \qquad (247)$$

To find this work in terms of p and v, we have $p\,v = R\,\tau$ as in equation (2), and $R = (\gamma - 1)\,C_v$, as in the answer to Exercise 7, page 59; $\therefore C_v\,\tau_0 = \frac{p_0\,v_0}{\gamma - 1}$, as at the top of page 65, and similarly for p_1, v_1, τ_1, &c.; hence,

$$U_{\text{lb.}} = \frac{1}{\gamma - 1}(p_1\,v_1 - p_2\,v_2 - p_0\,v_1 + p_3\,v_2). \qquad (248)$$

This may be further reduced by means of equation (42), and making $U_{\text{ft}} = U_{\text{lb}} \div v_2 =$ the work done per cubic foot of the mixture, $r = v_2 \div v_1$, the ratio of expansion, and $p_1 = p_2\,r^\gamma$, we have

$$U_{\text{ft}} = \frac{p_1 - p_0}{\gamma - 1} \cdot \frac{r^{\gamma - 1} - 1}{r^\gamma}. \qquad (249)$$

The mean effective pressure on a square foot of the piston, or the energy developed per foot of volume, distributed over four strokes, will be

$$p_e = \frac{U_{\text{lb.}}}{4(v_2 - v_1)} = \frac{\frac{1}{4}\,U_{\text{ft.}}\,r}{r - 1}. \qquad (250)$$

Let N be the number of revolutions per minute; S, the average piston speed; l, the length of stroke; HP, the horse-power; W, the work in foot-pounds developed by the piston per minute; w, the pounds of the mixture at each explosion; A, the area of the piston; then,

$$S = 2\,N\,l;$$

$$W = 33000\,HP = 2\,p_e\,l\,A\,N \tag{251}$$

$$w = \frac{2\,W}{N\,U}. \tag{252}$$

Work done per pound per single stroke will be

$$A\,l\,p_e = \frac{W}{2\,N}.$$

Volume swept over by the piston per stroke

$$A\,l = \frac{33000\,HP}{2\,N\,p_e}. \tag{253}$$

Volume swept over by the piston per pound of the mixture per stroke

$$v_2 - v_1 = \frac{U}{4\,p_e}. \tag{254}$$

131. The furnace. The smaller the combustion chamber, compared with the volume swept over by the piston, the more efficient will be the charge, as shown by equation (246), since $\tau_3 \div \tau_0$ will be smaller the smaller $v_0 \div v_3$ as shown by equation (42). But, on the other hand, the smaller $v_0 = v_1$ is when the explosion takes place, the greater will be p_1 and τ_1 after the explosion, as shown by equation (2); but as the temperature of explosion is very high in practice, there will be a practical inferior limit to the size of the combustion chamber, which must be determined by a protracted use of the engine. In the "Otto," with which experiments were made at the Institute, the combustion chamber was 0.38 of the entire volume of the cylinder, and about the same relation exists in some other engines.

The efficiency of the explosion, or, as we may say, the efficiency of the furnace, is not perfect. Experiments made by Mr. D. Clerk in closed vessels of fixed volume, on the supposition that the absolute temperature varied as the absolute pressure at constant volume, found that the heat evolved varied from 50 to 60 per cent of the theoretical (*The Gas Engine*, p. 182); the latter being nearly the highest value found in any case, while in many cases it is considerably less than the former. Thus, for a mixture of air and Oldham gas, he found

Fraction of gas, vol. $\frac{1}{15}$, $\frac{1}{14}$, $\frac{1}{12}$, $\frac{1}{10}$, $\frac{1}{8}$, $\frac{1}{7}$

Heating efficiency 0.40, 0.48, 0.50, 0.46, 0.40, 0.37.

This shows that the furnace efficiency diminishes with the richness of the gas when the gas exceeds $\frac{1}{12}$ of the volume of the mixture (*ibid.*, p. 113). The table in Article ~~128~~ 129 shows that the efficiency in that case varied from a little below to a little above 50 per cent, when the initial temperature was 17° C. The initial temperature in the engine will be considerably above this, which, added to the facts that—the pressure in the cylinder may be less than that of the atmosphere—a part of the products of combustion will be retained—possible leakage—and imperfect action—make it advisable, in the absence of actual measurements, to consider the efficiency of the explosion as not more than 0.45 of that indicated by the chemical composition. The cause of the large difference between the theoretical and actual heat developed is not well known. It is found that, generally, the best results are obtained when the volume of air is 6 or 7 times that of the gas, so that the volume of the gas for each charge will be $\frac{1}{7}$ or $\frac{1}{8}$ of the volume swept over by the piston in *one stroke*.

132. To find the work and efficiency in terms of the theoretical energy of the gas. Let k_c be the energy of the gas in thermal units, devel-

oped by the explosion of one pound of the gas as determined from its chemical composition, K_c the dynamic equivalent, m the coefficient of reduction of its efficiency, T_1, T_2, P_1, P_2, the temperatures and pressures, respectively, which would result at the states 1 and 2, if the efficiency, m, were unity, and the expansion adiabatic; and assuming that the entire energy of the explosion is communicated to the mixture, we have

$$K_c = J k_c = C_v (T_1 - \tau_0); \tag{255}$$

$$T_2 = T_1 \left(\frac{v_1}{v_2}\right)^{\gamma - 1}; \ \tau_3 = \tau_0 \left(\frac{v_1}{v_2}\right)^{\gamma - 1}$$

$$\therefore T_2 - \tau_3 = \frac{K_c}{C_v}\left(\frac{v_1}{v_2}\right)^{\gamma - 1} \tag{256}$$

and establishing an equation in the same manner as (247) we have, for the *indicated work per pound of the mixture* per stroke of the explosion, or per bi-revolution of the engine,

$$U_{lb} = m\, K_c \left[1 - \left(\frac{v_1}{v_2}\right)^{\gamma - 1}\right]; \tag{257}$$

hence, *the indicated efficiency of the plant* will be

$$E = m \left[1 - \left(\frac{v_1}{v_2}\right)^{\gamma - 1}\right]. \tag{258}$$

133 The expansion and compression curves. Since there is a flow of heat from the working fluid to the water jacket and the reverse, the curves will not be strictly adiabatic, neither will they be isothermal. Their character may be more accurately determined from a study of an actual diagram. Assuming that they follow the law

$$p\, v^x = constant,$$

Messrs. Brooks and Stewart found, for the curve between 6 and 7, Fig. 65, $x = 1.363$; and for the compression curve,

$x = 1.335$. Professors Ayrton and Perry, by an experiment which they confess was not as accurate as the above, found for the expansion curve, $x = 1.479$, and for the compression curve, $x = 1.304$.* These results show that it is much more nearly adiabatic than isothermal. The value of γ used in our analysis should be less than 1.4, the value for air, because the presence of the hydrocarbon of the mixture will reduce the ratio of the specific heats; but since the quantity of air predominates, we may, in the absence of actual measurements, use $\gamma = 1.4$. Air behaves so nearly like a perfect gas that this value would be practically constant, even for the highest temperatures, if the expansion were adiabatic.

134. Experimental results. Messrs. Brooks and Steward, during the summer of 1883, made a thorough test of an Otto engine,† from which we make the following abstract. Dimensions of the cylinder, 8½ inches diameter, 14 inches stroke. The air and gas used in mixtures were both measured by a gas metre, and it was found that when the volume of air used was 7.1 times that of the gas, the best indicated results were obtained. The diagram taken during the 19th test is shown in Fig. 65, with the linear dimensions reduced to one half their original value. During this test it was found that:

$$\frac{Vol.\ air}{Vol.\ gas} = 6.63\ ; \qquad \frac{Weight\ of\ air}{Weight\ of\ gas} = 13.68.$$

About one half the heat of explosion was carried away by the water jacket. The temperatures were computed by means of equation (2), the volumes and pressures being

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* *Phil. Mag.*, 1884, (2), 65.

† *Graduation Thesis at Stevens Institute of Technology*, 1883; Van Nostrand's *Engineering Magazine* 1884, Feb., pp. 90–104.

measured from the indicator diagram. The specific heats of the mixture were computed to be

$$c_p = 0.268\,; \quad c_v = 0.196\,; \quad \therefore \gamma = 1.37.$$

The complete combustion of the gas, determined from its chemical composition, gave 9070 calories per kilog., or 617.5 B. T. U. per cubic foot. From 23.5 to 25.6 cubic feet of this gas were used per indicated horse-power.

The mean effective pressure was about 58 pounds per square inch per stroke of the explosion; and if there was an explosion for every fourth stroke the average pressure was 14½ lbs. for every stroke. The average number of revolutions was nearly 155 per minute.

Temperature of the exhaust gases, from 720° F. to 778° F., as determined by a pyrometer.

The indicated efficiency was 18 per cent of the total heat of combustion of the gas, and the *effectual* efficiency for the plant, as determined by a brake, was 14½ per cent.

The experiments upon the "Otto," of various sizes and in distant parts of the world, have been numerous, giving efficiencies of 15, 16, 17, and 18 per cent.

From these results it will be seen that the explosive gas-engine gives the highest indicated efficiency for the plant of any system thus far considered. For intermittent work and cost of attendance, it has an advantage over the steam-engine. On the other hand, the engines are much larger for the same power, and are thus objectionable for very large powers, and the cost of fuel for steady running is greater, since a heat unit in the form of gas costs considerably more in the market than a heat unit in the form of coal. Circumstances must decide what class of engines is most economical for a particular case.

The following is an analysis by Professor T. B. Stillman, of Stevens Institute, of the gas used by Messrs. Brooks and Stewart. The gas was taken from the mains supplied by the Hoboken Gas Company:

		By volume.
H	Hydrogen	.395
CH_4	Marsh gas	.373
N	Nitrogen	.082
C_3H_6, Average	Heavy hydrocarbons	.066
CO	Carbonic oxide	.043
O	Oxygen	.014
H_2O_1, CO_2, H_2S, &c.	Impurities, &c.	.027
		1.000

By weight its composition is found to be :

	Cu. metres.		Densities.*		Kilos. per cu. m.	W't p. unit.
H	.395	×	.087	=	.035	.058
CH_4	.373	×	.694	=	.258	.426
N	.082	×	1.215	=	.099	.163
C_3H_6, Av'e	.066	×	1.84	=	.121	.200
CO	.043	×	1.215	=	.052	.086
O	.014	×	1.388	=	.019	.031
H_2O_1, &c.	.027	×	~.8	=	.022	.036
	1.000	×	.606	=	.606	1.000

Heating Power of the Gas.

Upon complete combustion the gas develops heat per cubic metre, as follows :

	Calories.				Calories.
from H	29060	×	.035	=	1020
" CH_4	11710	×	.258	=	3020
" C_3H_6, &c.	11000	×	.121	=	1330
" CO	2400	×	.052	=	125
					per cu. m. 5495c.

and per kilog. gas $\frac{5495}{.606} = 9070$ calories.

Expressed in British measures, one cubic foot of gas develops 617.5 heat units.

Air necessary for complete Combustion and the Products of Combustion.

In order to determine the amount of air to be supplied for complete

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* Schöttler : *Die Gasmaschine*, p. 77. By "density" is meant the weight of one cubic metre in kilogrammes. As will be seen from the above, one cubic metre of the gas in question weighs 0.606 kilos.

combustion, it is necessary to ascertain the quantity of oxygen that is taken into chemical combination by the several combustible constituents of the gas.

$$2H + O = H_2O$$
by volume $2 + 1 = 2$
by weight $2 + 16 = 18$

$$CH_4 + 4O = CO_2 + 2H_2O$$
by volume $2 + 4 = 2 + 4$
by weight $16 + 64 = 44 + 36$

$$C_3H_6 + 9O = 3CO_2 + 3H_2O$$
by volume $2 + 9 = 6 + 6$
by weight $42 + 144 = 132 + 54$

$$CO + O = CO_2$$
by volume $2 + 1 = 2$
by weight $28 + 16 = 44$

The combining proportions per unit of the several constituents is :

By volume—

$$1H + \tfrac{1}{2}O = 1H_2O$$
$$1CH_4 + 2O = 1CO_2 + 2H_2O$$
$$1C_3H_6 + 4\tfrac{1}{2}O = 3CO_2 + 3H_2O$$
$$1CO + \tfrac{1}{2}O = 1CO_2$$

By weight—

$$1H + 8O = 9H_2O$$
$$1CH_4 + 4O = \tfrac{11}{4}CO_2 + \tfrac{9}{4}H_2O$$
$$1C_3H_6 + \tfrac{24}{7}O = \tfrac{33}{7}CO_2 + \tfrac{9}{7}H_2O$$
$$1CO + \tfrac{4}{7}O = \tfrac{11}{7}CO_2$$

The volume of oxygen required for the combustion of 1 volume of gas is :

H	.395	× ½	= .197
CH_4	.373	× 2	= .746
C_3H_6	.066	× 4½	= .297
CO	.043	× ½	= .022
			1.262
less O in gas	.014		.014
			1.248

Taking oxygen as 21 per cent in atmospheric air, the volume of air required is

$$\frac{1.248}{.21} = 5.94 \text{ per volume gas,}$$

or the entire volume is 6.94 times the volume of gas.

Since air weighs 1.251 kilos. per cu. metre, the ratio by weight is

$$\frac{5.94 \times 1\,251}{1 \times .606} = 12.26 \text{ air to gas } 1.$$

From the combustion of 1 unit weight of gas with 12.26 air there results 13.26 units weight of a mixture the composition of which will be :

CO_2	(CH_4) .426 × $\frac{11}{4}$ = 1.171		
	(C_3H_6) .200 × $\frac{33}{7}$ = .629		1.93
	(CO) .086 × $\frac{11}{7}$ = .135		
H_2O	(H) .058 × 9 = .522		
	(CH_4) .426 × $\frac{9}{4}$ = .958		1.74
	(C_3H_6) .200 × $\frac{9}{7}$ = .257		
N	from the air	9.407	
	in gas itself	.163	9.57
Impurities in gas			0.03
			13.27

Per unit weight of mixture the composition will be :

CO_2	.146
H_2O	.131
N	.721
Impurities	.002
	1.000

The volume which 13.27 kilos. of products of combustion will occupy is found from the known volumes of the constituent gases as follows :

	kilos.	cu. m. per kilo.	cu. m.
CO_2	1.93 ×	.524	= 1.011
H_2O	1.74 ×	1,28	= 2.227
N	9.57 ×	.823	= 7.876
Impurities	.03 ×	~.9	= .027
			11.141

The products of combustion then occupy 11.141 cu. m. to every kilog. of gas. To find the ratio per cu. metre of gas we have simply to multiply by .606 the number of kilos. in a cubic metre, and we get 6.751 as the result. As there is necessary 6.94 cu. m. of mixture of air and gas to every cu. m. gas, it is seen that by combustion a contraction of 2.7 per cent takes place.

When there is an excess of air present, as is always the case in practice, the contraction becomes less in proportion, and may be considered to be about 2 per cent. In the following thermodynamic computations no account is taken of this contraction.

Specific Heats and their Ratio.

The specific heats of the products of combustion are determined from the specific heats of the several component gases as follows:

Specific heat at constant pressure (water = 1).

$$C_p = \left\{ \begin{array}{lll} .2169 \times .146\ (CO_2) & = & .0317 \\ .4805 \times .131\ (H_2O) & = & .0629 \\ .2438 \times .721\ (N) & = & .1758 \\ \sim.4 \times .002\ \text{(impurities)} & = & .0008 \end{array} \right\} .2712$$

Specific heat at constant volume (water 1).

$$C_v = \left\{ \begin{array}{lll} .1714 \times .146\ (CO_2) & = & .0250 \\ .3694 \times .131\ (H_2O) & = & .0484 \\ .1727 \times .721\ (N) & = & .1245 \\ \sim.3 \times .002\ \text{(impurities)} & = & .0006 \end{array} \right\} .1985$$

The ratio of these specific heats is the exponent of adiabatic expansion, and is found to be:

$$\gamma = \frac{C_p}{C_v} = \frac{.2712}{.1985} = 1.366.$$

Since there is always an excess of air present, these values will be somewhat modified by that fact. From the metre records of test 19 the ratio of air to gas by volume was found to be 6.63 to 1; by weight the ratio is

$$\frac{6.63 \times 1.251}{1 \times .606} = 13.68.$$

Since for complete combustion only 12.26 parts of air by weight are needed, there are 1.42 parts in excess. The specific heats of air being $C_p = .2375$ and $C_v = .1684$, the effect of the excess of air will be to reduce the specific heat slightly.

$$C_p = \frac{(.2712 \times 13.26) + (.2375 \times 1.42)}{14.68} = .268$$

$$C_v = \frac{(.1985 \times 13.26) + (.1684 \times 1.42)}{14.68} = .196$$

$$\gamma = \frac{C_p}{C_v} = \frac{.268}{.196} = 1.37$$

EXERCISES.

1. Required the horse-power and efficiency of a gas-engine whose cylinder is $8\frac{1}{2}$ inches diameter, stroke 14 inches, revolutions 160 per minute, charge every fourth stroke, combustion chamber 0.38 of the volume of the cylinder, heat of combustion of one pound of the gas 16326

B. T. U., volume of working air 7 times that of the gas, weight of gas say $\frac{1}{17}$ of that of the air, efficiency of the furnace 0.60 of the theoretical, $\gamma = 1.38$. (These are approximately the conditions of the engine and test in Brooks and Stewart's experiments.)

We find—

Area of the piston, sq. in..... $= 56.75$.
Piston displacement per stroke, cu. ft..... $= 0.46$.
Vol. of air taken in each fourth stroke, $\frac{7}{8}$ of $0.46 = 0.402$.
Pounds of air for each charge.....$0.402 \times 0.08 = 0.032$.
Pounds of gas for 80 charges...$\frac{1}{17} \times 0.032 \times 80 = 0.150$.
Work per lb. gas, eq. (257), ft.-lbs.....0.60×778
$\times 16326\ [1 - 0.38^{0.38}]$..... $= 2344970$.

IHP. for the 0.15 lbs. gas $\dfrac{2344970 \times 0.15}{33000}$..... $= 10.6$.

Indicated efficiency, (258).....$E = 0.185$.

If $m = 0.55$ we would have :—

Indicated IHP. for 0.15 lb. gas..... $= 9.7$.
Indicated efficiency.....$E = 0.169$.

These last results agree very nearly with the measured results of Brooks and Stewart—the horse-power being the same and the efficiency about one per cent less than their best result.

2. Required the horse-power, efficiency, pounds of air and of gas per minute, of a gas-engine having a 10-inch cylinder, 16-inch stroke, making 150 revolutions per minute, charge every fourth stroke, combustion chamber 0.4 of the volume of the whole cylinder, heat of combustion of one pound of the gas 18000 B. T. U., volume of working air $6\frac{1}{2}$ times that of the gas, weight of the gas 0.55 that of an equal volume of air, $12\frac{1}{2}$ cubic feet of air to weigh a pound, efficiency of furnace, 0.50, and $\gamma = 1.4$.

THE PETROLEUM ENGINE.

135. Naphtha Engine. In petroleum engines, the working fluid is either petroleum, or some of its products. Brayton's petroleum engine was a modification of that inventor's gas-engine. The products of combustion entered the cylinder, and in this respect was similar to the gas-engine; but combustion was gradual, the indicator diagram showing that it was made at nearly constant pressure up to the point of cut-off. Mr. Clerk concludes from his experiments with a 5 IIP. engine that it utilizes about 6 per cent of the heat of the petroleum. Professor Thurston, from an experiment with one of these engines, concluded that 32.06 cubic feet of gas were consumed per IIIP.; but Mr. Clerk concludes that the same experiment shows a consumption of 55.2 cu. ft. per IHP. per hour. (*The Gas Engine*, p. 158.)

We will consider only a recent form, called the Naphtha Engine, of which Fig. 67 is an external view. In this engine the products of combustion do not enter the cylinder, but the same substance is used for the fuel and working fluid. A small plunger pump near D, but not shown in the figure, worked by the engine, forces some liquid naphtha into the boiler F at each revolution, a part of which is conducted from the boiler down the tube H to the combustion chamber, or furnace, below the valve chest A, and is there burned. At E is an opening into the tube H through which air may be forced to increase the rate of combustion. The heat of the burning naphtha vaporizes that remaining in the boiler, and the vapor thus generated is used in the engine in precisely the same manner as if it were steam; and the *law* of its action in the engine is precisely the same as steam, as may be inferred from Fig. 68, which is a copy of an indicator diagram taken by Doty and Beyer,

in the experiments referred to below. The ratio of expansion, as here shown, is about 2, but it may be varied at pleasure. The drop, when the exhaust opens at the end of the stroke, is sudden, and the back pressure and compression lines are good. The depression of the steam line, showing initial expansion, is probably due to the setting of the valve, since the diagram from one of the cylinders was free from this defect.

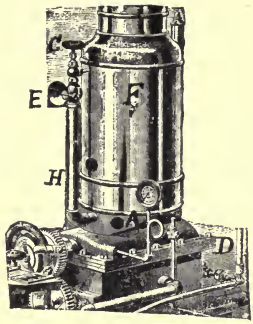

FIG. 67.

If the diagram be freed of its irregularities and of compression, it would be analyzed precisely like the steam-engine, and the solution in Articles 110, 111, and 112 would be applicable. Probably Article 110 represents the case more nearly, since the cylinders are very near the boiler, and receive heat continually from it. But the analysis cannot be carried out numerically, since the physical properties of Naphtha are not sufficiently well known. The latent heat of evaporation at varying pressures is not known, nor the value of R in the equation $p\ v = R\ \tau$—if indeed it is constant for the vapor. We will, however, after giving the results of some experiments, make an approximate solution.

FIG. 68.

136. Experiments. Messrs. Doty and Beyer made experiments upon a naphtha engine, of which the following is a summary of their report:*

Three single-acting trunk engines were connected to the

* *Graduation Thesis* of Paul Doty and Richard Beyer, Stevens Institute of Technology, Hoboken, 1888. *The Iron Age*, July, 1888.

crank shaft, the cranks making successive angles of 120° with each other. The lower ends of the coils constituting the boiler were connected with the pump, and the upper ends entered a common chamber.

Diameter of cylinders, each	$3\frac{1}{4}$ inches.
Stroke of pistons, each	$4\frac{1}{2}$ "
Piston displacement, each	$37\frac{1}{2}$ cu. in.
Admission ports, each	$\frac{7}{32} \times 2\frac{1}{16}$ inches.
Exhaust ports, each	$\frac{5}{16} \times 2\frac{1}{16}$ "
Diameter of pump	$1\frac{1}{4}$ "
Stroke of pump	$1\frac{3}{8}$ "
Travel of main valves, each	$\frac{5}{8}$ "
Clearance of cylinders, each	1.86 cu. in.
or $5\frac{1}{8}$ per cent of piston displacement.	
Boiler, seven spirals of four coils each.	
Coils, copper tubing outside diam	$\frac{3}{4}$ "
Height and diameter of coils, each	12 "
Burner had 26 openings, diam. of each	$\frac{3}{16}$ "
Heating surface	12 sq. ft.

RESULTS.

Average revolutions per minute, number	280.7.
Total Indicated HP. from the three cylinders	2.81.
Mean effective pressure, lbs. per sq. in., about	35.
Naphtha burned per IHP. per hour, lbs	3.53.
Price of naphtha, June 5th, cents per gallon	10.
Cost per IHP. per hour, cents	6.2.
Heating surface, square feet per IHP	4.3.
Water used to condense the exhaust naphtha, lbs. per IHP. per hour	25594.
Increase of temperature of condensing water, degrees F	3.9.
Naphtha passing through condenser per hour, lbs	421.
Temperature of the stack, degrees F., about	685.
Specific gravity of the naphtha, that of water being unity	0.683.
One gallon weighed, pounds	5.69.

Assuming that the vapor was saturated, three elements of this data give, for

$$\textit{the latent heat of evaporation} = \frac{25594 \times 3.9}{421} = 237$$

thermal units, at atmospheric pressure.

In order to determine the relation between the temperature and pressure of saturated vapor, these experimenters devised a special thermometer with which they determined the temperature of the vapor in the steam chest, and at the same time determined the pressure by means of a pressure gauge. These measurements gave

for 35 lbs. gauge pressure a temp. of 225° F.;
" 47 " " " " " " 242° F.;
" 60 " " " " " " 258° F.

Substituting in equation (80), page 97, these values of p reduced to their equivalent in pounds per square foot, and the corresponding temperatures reduced to the absolute scale, and finding the values of A, B, C, we have

$$log_{10} p = 6.4618 - \frac{889.4}{\tau} - \frac{625784}{\tau^2}; \qquad (259)$$

in which

$$log_{10} B = 2.949092; \qquad log_{10} C = 5.796469.$$

Making $p = 2116.2$ this formula gives $\tau = 602.62°$ or $T = 141.96°$ F. Naphtha has not a fixed boiling point. In an experiment it began to boil at 60° C., and as the more volatile parts passed off, the temperature gradually increased, and, in the course of twenty minutes, it raised to 68° C., giving a mean of 64° C. = 147° F. The value found by the formula agrees, approximately, with the lower observed temperature, 60° C. = 140° F.

To find the volume of one pound of the saturated vapor of naphtha at atmospheric pressure, we have, from equation (84), page 98,

$$v_2 = 0.0234 + \frac{237 \times 778}{24035.5} = 7.69 \text{ cu. ft.},$$

in which 0.0234 is the value assigned for one pound of liquid naphtha at 60° F.

It was found that the naphtha, exposed to an atmospheric pressure of 29.982 inches, at a temperature of 70° F., evaporated at the rate of 0.092094 lbs. per square foot per hour.

Some of the values found by these experimenters differ largely from those given by others. Thus, the boiling point, as found by them, is between 116° F. and 167° F., but Box *On Heat*, page 14, gives 306° F. on the authority of Ure. The latter must be an error. They found the latent heat of evaporation to be 237; while Box, page 18, gives 184 on the authority of Ure. If Ure's figures were reversed, giving 184° F. for the boiling point and 306 for the latent heat of vaporization, they would appear more rational; still we cannot say what is correct. They give 0.683 for the specific gravity at 60° F., while Rankine, in Table II. of the *Steam Engine*, gives 0.848 at 32° F.; and both cannot be correct. The discrepancies may be due in part to the difference in the chemical composition of the different specimens. The relation between the pressure and volume, given in equation (259), is considered only approximate, not only on account of the heterogeneous character of the substance, but also because it depends upon three experiments only, whereas there should be a larger range of experiments in order to test its accuracy and reliability. The results will, however, be subjects for comparison for future experimenters.

137. Efficiency of the Naphtha Engine. In the absence of a determination of the calorific power of the naphtha used in the above experiment, we will assume that the heat of combustion is 22000 thermal units per pound, since its value would be 22274 if the composition were $C_6 H_{14}$.

For,

$$\begin{array}{rl} 6 \times 12 \times 14544 = & 1047168 \\ 14 \times 1 \times 62032 = & 868448 \\ \hline 86 &)1915616(22274. \end{array}$$

There was consumed 3.53 pounds of naphtha per IHP. per hour, hence the indicated efficiency of the plant, including fuel, furnace, and engine, was

$$E = \frac{33000 \times 60}{3.53 \times 22000 \times 778} = 0.033,$$

or nearly $3\frac{1}{3}$ per cent. It is a good 3 horse-power engine, that—including fuel, furnace, boiler and engine—yields this efficiency. If the particular naphtha used were richer in hydrogen than that assumed, or rather if its chemical composition gave 23000 thermal units, the efficiency would be reduced to 3.1 per cent.

The cost of running, 6.2 cents per IHP. per hour, is not a measure of the efficiency, but of the economy. A steam-engine, run by the waste fuel of a saw-mill, may cost nothing for fuel; while the same engine run with anthracite coal may cost many dollars daily for this item, while as a heat engine the efficiency should be the same in the two cases.

138. Efficiency of fluid. Any solution of this part of the problem will necessarily be approximate, since some of the data must be assumed; and yet such a solution may give some idea of its probable efficiency, and hence of the efficiency of the furnace and boiler. Regnault found the specific heat of petroleum to be 0.434, and we will assume it to be the same for liquid naphtha. The latent heat of evaporation at atmospheric pressure is 237 B. T. U. per pound, as found above; but the law of change with temperature and pressure is not known. As this value approximates more nearly to that of acetic acid than any other substance now before us (see Article 74 of Addenda), we will assume, although otherwise quite arbitrary, that

$$h_e = 237 - 0.1\ T. \qquad (260)$$

Let the initial temperature of the liquid be 58° F.; the

initial absolute pressure in the cylinder 60 lbs., at which the temperature will be 239° F., Eq. (259); ratio of expansion, 2; back pressure, 16 pounds; and, in order to make as few other assumptions as possible, consider the law of expansion to be

$$p\, v^{1.6} = constant,$$

and since the ratio of expansion is small, this cannot lead to a large error.

Then,

$$p_1 = 60 \times 144 = 8640 \text{ lbs. per sq. ft.}$$

Eq. (86), $$v_1 = \frac{778\,(237 - 0.1 \times 239)}{2\,3026 \times 8640\,\left(\frac{459.4}{500} + \frac{2 \times 625000}{49000}\right)} = 2.18. \qquad (261)$$

Eq. (172), $v_2 = 2.18 \times 2 = 4.36$ cu. ft. (262)

Eq. (177), $U = 2.18 \times 8640\,(10 - 8.35) - 2304 \times 4.36 = 21033$ (263)

ft.-lbs. per pound of naphtha per stroke, or 26.40 thermal units.

The cut-off being one half, the weight of vapor in the three cylinders when half full will be

$$\frac{3 \times 37.5}{2 \times 1728 \times 2.18} = .01493 \text{ lbs.}$$

Hence per pound per revolution the work done will be

$$21033 \times 0.01493 = 314 \text{ foot-pounds.}$$

The mean work per revolution in the preceding experiment was

$$\frac{33000 \times 2.81}{280.7} = 330 \text{ ft.-lbs.}$$

which results agree sufficiently well when we consider that in the ideal diagram there is no compression or clearance.

Mean effective pressure, theory, (176),

$$p_e = 60\,(5 - 4.16) - 16 = 34.4 \text{ lbs. per sq. in.}$$

Mean effective pressure from the experiment—

$$p_e = \frac{330\ (\textit{work per revolution})}{\frac{3 \times 3\ 7 \cdot 5}{1\ 2}\ (\textit{piston displacement})}$$

$$= 35.2 \text{ lbs. per sq. inch,}$$

which is a very fair agreement under the circumstances.

The heat supplied per pound will be, in thermal units, equations (93) and (260),

$$h = c\,(T_1 - T_2) + h_e$$
$$= 0.434 \times 181 + 237 - 24 = 291. \qquad (264)$$

Efficiency of fluid, (263), (264)—

$$\frac{27.03}{291} = 0.0928,$$

which we will call *nine* per cent.

Efficiency of furnace—

$$\frac{0.033}{0.09} = 0.36,$$

or 36 per cent; that is, 36 per cent of the theoretical heat of combustion of the naphtha, as determined by its chemical composition, is utilized by the boiler. This is a good result for so small a boiler.

139. Remarks. In the design of this engine about four square feet of heating surface per horse-power was allowed. This is about one fourth of what would be allowed in the design of a steam boiler. In the former engine the boiler is completely filled with the flame of the burning fluid, and thus is made quite efficient.

The naphtha engine has met with much favor for propel-

ling steam launches and yachts. Their great compactness is apparent from the preceding report ; pressure is raised very quickly not only on account of the low boiling point of naphtha, but especially on account of its very volatile and highly inflammable character when used for fuel. As soon as the supply is cut off, the flame ceases, and no vapor is left in the boiler to be blown off or cooled down, as in the steam-engine. As a fuel it is more conveniently stored in pipes and vessels in the lower part of the boat than coal can be. It appears to be quite as efficient as a steam plant of this power—if not more so. But every power has disadvantages ; and this is objectionable for general purposes on account of the very volatile and inflammable character of the fluid, endangering as it might all combustible material in its vicinity, and becoming more dangerous the greater the quantity stored ; still it is especially adapted to launches.

140. Ammonia engines are those in which ammonia vapor is used instead of steam. These engines are condensing—a condition which is rendered necessary on account of the nature of the substance. Aqua-ammonia is introduced into the boiler ; vapor is generated in precisely the same manner as steam, and, after it is used in the engine, it is condensed and pumped back into the boiler, thus using it over and over.

Much interest has recently been excited in these engines from the fact that ammonia has been substituted for steam, using the same boiler and engine, and doing, it is claimed, the same work with less fuel. It is asserted by some writers that the science of *Thermodynamics* teaches that the efficiency of an engine is independent of the nature of the working fluid when used between the same limits of temperature ; and, hence, the above fact leads such to look with suspicion upon the correctness of this part of the science. The fact is, the bove statement is correct in only one very restricted case, and that case is never realized in practice ; we will,

therefore, state distinctly some of the principles established by this science, bearing upon this part of the subject.

The efficiency of an engine is independent of the fluid used when worked between the same limits of absolute temperature, PROVIDED ALL THE HEAT RECEIVED IS AT ONE TEMPERATURE AND ALL THAT IS REJECTED IS AT ONE LOWER TEMPERATURE, *the mass of fluid in the engine being constant.* (See p. 161.)

If the substance could be worked in this way, steam, ammonia, alcohol, &c., would be equally efficient. The fact that there is a latent heat of evaporation of the substance would not affect the truth of the statement. If water could be worked according to this law, beginning with a temperature of 60° F., evaporated at 250° F., and raised to 300° F., the efficiency would be the same as if the substance were a perfect gas, or

$$E = \frac{300 - 60}{300 + 460} = 0.314.$$

This will be proved for a vapor in the Addenda, Article 112, from which it may readily be inferred to be true for the more general proposition.

This gives the absolute maximum efficiency of any heat engine.

But no engine works according to this law. *In practical vapor engines, the mass of working fluid is variable.* In such engines, it has been shown on page 193, Eq. (o), that the effective work and the efficiency both depend upon the latent heat of evaporation and the specific heat of the substance; hence—

In practice, the efficiency of all vapor engines depends upon the nature of the working fluid, and involves both the latent heat of evaporation and the specific heat.

In a more complete theory of the vapor engine, involving incomplete expansion, the mean specific heat of the fluid and the latent heats of evaporation at the lower and higher

temperatures are involved in such a complex manner as to require a knowledge of their numerical values in order to determine whether, theoretically, one fluid will yield a higher efficiency than another.

Again, this science does *not* teach that the efficiency is independent of the working fluid when worked between the same limits of pressure.

To illustrate this principle, observe that in Articles 110 and 111, the work, equations (167) and (177), is expressed in terms of pressure only; hence, if the same pressures can be obtained at lower temperatures, there may be a gain of efficiency.

Saturated steam at 100 lbs. per sq. in. has a temp. of 327° F.
" " " 16 " " " " " " " " 216° F.
Dif. 111°

Saturated ammonia vapor at 100 lbs. per sq. in. has a temp. of ..... 57° F.
Saturated ammonia vapor at 16 lbs. per sq. in. has a temp. of ..... — 23° F.
Dif. 80°

The heat expended per pound of vapor above the lower temperature will be

for steam, Eq. (184),

$$\mathbf{H} = 111\,J + H_e;$$

and for ammonia

$$\mathbf{H} = 80\,C + H_e.$$

If the indicated work, U, is the same in both cases, since H_e is less for ammonia than for water and C also less than J, the efficiency of fluid will be greater for the former than the latter.

Again, *the formula for maximum efficiency*, $\frac{\tau_1 - \tau_2}{\tau_1}$, *is*

applicable only to a constant mass of fluid when working in the engine; and is not, in any sense, applicable to the plant.

The efficiency of the boiler and steam connections depends upon the absorption of heat by the fluid, radiation from the furnace, and radiation from the connections. It will be seen from the preceding remarks that a given pressure may be produced with ammonia at a much lower temperature than with steam. Such being the fact, a lower temperature may be maintained in the furnace, boiler, and connections, and hence less heat would be lost by radiation. Our knowledge of ammonia does not enable one to determine whether it would absorb more heat in the same time than would water; if it would, the per cent of heat escaping up the chimney would be less, and the direct efficiency of the boiler thus increased. Admitting that an ammonia plant is more efficient than a steam plant, we see that this science explains the cause. But,

Again, this science teaches that a condensing engine is more efficient than a non-condensing one, other conditions being the same.

In the cases above cited that have come to the knowledge of the author, where the ammonia plant proved superior to the steam plant, the steam-engines were non-condensing, while the ammonia engines which replaced them were condensing. Had the original steam-engines been changed to condensing engines there would have been an increase of efficiency; but a want of knowledge of certain physical constants of ammonia prevent this science from determining with certainty whether the efficiency would have been still further increased by substituting it for steam.

If the interest on the cost of a plant and the cost for repairs be considered, a small plant involving a condensing engine may not be as *economical* as with a non-condensing one, although the former may be the more *efficient*.

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Many conditions are involved in the choice of the working fluid, aside from its thermodynamic relations: as the first cost, its effect on the working parts, its safety or danger to life, and the character of the necessary mechanism.

141. A binary vapor-engine is worked by means of two fluids, one more volatile than the other, the fluids being worked in separate cylinders. If a surface condenser of a steam-engine be cooled by ether, the ether may be vaporized by the heat given up by the steam, and made to work a vapor-engine in precisely the same manner as steam drives a steam-engine. The exhausted vapor of the vapor-engine may be passed through a surface condenser cooled by water, and pumped in its cooled condition into the condenser of the steam-engine, when the former operation may be repeated.

A binary-engine is, theoretically, no more *efficient* than a properly designed steam-engine consuming the same amount of heat, and practically is not as *economical* on account of unavoidable waste and extra cost of the mechanism. But a steam-engine wasteful of heat may be improved by the addition of an ether-engine. (*Manuel du Conducteur des Machines à Vapeurs combinées*, M. du Tumbley, Lyons, 1850–51; *Institution of Civil Engineers*, February, 1859.)

142. The products of combustion *sometimes form the working fluid.* In these engines the entire products of combustion enter the cylinder. The principle of their analysis is similar to the Ericsson's hot-air engine. One of the most serious objections to this class of engines is the fact that the solid parts of the working fluid, dust and grit, wear out the working parts, with which they come in contact, rapidly. A recent attempt has been made by MM. Bermier Brothers, Paris, to overcome this objection, but as yet their engine is only an experiment. (*Scientific American Supplement*, 1889, p. 11099.)

THE STEAM-INJECTOR.

143. The injector is a device for feeding steam boilers, in which steam is taken from the boiler, and, by passing through the instrument, takes water with it, carrying the water and condensed steam in a steady stream back into the boiler. Fig. 69 shows an improved form of one of this class of instruments. A valve *W* is secured to the rod *B*, and has its seat on another valve *X*. *A* is a tube containing these valves, and the passage of steam through the

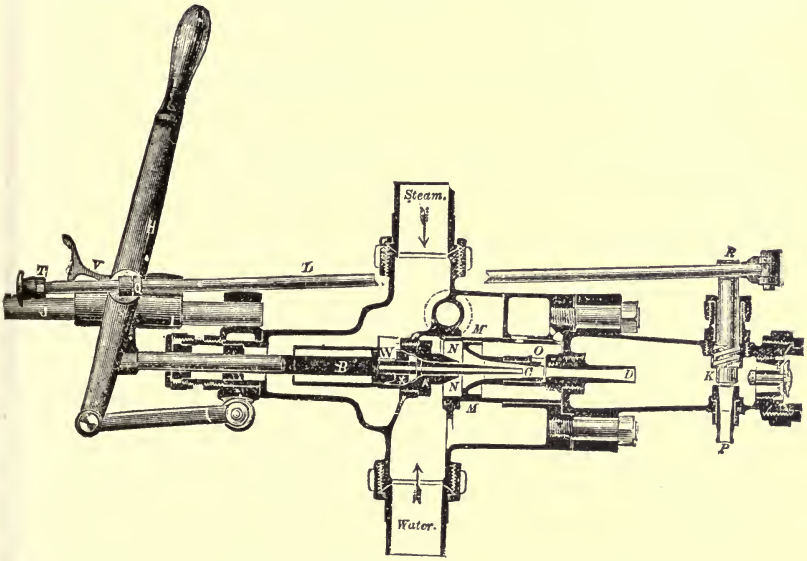

FIG. 69.

tube is controlled by the valve *X*. A hollow spindle, beginning with *W* and terminating at *C*, passes through the valve *X*, and may be moved, independently of the latter, for a short distance, by means of the lever *H*, thus admitting steam to the spindle without moving the valve *X*, but a further movement of the lever will unseat the latter valve. The chamber *M M* contains a piston *N N*, which terminates

in a gradually contracting nozzle at a point O, just beyond C. By a slight movement of the handle H, steam issues from the orifice C, and a partial vacuum will be formed in NN, into which water will be forced by outside pressure, and then forced through the delivery tube D, and at first escape through the waste orifice P, and as soon as a solid stream escapes, a further movement of the lever H closes the orifice P by the valve K, and opens the valve X, and a continuous flow of water will then pass the check-valve into the boiler.

If too much water passes C some will enter the chamber O and force the piston $N\,N$ back, thus throttling the water, and if sufficient water is not admitted the reduced pressure at O will cause the valve to move forward and permit more water to flow in.

144. Theory of the steam-injector.

Let W_0 = the weight of water required of the injector per unit of time,

W = weight of steam required to force W_0 into the boiler. The heat in the steam above that of the feed-water when forced into the boiler will be, in ordinary heat units, considering the specific heat of the water as uniform and equal to unity, equation (134),

$$W h = [\,(\tau_1 - \tau_2) + x\, h_e]\; W, \qquad (265)$$

and this will be the heat lost by this amount of steam in the injector and which is assumed to be imparted to the feed-water.

The heat imparted to the water, above that in the reservoir from which it is taken, will be

$$(\tau_2 - \tau_4)\; W_0, \qquad (266)$$

where

τ_4 = the absolute temperature of the feed-water in the tank,

τ_2 = the absolute temperature of the water just after it has passed the injector,

τ_1 = the absolute temperature of the steam in the boiler.

It will be shown hereafter that the work of lifting the water from the reservoir to the injector and of forcing it into the boiler together require only a small fractional part of the heat energy lost by the steam in having its temperature lowered from that of the boiler to that of the mixture of steam and water; and, neglecting these two elements, expressions (265) and (266) become equal, giving, in terms of ordinary scales of temperature,

$$W = \frac{T_2 - T_4}{T_1 - T_2 + x\,h_e} W_0. \tag{267}$$

For our present purpose it will be sufficiently accurate to assume that the steam supplied to the injector is pure saturated steam, or $x = 1$, and that equation (77) is sufficiently exact, or

$$h_e = 1114.4 - 0.7\ T_1.$$

To find the velocity of the water in the passage G, Fig. 70, let

p = the absolute pressure per unit in the boiler,
p_0 = " " " " " of the atmosphere,
V = the velocity of the water,
δ = weight of unity of volume of the water = 62.4 per cubic foot at ordinary temperatures,

then

$$V = \sqrt{2\,g\ \frac{p - p_0}{\delta}}. \tag{268}$$

The value of δ may be found with sufficient accuracy by means of the formula at the foot of page 102, thus

$$\delta = \frac{1}{v_1} = \frac{1}{0.008\left(\frac{\tau}{500} + \frac{500}{\tau}\right)}, \tag{269}$$

which, for 150° F., or 610° absolute, gives $\delta = 61.2$ pounds, and this value might properly be used in equation

(268), but as 62.4 pounds, the weight at ordinary temperatures, will not produce an error of 1 per cent in the velocity, and as by its use the resulting formula will be more generally applicable to ordinary cases, we retain the latter. Just after entering the chamber G, the water will be under atmospheric pressure, and $p_0 = 2116.2$ pounds per square foot, and $2\,g = 64.4$. With these values, equation (268) reduces to

$$V = 1.0158\,\sqrt{p - 2116.2}\text{ ft. per sec.} \tag{270}$$

If p be in pounds per square inch,

$$V = 12.1896\,\sqrt{p - 14.7}\text{ ft. per sec.} \tag{271}$$

If p be in atmospheres,

$$V = 46.7355\,\sqrt{p - 1}\text{ ft. per sec.} \tag{272}$$

If the diameter of the suction pipe F be n times that of the passage E, the velocity in it will be

$$V_2 = \frac{V}{n^2}. \tag{273}$$

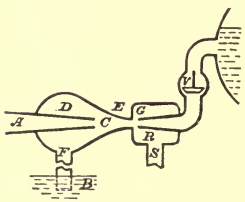

FIG. 70.

To find the area of the opening E for the passage of the water; consider that the steam passing through the injector will have been condensed to liquid water, then will the volume of the water and condensed steam passing the opening per second be $0.016\ (W + W_0)$ cubic feet, and if k be the area of this section, then

$$k = \frac{0.016\ (W + W_0)}{V}. \tag{274}$$

The diameter will be

$$d = 2\sqrt{\frac{k}{\pi}}. \tag{275}$$

To find the velocity of the steam issuing from the end of the passage C, it will be necessary to find the pressure in the condensing chamber. Let

p_2 be the pressure in the condensing chamber D,

p_0, the pressure of the atmosphere on the water in the tank B,

$h = C\,B$, the height of the condenser *above* the water in the tank,

V_2, the velocity of the water at F, entering the condensing chamber,

then

$$V_2^2 = 2\,g\left[\frac{p_0 - p_2}{\delta} - h\right], \tag{276}$$

in which h is *negative* because the water is *raised* instead of being a positive head. From this may be found

$$p_2 = p_0 - \delta\,h - \frac{\delta\,V_2^2}{2\,g}. \tag{277}$$

The velocity of the steam V_1 will be given by the general equation for V, following equation (62), page 82, and after substituting for τ_2 and τ_1 their values in terms of p and v, becomes

$$V_1^2 = 2\,g\,\frac{\gamma\,p\,v}{\gamma - 1}\left[1 - \left(\frac{p_2}{p}\right)^{\frac{\gamma-1}{\gamma}}\right], \tag{278}$$

in which γ has the value in equation (148), page 152. If the steam contains no moisture, this becomes

$$V_1 = 23.2687\sqrt{p\,v\left[1 - \left(\frac{p_2}{p}\right)^{0.1189}\right]}. \tag{279}$$

The area of the cross-section at C will be

$$F_1 = \frac{\textit{Volume of steam per sec.}}{V_1}, \tag{280}$$

and the diameter will be

$$d_1 = 2\sqrt{\frac{F_1}{\pi}}. \tag{281}$$

The work done by the injector will be that of forcing the mixture of steam and water against the boiler pressure p sufficiently far to make a displacement for $W + W_0$ pounds of water. Since the steam will be subjected, externally, to the atmosphere the resultant pressure against which the water is forced will be the gauge pressure, or $p - p_0$. Hence, if p be in pounds per square inch the work will be

$$U = 144\,(p - p_0)\,(W + W_0)\,0.016 \text{ ft. lbs.} \quad (282)$$

The efficiency as a force-pump will be

$$E = \frac{\textit{Work done}}{\textit{Heat expended}} = \frac{U}{(\tau_1 - \tau_2 + x\,h_e)\,W\,J}. \quad (283)$$

The efficiency of the plant. If 1 pound of coal is equivalent to q thermal units, and w pounds are required to generate W pounds of steam from the temperature of the feed-water, then

$$E' = \frac{U}{J\,q\,w}, \quad (284)$$

and if all the heat of the coal could be utilized for generating steam and the steam were pure saturated, E' would be the same as E. But there is always a waste of heat in the furnace and boiler. If the q thermal units would evaporate n pounds of water at and from 212° F., if there were no waste of heat, and in an actual boiler a pound of coal did evaporate n_1 pounds under the same conditions; then if T_4 be the temperature of the feed-water and $\mathbf{H}$ the total heat of steam at the temperature T_1 of the boiler, then

$$W\,(\mathbf{H} - T_4) = 966\,n_1\,w = 966\,n \cdot \frac{n_1}{n}\,w = q\,w\,\frac{n_1}{n}; \quad (285)$$

$$\therefore E' = \frac{U}{J\,W\,(\mathbf{H} - T_4)} \cdot \frac{n_1}{n}. \quad (286)$$

The value of n may be, theoretically, from 11 to 15, depending upon the composition of the coal, and n_1 from 6 to

11, depending upon the composition of the coal and the efficiency of the furnace.

The duty will be the work done per 100 pounds of coal, or,

$$D = 100 \times 144 \times \textit{gauge pressure} \times \left[\begin{array}{c}\textit{Volume of water injected per lb. of steam.}\end{array}\right] \times \left[\begin{array}{c}\textit{Pounds of steam evaporated per lb. of coal.}\end{array}\right]. \quad (287)$$

Effect of rejecting the work of raising the water and of forcing it into the boiler in the above analysis.

In the following exercise it will be seen that if the gauge pressure be 90 pounds per square inch, and other conditions as there given, there will be expended

$$(331 - 120 + 794) \times 0.05 = 50$$

thermal units in supplying 0.833 pounds of water to the boiler. To raise this weight of water 20 feet by suction—a distance too large to be realized in practice—would require

$$0.833 \times 20 \div 778 = 0.02$$

thermal units, which is only $\frac{1}{2500}$ of 50, and hence may be omitted in the computation.

The *work* of forcing 0.833 pounds into the boiler will be, equation (282), in thermal units,

$$U = 144 \times 90 \times 0.833 \times 0.016 \div 778 = 0.22,$$

which is also so small compared with 50 that it may be omitted. The theory above given, in which these two items are omitted, is, then, sufficiently accurate for engineering purposes.

EXERCISE.

If the steam pressure in a boiler is 90 pounds gauge per square inch, height of suction 4 feet, and the boiler is required to make 3000 pounds of steam per hour; required

the area of the section k of the passage E for the water, the velocity of the steam V, at C, the diameter of the suction-pipe, its section, being 5 times that of the section k (which is an average of actual values), the steam containing 10 per cent of moisture, the feed-water in the tank being 60°, the temperature of the mixture of water and condensed steam 120° before it is forced into the boiler; also the ratio of the velocity of the steam to that of the water, and the weight of water to that of the steam.

We have

$$p = 104.7,\ h = 4,\ T_4 = 60°,\ T_2 = 120°,\ n = 5.$$

From steam table, or equation (81), page 97, find $T_1 = 330°.9$ F.

$$W_0 = \frac{3000}{3600} = 0.833 \text{ lbs. per second.}$$

$$h_e = 1114.4 - 0.7 \times 330.9 = 882.6, \text{ Eq. (77).}$$

$$x_1 = 0.9\ ;\ \therefore x_1 h = 794.34.$$

$$W = \frac{(120 - 60)\,0.833}{331.13 - 120 + 794.34} = 0.04982 \text{ lbs., Eq. (267). Call this result } 0.05.$$

$$\frac{W_0}{W} = 16\,6.$$

$$V = 12.1896\sqrt{90} = 115.63 \text{ ft., vel. of water at } E \text{ and } G, \text{ Fig. 70, Eq. (271).}$$

$$k = \frac{0.016\,(0.05 + 0.833)}{115.63} = 0.00122 \text{ sq. ft.} = 0.017568 \text{ sq. in., Eq. (274).}$$

$$d = 0.149 \text{ in., diameter of water passage } E \text{ or } G, \text{ Eq. (275).}$$

$$p_2 = 2116.2 - 4 \times 62.4 - \frac{62.4 \times (115.63)^2}{625 \times 64.4} = 1845.87 \text{ lbs. per sq. ft.}$$

$$= 12.818 \text{ lbs. per sq. in. at } F, \text{ Eq. (277).}$$

$$v = 4.217, \text{ volume of one pound of steam at 104.7 lbs., Eq. (86).}$$

$$V_1 = 23.2687\sqrt{104.7 \times 144 \times 4.217\left[1 - \left(\frac{12.818}{104.7}\right)^{0.111}\right]}$$

$$= 2572.5 \text{ ft. per sec. velocity of the steam at } C. \text{ Eq. (278).}$$

$$F_1 = 4.22 \times 0.05 \times 144 \div 2572.5 = 0.0110 \text{ sq. in., Eq. (280).}$$

$$d_1 = 0.12, \text{ diameter of steam nozzle, Eq. (281).}$$

$$\frac{\textit{Vel. of steam}}{\textit{Vel. of water}} = \frac{2572.3}{115.63} = 22.$$

No allowance has been made in this computation for contraction or frictional resistances, and hence the diameters must be made larger than here found in order to deliver the assumed amounts. The diameters should be about 1.1 to 1.2 times those here found.

$U = 144 \times 90 \times (0.833 + 0.05) \times 0.016 = 183.099$ ft. lbs., Eq. (282).
$= 0.235$ thermal units.

$E = \frac{0.235}{1005 \times 0.05} = 0.0046$, Eq. (283), or the efficiency is less than one-half of one per cent.

If 10 pounds of coal evaporates 1 pound of water at and from 212° F., it will require, under the conditions of this exercise, to evaporate 0.05 pound of water,

$$w = \frac{0.05\ (1182.8 - 60)}{966 \times 10} = 0.00581 \text{ lb., Eq (285).}$$

If the coal be equivalent to pure carbon, it would evaporate, without loss of heat, $14500 \div 966 = 15$ lbs. at and from 212° F., and if one pound in the plant actually would evaporate 10 lbs., then would the efficiency of the plant be

$$E' = \frac{183.099}{778 \times 0.05\ (1183 - 60)} \cdot \frac{10}{15} = 0.00418 \times \frac{2}{3} = 0.00279, \text{ Eq. (286).}$$

$Duty = 1296000 \times (16.6 \times 0.016) \times 10 = 3442176$ ft. lbs., Eq. (287).

TABLE I.

GIVING CERTAIN RELATIONS WHEN THE DELIVERY INTO THE BOILER IS 1 POUND OF WATER PER SECOND, NEGLECTING LIFT AND WORK OF FORCING THE WATER INTO THE BOILER; TEMPERATURE OF THE FEED-WATER BEING 60° F., AND OF THE MIXTURE AND STEAM BEFORE ENTERING THE BOILER 160° F.

Gauge Pressure.	Diameter of Steam Nozzle in Inches. Eq. (281.)	Diameter of Water Nozzle in Inches. Eq. (275.)	Velocity of Steam, Ft. per Second. Eq. (279.)	Velocity of Steam and Water, Ft. per Second. Eq. (271.)	Ratio of Velocities. Col. (4) ÷ (5)	Ratio of Weight of Water to Steam. Eq. (267.)	Ratio of Volume of Steam to Water.
30	0.28	0.21	2007 9	66.7	30.	10.3	55.9
40	0.24	0.20	2178.8	77.1	28.	10.3	46.2
50	0.22	0.19	2213.5	86.2	25.	10.4	39.4
60	0.20	0.18	2428.8	94.4	25.	10.5	34.4
70	0.18	0.178	2522.3	101.2	25.	10.5	30.4
80	0.17	0.172	2554.1	108.0	24.	10.5	37.6
90	0.167	0.166	2590.6	115.6	22.	10.5	25.2
100	0.159	0.160	2735.8	121.8	22.	10.5	22.8
120	0.142	0.154	2842.7	133.5	21.	10.6	19.6
140	0.133	0.149	2922.3	144.2	20.	10.6	17.2
160	0 127	0.143	2999.7	154.2	19.	10.6	15 3
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

As the temperature 160° F. of the mixture of steam and water is near the higher limit of reliable working of the injector, we take another case of lower temperature.

TABLE II.

GIVING RESULTS FROM THE SAME DATA AS FOR TABLE I, EXCEPT THAT THE TEMPERATURE OF THE MIXTURE IS ASSUMED TO BE 140° F.

Gauge Pressure.	Diameter of Steam Nozzle in Inches. Eq. (281)	Diameter of Water Nozzle in Inches. Eq (275)	Velocity of Steam, Ft. per Second. Eq. (279.)	Velocity of Steam and Water, Ft. per Second. Eq. (271.)	Ratio of Velocities. Col. (4) ÷ (5).	Ratio of Weight of Water to Steam. Eq. (267.)	Ratio of Volume of Steam to Water.
30	.25	.18	2030	66.7	30.4	13.21	44.2
40	.218	.202	2211	77.1	28.0	13.29	36.3
50	.197	.190	2326	86.2	27.0	13.29	31.1
60	.158	.180	2431	94.4	25.6	13.33	27.1
70	.150	.175	2526	101.2	24.9	14.52	22.0
90	.146	.164	2677	115.6	23.1	13.42	19.6
100	.137	.160	2743	121.9	22.5	13.60	17.7
120	.125	.152	2845	133.5	21.3	13.49	15.4
140	.116	.147	2934	144.2	20.3	13.53	13.5
160	.108	.142	3008	154.2	19.5	13.58	12.0
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

145. Approximate Formulas. Certain general inferences will be apparent if we assume average conditions.

Let $T_1 = 60°$ F., $T_2 = 150°$ F., and $x = 1$, then, equation (267),

$$\frac{W_0}{W} = \frac{964.4}{90} + \frac{1}{300} T_1. \qquad (288)$$

If the gauge pressure be 80 pounds, then $T_1 = 323°$ F., and

$$\frac{W_0}{W} = 11.79;$$

that is, under ordinary conditions, 1 pound of steam will inject about 12 pounds of water into the boiler; or 13 pounds, including its own weight.

For the diameter of the cylindrical water-passage, E, equation (275),

$$d = 2\sqrt{\frac{0.016 \times \frac{13}{12}}{12.1896^{\pi}}}\ \frac{\sqrt{W_0}}{\sqrt[4]{p - 14.7}}; \qquad (289)$$

that is, the diameter will vary directly as the square root of the weight of water injected per second, and inversely as the fourth root of the gauge boiler pressure.

The velocity of the steam, according to the preceding tables, will be about half a mile per second.

The velocity of the water will be about 100 feet per second.

The duty will be, if gauge pressure = 80, and 9 pounds of steam be generated per pound of coal,

$$D = 1152000 \times 12 \times 0.016 \times 9 \text{ nearly} = 2000000 \text{ nearly}.$$

Since there will be some frictional resistance and radiation, and since 9 pounds of water are rarely evaporated at 80 pounds gauge, the duty would be somewhat less than 2000000.

Efficiency, equation (283),

$$E = \frac{144 \times 80 \times 13 \times 0.016}{778 \times 1059 \times 1} = 0.00293, \qquad (300)$$

which is about $\frac{3}{10}$ of 1 per cent. The efficiency of the plant would be about $\frac{1}{5}$ of 1 per cent as a pump.

146. Injector compared with Direct-Acting Pump. By comparing these results with those on page 182 it will be seen that the efficiency and duty of the injector are much less than that of a direct-acting pump—being about $\frac{1}{5}$ as efficient. This is for service as a pump. But as a heat device, if there be no radiation nor lift of feed-water the efficiency of the injector will be *perfect;* similarly, if

all the exhaust heat from the direct-acting pump be returned to the boiler, and there be no radiation, the heat efficiency of the pump will also be *perfect;* and hence in either case *would cost nothing for fuel.* In both cases the furnace (or boiler) heats the water from the temperature of the feed to that of the boiler. If there be no losses from radiation, the difference in the cost for fuel in running the two devices will be that which furnishes the steam for running the pump for doing the same work, if this steam be wasted at the exhaust. To illustrate: the work done by 1 pound of steam in the approximate cases above is that of forcing 13 pounds of water against 80 pounds pressure, and is

$$U = 144 \times 80 \times 13 \times 0.016 = 2396 \text{ ft. lbs.}$$

One pound of steam in the direct-acting pump will, at about 70 or 80 lbs. boiler pressure, do the actual work of

$$10{,}000 \text{ foot-pounds;}$$

hence, to do 2396 foot-pounds will require

$$2396 \div 10000 = 0.24 \text{ lbs., nearly,}$$

of steam; hence, it requires, in this case, about 24 hundredths as much steam to feed the boiler with a direct-acting pump as with an injector. But this steam is saved by the injector, and, we assume, is wasted by the pump. If 1 pound of coal generate $8\frac{1}{2}$ pounds of steam under a pressure of 80 lbs. gauge, this waste will require $0.24 \div 8.5 = 0.0282$ pounds of coal for every 12 pounds of feed-water forced into the boiler. To evaporate this 12 pounds of water will require $12 \div 8.5 = 1.41$ pounds of coal; hence, the fractional part of the fuel required by the pump will be

$$0.0282 \div 1.41 = 0.02,$$

or about 2 per cent of the fuel burned in the furnace.

Or, if the engine requires 30 pounds of feed-water per horse-power per hour, it will require

$$0.0282 \times \tfrac{30}{12} = 0.0705$$

pounds of coal to work the feed-pump per hour per horse-power; and if the plant requires $3\frac{1}{2}$ pounds of coal per horse-power per hour, then will the fractional part of the fuel required by the feed-pump be

$$\frac{0705}{35000} = 0.0201,$$

or about 2 per cent of the fuel burned, as before found.

The low efficiency of the injector, as a pump, is due to the fact that the high velocity of the steam is very suddenly reduced to a comparatively low one by its impact against the non-elastic water, and the kinetic energy lost by the steam will be as the difference of the squares of the velocity before impact and that after.

Considering the velocity of the steam as 25 times that of the mixture, and the weight of the mixture as 13 times that of the steam, the kinetic energy of the mixture will be

$$13\,(\tfrac{1}{25})^2 = 0.0208$$

of the initial energy of the steam; or 98 per cent of the initial energy is lost by the change of velocity at E. The 2 per cent remaining is gradually diminished on account of the decreasing velocity in the passage from G to V.

The thermodynamic theory of the inspirator is the same as that of the injector.

Steam-injectors are also used as pumps where intermittent action is required, as in the hold of a ship, and in mines; also as ejector-condensers when attached to the escape-pipe of a condensing engine to avoid the use of an air-pump; also as a gas-pump where it was more efficient than as a water-pump; also as a steam-blower; and also in the well-known case of the locomotive exhaust.

THE PULSOMETER.

147. The Pulsometer is a pump consisting principally of two bottle-shaped chambers, A, A, joined together side by side, with tapering necks bent toward each other, uniting in one common upright passage, into which a small ball, C, is fitted so as to oscillate with a slight rolling motion between seats formed in the junction.

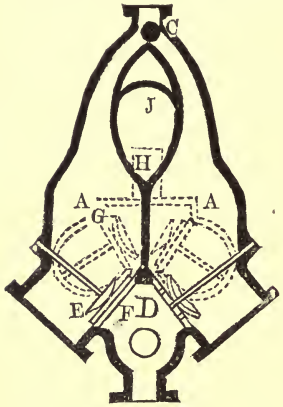

FIG. 71.

These chambers also connect by means of openings with the vertical induction passage, D, having valves, E, E, and their seats, F, F.

The delivery passage, H, which is common to both chambers, is also constructed so that in the openings that communicate with each cylinder are placed valve-seats fitted for the reception of the same style of valves, G, G, as in the induction passage.

J represents the air chamber, cast with and between the necks of chambers A, A, and connects only with the induction passage below the valves E, E.

A small brass air check-valve is screwed into the neck of each chamber, A, A, and one into the vacuum chamber J, so that their stems hang downward. Those in the chamber allow a small quantity of air to enter above the water, to prevent the steam from agitating it on its first entrance.

Conceive that the left chamber is full of water; steam passes to the left of the valve C, and acting by its pressure directly upon the upper surface of the water, forces the water through the valve G and into the air chamber J. During this operation the chamber A is being filled, and water by its momentum finally drives the valve C to the

left, thus cutting off steam communication with the left chamber in which the steam condenses, forming a vacuum, when water will be forced through the valve E by atmospheric pressure into the left chamber, while the steam is forcing the water out of the right chamber. All the steam entering the pump is condensed and forced out with the other water; and the temperature of the discharged water will be higher than that entering the pump.

148. Analysis. The work done by the pulsometer will be that of lifting the water from the source to the pump by the operation usually called "suction," and of lifting this water and the condensed steam to the point of delivery—neglecting losses, such as friction, contractions, etc.

Let

W_0 be the weight of water raised in a unit of time,
W, the weight of steam used in the same time,
T_0, the temperature of the water at the source,
T_1, " " " " mixture,
T, " " " " steam,
h_e, the latent heat of evaporation of the steam,
h_0, the height of the pump above the source,
h_1, " " " " delivery above the pump,
h, the total height $= h_0 + h_1$.

Considering the specific heat of water as constant and equal to unity, the heat lost by the steam will be

$$W(T - T_1 + h_e),$$

and the heat gained by the water will be

$$W_0(T_1 - T_0),$$

and no allowance being made for radiation, these will be equal;

$$\therefore \frac{W_0}{W} = \frac{T - T_1 + h_e}{T_1 - T_0}. \tag{301}$$

Observing the boiler pressure, and the temperature of the water before and after mixture, the ratio of the weight of the steam to that of the water may be determined.

The work will be

$$U = W_0 h_0 + (W_0 + W) h_1 \tag{302}$$

If the temperature of the feed-water be the same as that of the source, or T_0, then will the heat expended be

$$\mathbf{H} = J W (T - T_1 + h_e); \tag{303}$$

hence the efficiency will be

$$E = \frac{U}{\mathbf{H}} = \frac{W_0 h_0 + (W_0 + W) h_1}{J W (T - T_1 + h_e)}. \tag{304}$$

If the work of lifting the condensed steam and frictional resistances be neglected, then

$$E = \frac{h}{J (T_1 - T_0)}, \text{ nearly.} \tag{305}$$

EXERCISES.

1. By actual measurement 105000 gallons of water were raised in ten hours with 274 pounds of coal a height of 38 feet, and drawn horizontally 600 feet. If 10 per cent be allowed for resistances, find the work done in ten hours, the weight of water raised per pound of coal and the horse-power; and if a pound of coal evaporated $7\frac{1}{3}$ pounds of water, find the pounds of coal required per horse-power per hour, the weight of water raised per pound of steam, the increase of temperature of the water pumped, assuming its initial value to be 60° F., the gauge pressure 50 pounds, and the efficiency, the feed-water also being 60° F.

If a gallon be 231 cubic inches, and a cubic foot be 62.2 pounds, then

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Weight of water, $105000 \times \frac{231}{1728} \times 62.2$ lbs. = 873000.
Work for 10 hours, $873000 \times 38 \times 1.10$ ft. lbs. = 36491400.
" " 1 hour, ... 3649140.
Horse-power, ... 1.84.
Coal per horse-power per hour, lbs., . . . 14.8.
Water raised per pound of coal, lbs., . . 3186.
Pounds of steam, $274 \times 7\frac{1}{3}$, ... 2000.
Water raised per pound of steam, lbs., . . 436.5.
Work done per pound of steam, ft. lbs., . . 18246.
Heat in the steam above 60° F., B. T. U., . 1137.
Increased temp. of water, $1137 \div 436.5$, Deg. F., 2.6.

Efficiency, $\frac{16587}{1137 \times 778} =$ 0.0187

Efficiency, Eq. (305), ... 0.0180

or less than 2 per cent. (See page 452.)

The assumption in regard to the evaporating power of the furnace would make the efficiency of the furnace about 56 per cent, making the efficiency of the entire plant over 1 per cent.

Diameter of discharge pipe, if the coefficient of discharge be 0.8 and velocity 4 feet per second,

$$d = 12\sqrt{\frac{105000 \times \frac{231}{1728} \div 36000}{4 \times 0.7854 \times 0.8}} \text{ inches} \ldots\ldots\ldots 3.01.$$

A three-inch pipe was used.

Pressure producing a velocity of 5 feet per minute against the atmosphere and a head of 38 feet of water, pounds per square inch. 31

2. If the temperature of the source be 60° F., of the mixture 65° F., the gauge pressure 60 pounds, lift by suction 5 feet and lift above the pump 15 feet; required the number of pounds of water raised per pound of steam, the efficiency; also the horse-power if 300 pounds of steam are used per hour. (These quantities are ideal.)

COMPRESSED AIR-ENGINE.

149. A Compressed Air-Engine is an engine in which the working fluid is common air under a high tension. The air is usually compressed by a machine called an air-compressor, to a tension of from 40 to 1000 pounds to the square inch, and stored in an air reservoir, called a receiver, from which it is taken for driving an engine. Any ordinary steam-engine may be run by compressed air; the only practical difficulty being the tendency of the moisture in the air to freeze, and thus choke the exhaust. The freezing may be prevented by causing a circulation of warm air about the exhaust passage through channels especially provided; and without this the evil may be mitigated in a measure by a proper form of the exhaust passage—gradually enlarging it as it goes outward—and making it smooth, so that the ice, if formed, will not adhere so firmly.

150. Analysis. We assume that the cylinder is filled with air of uniform pressure and temperature up to the point of cut-off, that then it expands according to an assumed law, then exhausts and a uniform back pressure during the back stroke; also that there is no clearance. The diagram cleared from irregularities and clearance will be similar to $A\ B\ C\ E\ F\ A$, Fig. 72.

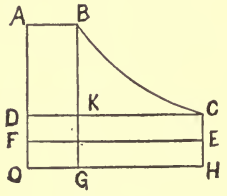

FIG. 72.

Let p_1 be the absolute pressure $O\ A$,
p_2, the absolute pressure $H\ C$ at the end of expansion,
p_3, the absolute back pressure $H\ E$,
v_1 the volume of a pound of air at the pressure p_1,
v_2 " " " " " " " ". p_2.

(*a*) *Adiabatic expansion—incomplete.* The work done per pound at full pressure will be

$$p_1 v_1.$$

The work done per pound during expansion will be, Exercise 3, page 64, or the second equation in Article 56,

$$\int_{v_1}^{v_2} p\,dv = -C_v \int_{\tau_1}^{\tau_2} d\tau = C_v(\tau_1 - \tau_2).$$

The negative work during the return stroke will be

$$p_3 v_2;$$

hence, in the cycle, the work done per pound will be

$$U = A\,B\,C\,E\,F = C_v(\tau_1 - \tau_2) + p_1 v_1 - p_3 v_2. \quad (306)$$

Since the fluid is considered perfect, we have, equations (2) and (29),

$$p_1 v_1 = R\tau_1 = (C_p - C_v)\tau_1,$$
$$p_2 v_2 = R\tau_2 = (C_p - C_v)\tau_2;$$

$$\therefore U = C_v(\tau_1 - \tau_2) + (C_p - C_v)\left(\tau_1 - \tau_2 \frac{p_3}{p_2}\right). \quad (307)$$

(*b*) *Adiabatic expansion—complete.* The back pressure will be along $C\,D$, and $p_3 = p_2$, $\tau_2 = \tau_3$, and

$$U = A\,B\,C\,D = C_p(\tau_1 - \tau_2) = J\gamma c_v(\tau_1 - \tau_2). \quad (308)$$

(*c*) *If there be no expansion*, $p_2 = p_1$, $\tau_1 = \tau_2$, and

$$U = (C_p - C_v)\left(1 - \frac{p_3}{p_2}\right)\tau_1. \quad (309)$$

Equations (307) and (309) may be put in a more symmetrical form by introducing an auxiliary τ_x, thus:

$$C_v(\tau_1 - \tau_2) + (C_p - C_v)\left(\tau_1 - \tau_2 \frac{p_3}{p_2}\right) = C_p(\tau_1 - \tau_x);$$

$$\therefore \frac{\tau_x}{\tau_2} = \frac{C_v}{C_p} + \frac{C_p - C_v}{C_p} \cdot \frac{p_3}{p_2} = \frac{1}{\gamma} + \frac{\gamma - 1}{\gamma}\frac{p_3}{p_2}. \quad (310)$$

Equation (309) will reduce to precisely the same value; hence (307) and (309) become

$$U = C_p\left(1 - \frac{\tau_x}{\tau_1}\right)\tau_1. \quad (311)$$

Making a table of values of $\frac{\tau_x}{\tau_2}$, having for argument $\frac{p_2}{p_3}$, the computation for the work may be much facilitated. Let $\gamma = 1.41$, then :

$\frac{p_2}{p_3}$	$\frac{\tau_x}{\tau_2}$	$\frac{p_2}{p_3}$	$\frac{\tau_x}{\tau_2}$
2	0.855	9	0 742
3	0.807	10	0.739
4	0.783	11	0.736
5	0.768	12	0.734
6	0.758	13	0.732
7	0.732	14	0.731
8	0.746	15	0.730

Final temperatures. Knowing the initial temperature, the final may be found from equation (41), which is

$$\tau = \tau_1 \left(\frac{p_2}{p_1}\right)^{\frac{2}{7}} \tag{312}$$

Final Temperatures, the initials being $T_1 = 68°$ F. or $\tau_1 = 528°.66$.

$\frac{p_1}{p_2}$	Final Temp. Deg. F.	$\frac{p_1}{p_2}$	Final Temp. Deg. F.
2	− 28	9	−181
3	− 76	10	−189
4	−107	11	−197
5	−129	12	−203
6	−148	13	−209
7	−160	14	−214
8	−171	15	−219

Such low temperatures are fatal to successful working if moisture be present in the working air, as ice would be formed in the exhaust. Either the initial temperature must be considerably higher or the range of pressures must be small, or adiabatic expansion avoided, unless the air be thoroughly dry.

(*d*) *Let the expansion be isothermal and incomplete.*

Then

$$p_1 v_1 = p_2 v_2 = R \tau_1, \tag{313}$$

and

$$U_2 = A\ B\ C\ E\ F = p_1 v_1 + \int p\, d v - p_3 v_2$$

$$= p_1 v_1 \left[1 + log_e \frac{v_2}{v_1} - \frac{p_3}{p_2} \right]. \tag{314}$$

For the same cut-off, the isothermal will lie wholly above the adiabatic, and U_2 in equation (314) will exceed U in equation (306). In order to secure isothermal expansion it is necessary to heat the air in the cylinder during expansion. It is not, however, practical to maintain a uniform temperature in this way. The very low final temperature has, however, in practice been prevented by working the air in one cylinder through a part of the full range of pressures, then exhausting into a receiver and there heating it, after which it is worked in a second cylinder.

(*e*) *Expansion isothermal and complete.* In this case the terminal pressure *H C*, Fig. 72, will equal the back pressure *H E*; or $p_3 = p_2$ in equation (314);

$$\therefore U = D\ A\ B\ C = G\ B\ C\ H$$

$$= p_1 v_1 log_e r = 122.5\ \tau_1 log_{10} r. \tag{315}$$

Weight of air per minute. Let W be the number of pounds of working air necessary to deliver N horse-powers per minute, then, since U is the work in foot-pounds per pound of air,

$$W U = 33000\ N;$$

$$\therefore W = \frac{33000\ N}{U}. \tag{316}$$

Volume of the cylinder. If p_0, v_0, be respectively the pressure and specific volume of the air before compression, and if the temperature of the air entering the engine be the same as before compression, then $p_1 v_1 = p_0 v_0$, and the final

specific volume v_2 may be found when the law and amount of expansion are fixed. The terminal volume of W pounds will be $W v_2$, and this will equal the volume swept through by the piston per minute, if there be no clearance. Let V be the volume of the cylinder and n the number of single strokes of the piston per minute, then for a double-acting engine,

$$2\,n\,V = W v_2 = W R \frac{\tau_2}{p_2};$$

$$\therefore\ V = \frac{33000\ N\ R\ \tau_2}{2\,n\,U\,p_2}. \tag{317}$$

Efficiency. In order to determine the efficiency, the full cycle of operations must be known, and this involves the law of compression, which will be considered in the discussion of the air-compressor. We know, however, if air were compressed according to any law and expanded according to the same law, there being no escape of heat by radiation between the states of expansion and compression, that the efficiency would be unity; but there would be no resultant work, even neglecting the friction of the engine.

The above formulas being for perfect conditions must be modified in order to conform to practice. Pernolet determined that the moisture in the air, when converted into vapor, did not materially affect the theoretical results of considering the air as dry. The weight of air as determined from equation (316) must be increased to allow for clearance, leakage, and imperfect working, as is done with the steam-engine; and this must be still further increased in determining the weight of air before it enters the compressor, to allow for the imperfect working of the compressor.

Compressed air-engines are frequently used where if steam were used there would be excessive condensation, as, in mines and other underground work, for driving drills, pumps, hoisting engines and locomotives; also for small intermittent powers.

THE COMPRESSOR.

151. An air-compressor is a kind of air-pump for receiving air at ordinary conditions, and after compressing it to a higher tension, forcing it into a vessel called a receiver. It is not a motor, but requires a motor for driving it. The principles of construction are substantially the same as for an hydraulic pump, although in detail clearance spaces must be as small as possible and the valves be so made as to work with certainty. *The valves* are the most important details, and have received a large amount of attention from inventors and practical men. The best condition for the proper working of the air valves, both inlet and exit, is to have them open and close by moving vertically and automatically; and for this reason the compression cylinder has often been placed vertically, although vertically moving valves are used with horizontal cylinders. In the latter case, at least two valves at each end of the cylinder are commonly used—one for inlet, the other for outlet. When the cylinders are vertical, the compression cylinders are frequently single acting, and are driven by a double-acting steam cylinder. The steam cylinder may be vertical or horizontal. In some cases the axes of the cylinders have been inclined to each other, but the horizontal types are most common. Other fluids than air may be compressed in such a compressor.

Fig. 73 is a view of a duplex air-compressor made by the author and worked in a silver mine in Colorado. The two steam cylinders are at the left hand, and the other two are the compression cylinders.

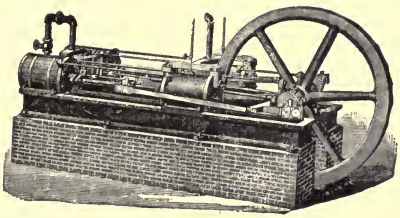

FIG. 73.

The cranks are so set that the steam in one cylinder will

be at full pressure when the piston in the air cylinder on the other side will be near the end of its stroke where resistance is greatest.

152. Analysis. During the back stroke of the piston the air flows into the cylinder; assume that it has the uniform pressure $O\ D$, Fig. 74. During the return stroke the pressure rises from C to B, and the air is then forced into a receiver at a pressure which we assume to be uniform and equal to $O\ A$.

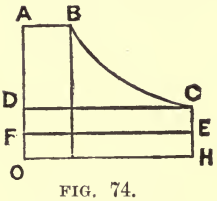

FIG. 74.

Let p_2', v_2' τ_2' represent state C,

p_1', v_1', τ_1' " " B, the subscripts denoting the states ordinarily used in this work, and the accents distinguishing them from the notation of an engine.

a. Adiabatic compression. The work will be, equation (308),

$$U' = A\ B\ C\ D = C_p\ (\tau_1' - \tau_2'). \qquad (318)$$

For air $C_p = 184.77$ (p. 53). We have, equation (42), page 61,

$$\tau_1' = \tau_2' \left(\frac{p_1'}{p_2'}\right)^{\frac{2}{7}}, \qquad (319)$$

where $\gamma = 1.4$. From this the final temperature due to compression may be found. Thus:

FINAL TEMPERATURES, THE INITIAL TEMPERATURE = 68° F., OR $\tau = 528.66°$.

$\frac{p_1'}{p_2'}$	Final Temp. Deg. F.	$\frac{p_1'}{p_2'}$	Final Temp. Deg. F.
2	186	9	538
3	266	10	569
4	329	11	599
5	382	12	625
6	427	13	650
7	468	14	675
8	505	15	700

The temperature exceeds that of boiling water under the corresponding pressure before the tension reaches four atmospheres.

b. Isothermal compression. Here $p_1' v_1' = p_2' v_2'$; and

$$U_2' = A B C D = p_1' v_1' - p_2' v_2' + \int_{v_1'}^{v_2'} p \, dv = 122.5 \, \tau_2 \log_{10} \frac{v_2'}{v_1'} \quad (320)$$

which is equation (~~309~~ 315). Equations (318), (319), (320), give

$$\frac{U_2'}{U'} = \frac{2}{3} \cdot \frac{\log_{10} \frac{p_1'}{p_2'}}{\left(\frac{p_1'}{p_2'}\right)^{\frac{2}{7}} - 1}, \text{ nearly,} \quad (321)$$

which is less than unity for all practical cases; hence *isothermal compression requires less work than adiabatic compression between the same pressures.*

Isothermal compression is secured approximately by injecting water into the cylinder in the form of a fine spray in sufficient quantity to absorb the heat due to compression. The same result is secured less efficiently by performing part of the compression in one cylinder, and allowing the air to cool in a receiver, after which the compression is completed in another cylinder. If air is compressed adiabatically, the heat lost between the compressor and the motor represents lost energy. If no heat were thus lost, adiabatic compression would be desirable.

Energy lost by radiation. If τ_1 be the final temperature in the compressor and τ_3 the temperature at the motor, then will the energy lost per pound be

$$C_p (\tau_1 - \tau_3). \quad (322)$$

Weight of water to be injected in order to reduce the temperature a given amount. In order to secure isothermal compression, the refrigerant would necessarily be indefinitely large, for otherwise its temperature would be raised. Let W be the pounds of water which must be injected to

reduce the temperature of one pound of air from T_1^0 to T_2^0, the temperature of the water being raised from T_4^0 to T_3^0. Since the specific heat of water will be considered as unity, we have

$$W(T_3 - T_4) = C_p(T_1 - T_2). \tag{323}$$

Volume of the compressing cylinder. If there were no losses, the volume of the compressor cylinder would be the same as that of a compressed-air motor doing the same work as expended in compression, working under the same law of expansion. Hence, if V' be the volume of the compression cylinder, W' the pounds of air compressed by N' horse-powers per minute, U' the work necessary to compress one pound under an assumed law and force it into a receiver, n' the number of single strokes of the engine, and τ_2' the initial temperature of the air, then equation (317) gives

$$V' = \frac{33000\ N'\ R\ \tau_2'}{2\ n'\ U\ p_2'} = \frac{W'\ R\ \tau_2'}{2\ n'\ p_2'}. \tag{324}$$

Modifications. The initial pressure in the cylinder, p_2', will be less than that of the external air, for the valves will offer some resistance to the inflow of air, and it would take a short time to establish equilibrium, and the temperature of the cylinder may expand the air. If there be any clearance, all of the compressed air would not be forced into the receiver. For these reasons, and also on account of the heat lost by radiation, the volume of the cylinder should be considerably larger than that found from equation (324). This would be secured by assuming the horse-powers, N', expended in compressing the air as proportionately larger than the horse-powers N, to be delivered by the motor, but so many contingencies arise in practice that a definite rule cannot be stated beforehand. Deficiency in size in the construction may often be overcome in practice by increasing the piston speed.

153. Efficiency *of compressor and engine.* For complete *adiabatic* expansion, equations (308) and (318) give

$$E = \frac{\tau_1 - \tau_2}{\tau_1' - \tau_2'} = \frac{\tau_1}{\tau_1'} \frac{\left[1 - \frac{\tau_2}{\tau_1}\right]}{\left[1 - \frac{\tau_2'}{\tau_1'}\right]} = \frac{\tau_1}{\tau_1'}, \text{ approx.} \quad (325)$$

This operation may be illustrated by Fig. 75. The air will be taken into the compressor at the absolute temperature τ_2' at C, then compressed adiabatically to temperature τ_1' at B, then forced into the receiver at the constant temperature τ_1' and pressure p_1'. The work done by the compressor per pound of air will be $I\ C\ B\ H$. The air then loses heat and enters the engine at A under the pressure $p_1 = p_1'$ and a temperature τ_1 and expands adiabatically to D, the temperature being reduced to τ_2, where it is exhausted. The work of the engine will be $I\ D\ A\ H$. The resultant work will be $A\ B\ C\ D$. If no heat were lost, the temperature at A would equal that at B, and that at D equal that at C, or $\tau_1 = \tau_1'$, $\tau_2 = \tau_2'$; $\therefore E = 1$, or the efficiency would be perfect. In this case, however, $A\ D$ will fall on $B\ C$, and the resultant work will be zero. The compression might be along $D\ A$ and expansion along $B\ C$. Equation (325) expresses the efficiency if the air enters the motor at a less pressure than that of $H\ A$, and exhausts it at a higher or lower pressure than that of C. In this case the cycle will not be complete.

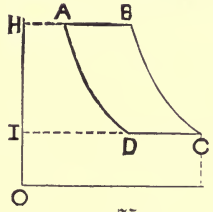

FIG. 75.

Mass of fluid constant. In some operations, especially in refrigerating machines, the mass of working fluid is constant, the operation $B\ A$ being effected by abstracting heat, and $D\ C$ by supplying heat. In this case, if $A\ D$ and $B\ C$ are adiabatics, the heat supplied along $D\ C$ will be, per pound,

$$H_2 = C_p (\tau_2' - \tau_2),$$

and abstracted along $B\ A$,

$$H_1 = C_p (\tau_1' - \tau_1);$$

also

$$\frac{\tau_1}{\tau_2} = \frac{\tau_1'}{\tau_2'};$$

and the efficiency of fluid will be

$$E = \frac{-U}{H_1} = \frac{C_p(\tau_2' - \tau_2) - C_p(\tau_1' - \tau_1)}{C_p(\tau_1' - \tau_1)} = -\frac{\tau_1' - \tau_2'}{\tau_1'}, \quad (326)$$

the same as for the perfect elementary engine; and is the fraction of the work which is transmuted into heat. If the operation were in a reversed direction, the result would be positive and would be the fraction of the heat absorbed which would be transmuted into work.

154. Friction of air in pipes. The experiments at Mont Cenis gave the formula

$$F = 0.00936 \frac{n^2 l}{d}, \quad (327)$$

in which d is the diameter of the pipe in inches, l the length in feet, n the velocity in feet per second, and F the loss of pressure in pounds.

EXERCISES.

1. Required the volume of the cylinder of a double-acting air-compressor making 50 revolutions per minute to deliver to a compressed air-engine, making 100 revolutions, sufficient air to give 5 indicated horse-powers, allowing fifty per cent lost in the power of the compressed air. Let the initial temperature of the air be 60° F., and compressed isothermally to 5 atmospheres absolute, the initial pressure in the engine also 5 atmospheres at the same temperature.

Find the weight of water which must be injected per stroke at 55° F., that the temperature may not exceed 65° F.; the volume of the cylinder of the engine, the point of cut-off of the engine that the expansion shall be complete, the final temperature at the end of the stroke, and the efficiency of the system.

First find the work which one pound of compressed air will do. We have

$$\tau_1 = 60 + 460 = 520,\ p_1 = 14.7 \times 5 = 73.5,\ p_2 = 14.7.$$

Final temperature, τ_2, $520 \times (\frac{1}{5})^{\frac{2}{7}}$, degrees absolute	328.
" " degrees F	−132.
Difference of temperatures, $\tau_1 - \tau_2$	192.
Work per lb. of air, Eq. (308), 184.77×192, ft.-lbs	35476.
Work required per minute, 33000×5, ft.-lbs	165000.
Air required per minute, lbs	4.65.
Vol. cyl., Eq. (317), $4.65 \times 53.21 \times 328 \div (200 \times 2116.3)$ cu. ft.	0.192.
Diameter, if stroke is $1\frac{1}{2}$ times the diameter, inches	6.55.
Ratio of expansion, $\frac{v_2}{v_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{1.4}} = (5)^{\frac{5}{7}}$	3.15.
Air to be supplied to compressor, 4.65×2, lbs. per minute	9.30.
" " " " " per stroke, lbs	0.093.
Vol. air cyl., Eq. (324), $\frac{9.3 \times 53.21 \times 520}{2 \times 50 \times 2116.3}$, cu. ft	1.22.
Diameter, if stroke is $1\frac{1}{2}$ times the diameter, inches	12.1.
Work per lb., Eq. (320), $122.5 \times 520 \times 0.699$, ft.-lbs	44526.
Work per minute, 44526×9.3, ft.-lbs	414092.
Efficiency, $165000 \div 414092 =$	0.40.
Water injected, $0.24 \times 192 \div 10$, lbs. per lb. of air	4.61.
" " lbs. per stroke 4.61×0.093	0.43.
" " cu. in. per stroke $0.43 \times 0.016 \times 1728$	11.89.

If the temperature be limited to 100° F., it would require less than three cubic inches of water per stroke.

2. In the preceding exercise if the compression were adiabatic, find the final temperature of compression, the final pressure being 5 atmospheres, initial temperature of the air, 65° F.; also the efficiency.

THE STEAM TURBINE.

155. Steam turbines act on the same general principles as hydraulic turbines; an essential difference being that water is considered non-compressible, while steam and other vapors are compressible. A more general term for this class of turbines would be *elastic vapor turbines.* They may be reacting, like the Barker mill, Whitelaw or Scottish turbine, parallel flow, outward or inward flow. One is described in the Pneumatics of Hero of Alexandria. Rankine also mentions an inward-flow turbine which was used at the Glasgow City Saw Mills, and was considered equal in efficiency to an ordinary high-pressure engine (*Steam Engine*, p. 538). The *claim*, however, is not sustained by any authentic experiments. Very few of these turbines appear to have been in use until quite recently; now they are being used to drive electrical dynamos, chiefly on account of the very small space occupied by them and the ease with which they may be located wherever desired. In many cases they are wasteful of steam on account of the clearance spaces permitting a part of the steam to pass through the engine without doing work, but one quite recently invented by Messrs. Dow appears to be a great improvement on previous engines of this class.

156. Balanced outward-flow steam turbine. The turbine shown in Fig. 76 is the joint invention of J. H. Dow and H. H. Dow, of Cleveland, Ohio. *A A* represents the casing, or stationary part of the engine; *B B* the rotating wheel firmly secured to the shaft *C*, and containing the buckets or floats shown in the section *D D*, which are arranged in concentric circles; and concentric with these and between them are rings projecting from the stationary part of the engine through which are cut steam passages or guides. Steam entering through the stationary part at *E*,

passes both sides of the rotating disk FF, into the annular cavity near the centre, thence outward through passages in the annular spaces between the buckets, and through the buckets, finally escaping at the outer circumference at D, and is conducted away at the exhaust at G.

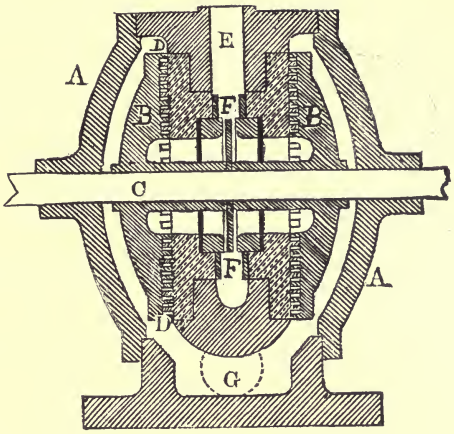

FIG. 76

It is balanced laterally by means of the disk FF, which is firmly secured to the shaft C; so that if there be a lateral movement, however small, the space on one side of the disk will be reduced and on the other side enlarged, so that the increased amount of steam entering the latter will force it back to its normal position. It is claimed that this movement may be limited to 0.002 of an inch. The energy of the steam is gradually absorbed by the wheel as it passes through it, thereby diminishing its pressure and causing expansion, similar to that in a multiple-expansion engine, there being six compoundings in this wheel.

157. Analysis. *Reaction Turbine.* These may be constructed like the Barker mill, Scottish or Whitelaw turbines, or other hydraulic turbines of this class. The section of the orifices is very much smaller than that of the arms. The reaction of the steam as it escapes from the arms imparts to them a rotary motion, and, consequently, as the fluid passes outward in the arms a rotary motion is imparted

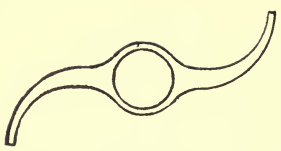

FIG. 77.

to it in common with the arms; the fluid escaping in a backward direction relative to the motion of the orifices. The velocity of exit will depend upon the three elements:

1. The pressure at the orifices due to the boiler pressure, as if the arms were at rest;

2. The additional pressure due to rotation, as if the orifices were closed; and

3. The velocity of the orifice relative to the earth.

The velocity of discharge relative to the earth will be the resultant due to these three causes acting simultaneously. On account of the compressibility of the fluid and the centrifugal action, the density of the steam will increase from the axis of rotation outward. The centrifugal force of the liquid, if any, in the vapor will cause the liquid to flow outward more rapidly than the vapor, and thus greatly complicate the solution; and it would be still further complicated by considering the change of temperature and of re-evaporation in passing outward along the arm. We will assume that the steam is dry saturated or slightly superheated and the temperature uniform.

Let h_1 be a head producing the pressure at the entrance to the arms where the weight of a unit of volume is w_1 and pressure p_1. At any distance ρ let the pressure be p and w the weight of unity of volume; then, since the weight will be directly as the pressure, we have

$$p_1 = h_1 w_1, \qquad p = h_1 w. \tag{328}$$

The variation of pressure will be due to the centrifugal force of an element whose thickness is $d\rho$ and base unity;

$$\therefore d p = \frac{w}{g} d \rho . \omega^2 \rho, \tag{329}$$

where ω is the angular velocity per second. These equations give

$$\int_{p_1}^{p_2} \frac{d\,p}{p} = \frac{\omega^2}{g\ h_1} \int_0^r \rho\ d\ \rho;$$

$$\therefore p_2 = p_1\, e^{\frac{\omega^2\ r^2}{2\,g\,h_1}} = p_1\, e^{\frac{V_3^2}{V^2}}, \tag{330}$$

where p_2 is the pressure at the orifice, V_3 the velocity of the orifice, and V the velocity due to the head h_1. If $V_3 = 0$, $p_1 = p_2$, as it should. The pressure due to the centrifugal force will be

$$p_2 - p_1 = p_1 \left(e^{\frac{V_3^2}{V^2}} - 1 \right). \tag{331}$$

The interior pressure ready to produce velocity will be p_2; now, if the orifice be opened into the atmosphere, the resultant pressure will be $p_2 - p_0$, when p_0 is the pressure of one atmosphere. The velocity of exit will be found from equation (278), page 283, after making $\gamma = 1.3$, as given in equation (145), page 151; hence

$$V_2 = 16.705 \sqrt{p_2\, v_2 \left[1 - \left(\frac{p_0}{p_2} \right)^{0.2307} \right]}. \tag{332}$$

Without rotation, the velocity relative to the orifice, or the earth, would be

$$V_1 = 16.705 \sqrt{p_1\, v_1 \left[1 - \left(\frac{p_0}{p_1} \right)^{0.2307} \right]}. \tag{333}$$

The orifices have a velocity $\omega\ \rho = V_3$ opposite to V_2; hence the velocity of discharge relative to the earth will be

$$V = V_2 - V_3. \tag{334}$$

The pounds of steam discharged per second will be, equation (64), value of R, p. 103;

$$W = w_2\, k\, s\, V = 1.8295\, k\, s\, p_1 \left(\frac{\tau_2}{\tau_1}\right)^{\frac{1.0}{3}} \sqrt{\frac{\tau_1 - \tau_2}{\tau_1^{\,2}}}, \qquad (335)$$

in which k is the coefficient of discharge, p_1 the pressure in the arm at the orifice and is p_2 in equation (330), τ_2 the temperature outside and τ_1 the temperature in the arm, w_2 the weight of unity of volume at the section of greatest contraction, and W the weight discharged per second at that point.

The work done by the reaction per second will be

$$\frac{W}{g}(V_2 - V_3)\ V_3,$$

or per pound,

$$U = \frac{1}{g}(V_2 - V_3)\ V_3. \qquad (336)$$

The energy expended will be that in the steam above the temperature of the feed-water, and per pound will be, equations (93) and (77),

$$\mathbf{H} = 778\ (1114.4 + 0.305\ T_1 - T_2), \qquad (337)$$

where T_1 is the temperature at the boiler in degrees Fahrenheit and T_2 that of the feed.

The efficiency will be

$$E = \frac{U}{\mathbf{H}}. \qquad (338)$$

The horse-powers will be

$$H\ P = \frac{W\ U}{33000}. \qquad (339)$$

EXERCISE.

1. In a reaction turbine having orifices 12 inches from the axis of rotation, if the boiler pressure be 50 pounds (gauge), section of all the orifices 0.02 of a square inch, coeffi-

cient of discharge .50, velocity of the orifices one fourth the theoretical velocity due to the steam pressure when the engine is at rest; find the work per second, the efficiency, the temperature of the feed-water being 60° F., the horse-power and the pounds of water used per horse-power per hour.

From Eq. (84) or from steam tables, volume of 1 lb. of steam at 64.7 lbs. absolute, will be, cu. ft.	6.58.
Weight of a cu. ft., lbs.	0.152.
Vel. of steam, no rotation (333), V_1 ft. per sec.	2223.
Velocity of orifices $\frac{1}{4}$ of 2223, V_3	555.7.
Revolutions per second	89.3.
" " minute	5358.
Pressure at axis where steam enters the arms	64.7.
Pressure at orifices, (330), p_2, lbs. per sq. in	140.
Vol. of 1 lb. steam at orifices $6.58 \times 64.7 \div 140$,	3.04.
Vol. discharge relative to orifice, V_2, (332)	2639.
" " " " the earth, $V_2 - V_3$	2084.
Work per pound, U, (336), ft. lbs	35952.
Energy expended, **H**, (337), ($T_1 = 297$, $T_2 = 60$)	890732.
Efficiency (338), per cent	4.0.
Discharged per sec. if $T_2 = 220°$, Eq. (335), lbs.	0.0195.
Horse-powers	1.27.
Steam per H. P. per hour, lbs	55.

This result appears to make the engine quite as efficient as small non-condensing engines. Theoretically, the efficiency would be high if the velocity of the orifices were half that of the theoretical velocity of discharge, or twice that assumed above, but the resistance of the air to the motion of the arms would be so great as to consume the power of the engine. At the velocity in this example the speed of the orifices is over seven miles per minute. At 200 feet per second, or about ten times the average highest piston speeds, the efficiency would be very low, not even considering

prejudicial resistances. To be efficient the speed must be high.

158. Outward-flow turbine of Fig. 76. The best speed for the turbine requires that the fluid shall be discharged with the least velocity—just sufficient to escape from the wheel. To accomplish this the steam must expand down nearly to the pressure of the atmosphere.

If the wheel is so constructed and operated that the steam will expand without transmission of heat, the method of Article 112 will be applicable, and work done per pound of steam would be

$$U = J\left[\tau_1 - \tau_2\left(1 + log_e\,\frac{\tau_1}{\tau_2}\right) + \frac{\tau_1 - \tau_2}{\tau_1}h_{e1}\right] + (p_2 - p_1)\,u_2, \quad (340)$$

if there were no losses from friction, contraction, eddies or clearances.

To determine the speed requires a definite knowledge of its construction. A properly constructed wheel must run at a definite speed for maximum efficiency, and it cannot be correctly analyzed for speeds differing much from that, on account of eddies or whirls being induced, the effect of which cannot be formulated.

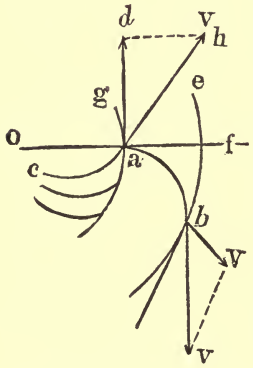

FIG. 78.

Let O, Fig. 78, be the centre of the wheel, $a\ g$ the inner rim, $b\ e$ the outer, $c\ a$ a guide, $a\ b$ a bucket. If there were no friction or eddies, the analysis for several concentric circles of buckets would be the same as if all the work were done in one series of buckets; so we treat the case as if the several series were developed into one. Let the initial ele-

ment of the bucket at a be tangent to the radius $O\,a\,f$* of the wheel, $a\,d$ a tangent to the inner rim, $a\,h$ tangent to the guide $c\,a$; also

$d\,a\,h = \alpha$, the angle between the terminal element of a guide and the inner rim of the wheel,
$r_1 = O\,a$, the inner radius of the wheel,
V, the velocity of the steam at a in the direction $a\,h$,
v_t, the tangential component of V,
v_r, the radial component of V,

and the same letters with accents to indicate similar quantities at b, the point of discharge: also ω, the angular velocity of the wheel, and $M = W \div g$, the mass of steam flowing through the wheel per second; then

$$V \cos \alpha = v_t, \qquad v_t \tan \alpha = v_r;$$
$$V' \cos \alpha' = v_t', \qquad v_t' \tan \alpha' = v_r'.$$

The rim velocities will be, respectively,

$$r\,\omega, \qquad r'\,\omega.$$

According to the principle of mechanics—*the difference of the moments of momenta of the fluid upon entering and quitting the wheel into the angular velocity of the wheel equals the energy imparted to the wheel*—we have for the work per second,

$$M\,(v_t\,r - v_t'\,r')\,\omega,$$

or per pound,

$$U = (v_t\,r - v_t'\,r')\,\frac{\omega}{g}, \tag{341}$$

and this should be approximately the value of equation (340). In the former, much the greater part of the heat of the steam is lost at the exhaust, while in the latter considerable energy may be lost in the kinetic energy of the steam

* In the Dow engine the buckets are straight and the angle at a acute.

as well as that in the exhaust. In order that there be no impact on entering the wheel, we have $v_t = w\ r$, and if the steam were discharged from the wheel radially, $v_t'\ r'$ would be zero; as a rough approximation let it be zero, then

$$U = \frac{\omega^2\ r^2}{g}; \tag{342}$$

$$\therefore\ \omega\ r = \sqrt{g\ U}.$$

Let n be the ratio of the actual to the theoretical value of U, then

$$\omega\ r = \sqrt{n\ g\ U}. \tag{343}$$

On account of condensation, clearance and friction, n for non-condensing engines is from $\frac{1}{2}$ to $\frac{1}{3}$.

If the wheel does not run at best velocity, $v_t'\ r'$ will not be zero; let it be $\eta\ v_t\ r$, in which η will have a different value for every different velocity; also $\omega\ r$ will not equal v_t, let it be $\varepsilon\ \omega r$; then

$$\varepsilon\ (1 - \eta)\ \frac{\omega^2\ r^2}{g} = n\ U$$

$$\therefore\ \omega\ r = \sqrt{\frac{n\ g\ U}{\varepsilon\ (1 - \eta)}}. \tag{344}$$

The number of revolutions per minute will be

$$N = \frac{30\ \omega}{\pi}. \tag{345}$$

EXERCISES.

1. Consider a Dow steam turbine run with steam at 70 pounds boiler pressure (gauge), using 600 pounds of steam per hour, the efficiency being one fourth the theoretical.

Assume that the gauge pressure at the engine is 67 pounds, or about 4 per cent less than the boiler pressure, that the terminal pressure is 17 pounds per square inch, the inner radius, $r = 1\frac{1}{4}$. We have

$\tau_1 = 773$, $\tau_2 = 679$, $H_{e_1} = 778 \times 894\frac{1}{2} = 696000$,
$u_2 = 21.9$, eq. (150), $u_2 \div v_1 = 4.05$ = ratio of expansion.
Work per lb. of steam, Eq. (340), ft. lbs., approx.. 88000.
Work per lb. as per hypothesis, ft. lbs..... 22000.
Steam per H. P. per hour, 1980000 ÷ 25075 lbs . 90.
Vel. of inner rim if $\eta = 0.1$, $n = \frac{1}{4}$, $\varepsilon = 4$, ft. per sec. 223
Revolutions per minute..... ..... 20452.
Horse-power, 600 ÷ 90..... 6.7.

An engine made by the Messrs. Dow had turbine wheels $5\frac{3}{8}$ inches in diameter; shaft, $\frac{5}{8}$ inch diameter; depth of buckets, $\frac{3}{16}$ inch; depth of guides also $\frac{3}{16}$ inch; weight of moving parts, 7 pounds, 7 ounces; weight, including casing, 68 pounds; highest measured speed with 70 pounds steam, 35000 revolutions per minute; so that the velocity of the circumference was nearly nine miles per minute.

An approximate computation of its regular daily performance at 70 pounds pressure gave about 8 horse-power with about 75 pounds of steam per horse-power per hour, the speed being about 25000 revolutions per minute. Accurate measurements will doubtless modify these results.

2. Required the number of revolutions per minute necessary to burst a cast-iron disk from the centrifugal force, the modulus of tenacity, T being 20000 and the diameter of the disk 6 inches, there being a hole 1 inch in diameter at the centre for the shaft, weight of a cubic inch $\frac{1}{4}$ of a pound.

Assume the centre of gravity of each half to be at $\bar{r} = 1.3$ inches from the centre.

$$N = \frac{30}{\pi}\sqrt{\frac{5\,g\,T}{W\,\bar{r}}} = 30\sqrt{\frac{5 \times 32.2 \times 20000 \times 12}{1.3 \times \frac{1}{4}(9 - \frac{1}{4})\,\pi^3}} = 71430.$$

CHAPTER V.

REFRIGERATION.

159. A refrigerating machine is a device for producing relative cold. It has been repeatedly shown in the preceding pages that in any fluid doing work by expansion, without transmission of heat, the temperature is lowered. Advantage may be taken of this fact to produce a low temperature. Let $M\ N$, Fig. 79, be the volume of a pound of the fluid when the cylinder of a compressor is full; let it be compressed adiabatically to B and at constant pressure to A; thence expanded adiabatically to J and at constant pressure to C. If the fluid be a compressible gas, the temperature will decrease from B to A and increase from J to C; but if it be a vapor the temperature will be constant at constant pressure —some or all of the vapor being condensed during compression, and evaporation taking place during expansion. In both cases heat must be abstracted from the working fluid during the operation $A\ B$—the heat being carried away by the cooling substance; and absorbed by the working fluid during the operation $J\ C$—being taken from surrounding substances. The latter result is the one sought, and is made practical by placing the articles to be chilled in a room whose walls are made practically impermeable to the passage of heat, and abstracting heat from the room by repeated operations like the one just described, the heat so

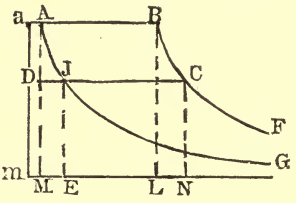

FIG. 79.

carried out by the working fluid being imparted to objects outside said room.

160. Practical operation. The practical operation is shown in Fig. 80, which represents a vapor plant. Omit-

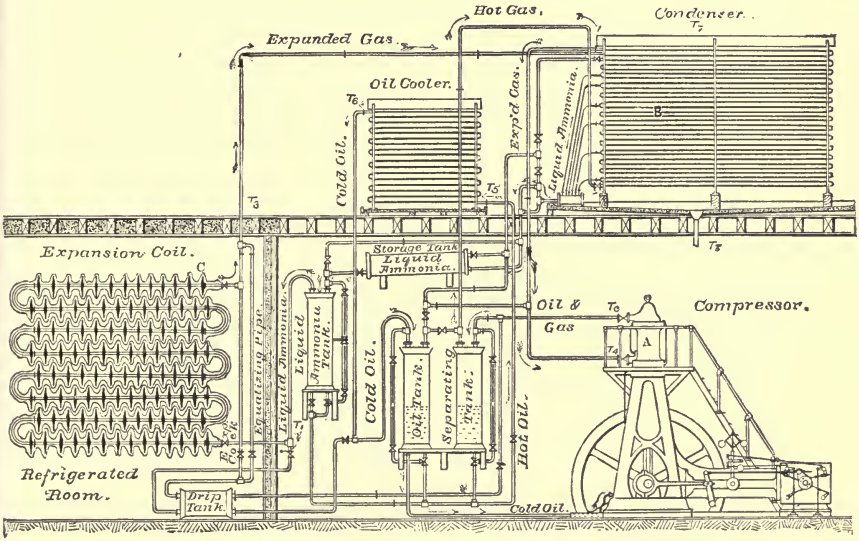

FIG. 80.

ting minor details, it is as follows: The working fluid is taken into one end of the compressor A during the back stroke of the piston, the operation being represented by $J\ C$, Fig. 79, the volume of a pound being N when the cylinder is full; during the forward stroke of the piston the fluid is compressed, the operation being represented by $C\ B$, and at B the valve is opened and the fluid forced into the coils of the condenser B, Fig. 80. Water flows over the coils, reducing the temperature, if the fluid be a permanent gas, and liquefying it if it be a vapor, the operation being represented by $B\ A$.

The heat absorbed by the water is wasted unless the water is used for other purposes. At the left of Fig. 80 is the refrigerating room C, which should be enclosed on all sides, including roof and floor, with several inches in thickness of sawdust, felt, or other non-conductor of heat. This room contains many coils of pipe through which the fluid is made to flow, the coils being in the centre of the room, or, as is often the case, arranged about its walls. The fluid passes from the condenser B to this room, where, by properly adjusted cocks, it expands against a pressure, reducing the temperature and pressure until the latter is that of the initial in the compressor; the operation being represented by $A\ J$, Fig. 79. During the back or return stroke of the piston the fluid flows into the compressor at constant pressure, the pressure being maintained by the heat in the refrigerating room, the operation being $J\ C$. If the fluid be a gas, the heat of the refrigerating room increases the heat of the gas, the temperature at J being lower than that of the room; but for a vapor the pressure and temperature are maintained constant by the evaporation of the liquid, its volume being increased from J to C.

It will be seen that only a part of the changes here described are made in the compressor; however, the indicator diagram $C\ B\ A\ J$ represents the changes passed through by the circulating fluid, and represents the work done by the compressor.

Let the adiabatics $B\ C$ and $A\ J$ be extended indefinitely to the right; then will the heat taken from the working fluid and carried away by the condenser be $G\ A\ B\ F$; and that taken from the refrigerating room will be $G\ J\ C\ F$. The operation is in effect that of taking the heat out of the refrigerating room, adding heat to that by the compressor, and finally causing both heats to be carried away by the water which passes through the condenser.

The operation of all refrigerating machines is essentially

the same in principle: condense the gas or vapor, deprive it of heat diminishing its volume, lower its temperature by doing work, then expand it; during the last operation heat is supplied by the articles to be cooled, and produces the refrigerating effect.

The heat of the refrigerating room is carried out by the circulating fluid to the condenser, where it is carried away by the water of the condenser. During its passage thither heat is added to the fluid by the work done upon it by the compressor raising its temperature, and by removing both heats at the condenser, the circulating fluid is put into a condition to take up heat again as it passes through the refrigerating room, so that the mass of circulating fluid may be constant. The mechanical operation of transferring the heat may be illustrated by the removal of water from a chamber at a lowel level than that of surrounding objects. For instance, conceive a mine having springs of water, and that the water is to be kept at a low level; or conceive a room nearly but not quite water-tight submerged in a lake, and that the water in the room is to be kept at a low level. By placing a pump in the room the water may be raised, as fast as it accumulates, to a higher level than that of surrounding objects, from which point it will flow away naturally. If it be not raised sufficiently high it will not *flow* away. In the refrigerating apparatus the compressor raises the temperature of the fluid to a higher value than that of surrounding objects, thus enabling the heat to flow away; and by exposing it for a sufficient time it would escape by radiation without the use of water; but the condensing water hastens the process.

161. Efficiency. It will be seen that a refrigerating machine is a heat engine reversed. Instead of transmuting heat into work, work is transmuted into heat.

Let $H_1 = F\,B\,A\,G$, Fig. 79, be the heat carried away by the condenser $H_2 = F\,C\,J\,G$ the heat taken from

the refrigerating room and absorbed by the circulating fluid; then the work done by the compressor upon the fluid will be $H_1 - H_2$.

The general expression for the efficiency is

$$E = \frac{\textit{Energy obtained (or work done)}}{\textit{Energy expended}}. \qquad (346)$$

If the energy obtained be the heat removed from the refrigerating room, and the energy expended be the work done on the fluid, then representing this efficiency by E_1, we have

$$E_1 = \frac{H_2}{H_1 - H_2}. \qquad (347)$$

In practice this will exceed unity, a result due to the peculiar unit to which the energy sought is referred. In most cases the energy obtained is a part of the energy expended, which is not the case in the above assumption. If the energy obtained be referred to the heat expended, the expression will be less than unity. Thus, let

E' be the efficiency of the furnace compared with the heat of combustion of the fuel,

E'', the efficiency of the engine, compared with the heat energy delivered to it by the furnace,

E''', the efficiency of the compressor referred to the engine as unity,

E_1, the efficiency of the refrigerating system compared with the compressor as unity; then will the efficiency of the system be

$$E = E'.\ E''.\ E'''.\ E_1. \qquad (348)$$

If the cycles were Carnot's, and no losses from clearance, friction or leakage in the engine and compressor, and the efficiency of the furnace be 0.70, then

$$E = 0.70\ \frac{\tau_1 - \tau_2}{\tau_1} \cdot \frac{\tau_3}{\tau_4 - \tau_3}, \qquad (349)$$

in which τ_1 is the absolute temperature of the steam at the furnace, τ_2 the temperature of the refrigerator of the engine, τ_4 the temperature of the condenser, and τ_3 the temperature of the refrigerating room. The efficiency realized is far less than this. It appears that *the efficiency of a refrigerating machine will increase as the temperature of the condenser decreases, and also as the temperature of the refrigerating room increases.* This is also apparent from general considerations, for the higher the temperature of the refrigerating room is allowed to be, the greater amount of heat will be carried away by a pound of the circulating fluid in expanding at constant pressure, and the lower the temperature of the condenser the less the work required of the compressor in raising the temperature from τ_3 to τ_4.

EXERCISE.

1. Let the efficiency of the boiler be 0.75, of the steam utilized by the engine 0.15, of the engine compared with one without friction or other waste 0.50, of the compressor compared with one without waste 0.70, temperature of the refrigerating fluid when it leaves the condenser 75° F., when it leaves the refrigerating room 5° F., and that 15 per cent of the latter energy is lost, required the efficiency of the plant. And if 1 pound of coal fed to the furnace develops 12300 thermal units when completely burned, how many pounds of ice at 32° F. may be formed from water also at 32° F. for each pound of coal burned?

We have

$$E = 0.75 \times 0.15 \times 0.50 \times 0.70 \times \frac{460 + 5}{70} \times 0.85 = 0.222,$$

or the efficiency is 22.3 per cent; that is, for each thermal unit contained in the coal fed to the furnace, 0.223 of a thermal unit will be taken from the refrigerating room.

If each pound of coal contains 12300 thermal units, then for each pound burned there will be

$$12300 \times 0.222 = 2731$$

thermal units taken from the refrigerating room, and as 144 thermal units are required to congeal 1 pound of water at 32° (page 89), there may be congealed

$$2731 \div 144 = 18.96 \text{ pounds.}$$

In this solution it is assumed that a Carnot's cycle is performed. If 25 per cent of the energy were lost instead of 15, the result would have been 16.8 pounds, and this is in the vicinity of actual values. Later we will show how purely theoretical results may be found. If this engine developed a horse-power with $3\frac{1}{2}$ pounds of coal per hour, then would 66.40 pounds of ice be made per horse-power per hour from water at 32° F.

Compared with the work done by the compressor on the circulating fluid, the efficiency would be

$$E_1 = 0.85 \frac{460 + 5}{70} = 5.646;$$

that is, for every thermal unit of work done by the compressor more than 5.6 thermal units would be removed from the refrigerating room.

162. The circulating fluid. Thermodynamically, any fluid may be the working fluid; but there are certain physical and practical considerations which determine a choice. It must admit of a low temperature without congealing. Air offers the advantage of being abundant, without cost, and admitting of any desired range of temperature; but its density being small, the required apparatus must be correspondingly large. If vapors are used they must be capable of vaporizing at low temperatures. Among the substances

used are ammonia, $N\ H_3$, sulphur dioxide, S_2O_2, methylic ether, $C_2\ H_6\ O$, and sulphuric ether; the first two of which are the most common, and of these we will consider ammonia especially. The general formulas will be applicable to any vapor.

Generally *brine*—water thoroughly saturated with salt—circulates in the coils, the brine being cooled in a tank by the ammonia, as above described. This saves a large amount of ammonia. Brine may be produced that will not congeal until the temperature is below zero Fahrenheit.

163. Some properties of ammonia. Certain properties of ammonia have been determined by Regnault, but his determination of the latent heat of vaporization and the specific heat of liquefied ammonia were lost during the reign of the Commune, in 1870; and these we will determine by computation founded on the results of experiment and certain thermodynamic principles.

In *Relation des Expériences*, Vol. II., pp. 598–607, are the results of Regnault's experiments upon temperature and corresponding pressure of saturated ammonia. These we have plotted in Fig. 81, the ordinates to the dots representing the pressures, and the abscissas, temperatures. If the law be represented by Rankine's formula, equation (80), p. 97, the value of C will be so small that its effect will be inappreciable, and the formula

$$\text{com. } log\, p = 8.4079 - \frac{2196}{\tau}; \qquad (350)$$

or, if p be pounds per square inch,

$$log_{10}\, p = 6.2495 - \frac{2196}{\tau}$$

represents the results of the experiments with much accu-

racy from about — 20° F. to 100° F., or from about 18 pounds per square inch to 215 pounds.*

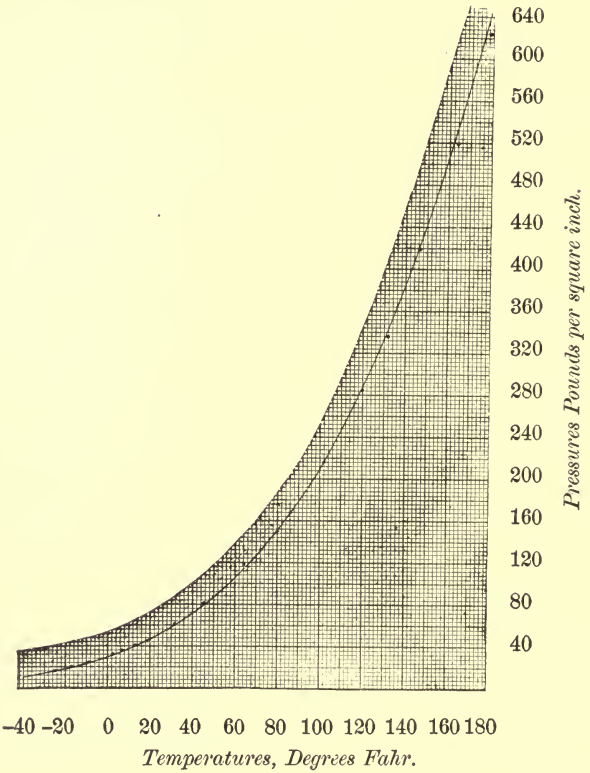

FIG. 81.

The specific heat of ammonia gas is 0.50836, which is a little more than for steam (*Rel. des Exp.*, II., p. 162).

Density of liquefied ammonia, that of water being unity.

* In the *Transactions* of the American Society of Mechanical Engineers for 1889, I used the formula $\log p = 6.2469 - \frac{2200}{\tau}$, and showed the difference between the computed and observed values. This formula is nearer correct for higher pressures.

Temp.	Density.	Dif.	Authority.
At 15.5° C.	0.731		Faraday.
— 10	0.6492		D'Andreeff: An. (3), 56, 317 ("Smithsonian Miscellaneous Collections," Vol. XXXII., 1888).
		—63	
— 5	0.6429		
		—65	
0	0.6364		
		—66	
5	0.6298		
		—68	
10	0.6230		
		—70	
15	0.6160		
		—71	
20	0.6089		

These may be expressed very nearly by the formula

$$\begin{aligned} \delta &= 0.6364 - 0.0014\, t \\ &= 0.6502 - 0.000777\, T, \end{aligned} \qquad (351)$$

when t is degrees centigrade and T degrees Fahrenheit.

Density of the gas.—Regnault gives, for the theoretical density of the gas, 0.5894 (*Rel. des Exp.*, Vol. II., p. 162), but he also says: "The real density of ammonia gas is certainly higher than the theoretical; the only experimental density of which I have knowledge gives 0.596" (*Ibid.*, Vol. III., p. 193). We will use the latter value.

Volume of a pound of the gas at the melting-point of ice. We have

Weight of litre of air at 0° C., 760^{m}. . . . 1.293187 grammes, or weight of cu. metre of air at 0° C., 760^{m}..1.293187 kilog.

(*Ibid.*, Vol. I., p. 162.) Hence the weight of one litre of the gas at 0° C., 760^{mm}, will be

$$1.293187 \times 0.596 = 0.770739 \text{ grammes},$$

and the volume of one gramme of the gas will be

$$\frac{1}{0.770739} = 1.2973 \text{ litres}.$$

Reducing this to the equivalent of one pound and cubic feet gives

$$1.2973 \frac{35.3161}{2.2046} = 20.7985 \text{ cu. ft. per lb.} = v_0.$$

Value of R.

$$R = \frac{p_0 v_0}{\tau_0} = \frac{2116.3 \times 20.7985 \times}{492.66} = 89.343. \quad (352)$$

This is $89.343 \div 778 = 0.11483$ of a thermal unit; hence, at this state, equation (28), p. 49,

$$k_v = 0.50836 - 0.11483 = 0.39352; \quad (353)$$

and, equation (31)

$$\gamma = 1.292; \quad (354)$$

and, although γ will not be constant, it will practically be so for the superheated gas.

164. To find the latent heat of evaporation *of Ammonia.* From equation (74), p. 95,

$$h_e = \tau (v_2 - v_1) \frac{dp}{d\tau} \div J,$$

in which v_1 is the volume of a pound of the liquid; and as this is small compared with the volume of a pound of the vapor it may be omitted, and we have, omitting also the subscripts,

$$h_e = \tau v \frac{dp}{d\tau} \div 778. \quad (355)$$

From equation (350) we have

$$\frac{dp}{d\tau} = 2196 \times 2.3026 \frac{p}{\tau^2};$$

$$\therefore h_e = 6.49922 \frac{pv}{\tau}. \quad (356)$$

At the state when

$$\frac{pv}{\tau} = \frac{p_0 v_0}{\tau_0} = 89.343, \text{ we have}$$

$$h_e = 580.66.$$

This result must be for a state where $v > v_0$; for the general theory of imperfect gases shows that for the same volume $p + \tau$ is less for a smaller pressure, and in this case at the pressure p_0 the gas is superheated, and at the point of saturation p will be less than p_0 ; hence *the latent heat of evaporation of ammonia must be less than* 580 66 *when the specific volume is* 20.7985 *cubic feet.**

The general value of $p\,v + \tau$ will be found from the equation of the gas. In Vol. II. of *Expériences*, p 152, Regnault has given the results of his experiments upon the elastic resistance of ammonia at the constant temperature of 8.1° C. (46.58° F.). These give the relations between the pressures and volumes of the actual isothermal *A C*, Fig. 82 ; the isothermal of the gas passing through *A*, if perfect, being *A B*. These experiments reduced to volumes in cubic feet per pound, and pressures in pounds per square foot, are given in the following table :

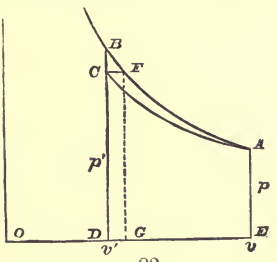

FIG. 82.

TABLE.

RELATIONS BETWEEN VOLUMES AND PRESSURES OF AMMONIA GAS AT THE TEMPERATURE 46.58° F.†

Volumes cu. ft. per lb.	Pressures.	
	Lbs. per sq. ft.	Lbs. per sq. in.
24.3716	1862.706	12.93
23.157	1958 976	13.60
21.944	2064.096	14.33
20.7985	2178.960	15.13
19.563	2311.200	16.05
18.365	2458 800	17.08
17 160	2618.784	18.19
15.961	2822.544	19.60
14.762	3042.288	21.13
13.557	3303.648	22.94
12.355	3617.768	25 12
11.1412	3996.820	27.76

* This shows that the latent heat found by Ledoux is erroneous (*Ice-Making Machines*, by M. Ledoux, Van Nostrand's *Science Series*, No. 40). For the volume 20.8 cubic feet Ledoux gives about 600 B. T. U.

† Trans. Am. Soc. Mech. Engineers, 1889.

Assuming in equation (4), p. 13, $a_0 = o$, and neglecting all terms after a_1, it may be written in the form

$$p\,v = a\,\tau - \frac{b}{\tau\,v^n}.$$

The first, sixth, and last experiments of the preceding table give for the products $p\,v$, and the corresponding values v,

$$a\,\tau - \frac{b}{\tau\,(24.3716)^n} = p\,v = 45397.$$

$$a\,\tau - \frac{b}{\tau\,(18.365)^n} = p\,v = 45156.$$

$$a\,\tau - \frac{b}{\tau\,(11.141)^n} = p\,v = 44529.$$

In these equations $\tau = 507.24$, and they give

$a = 91\,005$, $b = 16921\,\tau$, $n = 0.97$.

Letting $a = 91$, $b = 16920\,\tau$, $n = 0.97$.

The equation of the gas will be

$$\frac{p\,v}{\tau} = 91 - \frac{16920}{\tau\,v^{0.97}}, \tag{357}$$

and hence, equation (356),

The latent heat of ammonia is

$$h_e = \frac{5065.7}{778}\left(91 - \frac{16920}{\tau\,v^{0.97}}\right)$$

$$= 592.52\left(1 - \frac{185.93}{\tau\,v^{0.97}}\right) \tag{358}$$

We now proceed to find the latent heat for certain states of the fluid. In Fig. 83, a represents the state of ammonia gas at the temperature of melting ice under the pressure of one atmosphere, for which the volume, as found above, is 20.7985 cubic feet per pound ; that is,

$$o\,h = 20.7985, \qquad h\,a = 2116.3, \qquad \tau = 492.66.$$

Let state s represent the pressure and volume of the first experiment in the preceding table, for which

$$o\,t = 24.3716, \qquad t\,s = 1862.7 \text{ lbs. per sq. ft.}$$

State e is the last in the table, for which

$$o\,j = 11.141, \qquad j\,e = 3996.8.$$

From s to e is the actual isothermal of the gas as determined by Regnault's experiments for the temperature of 46.68° F. As this was the temperature of the water surrounding the tube, the temperature of the gas may have been somewhat less; but we use this value as exact. The isothermal prolonged intersects the curve of saturation in m.

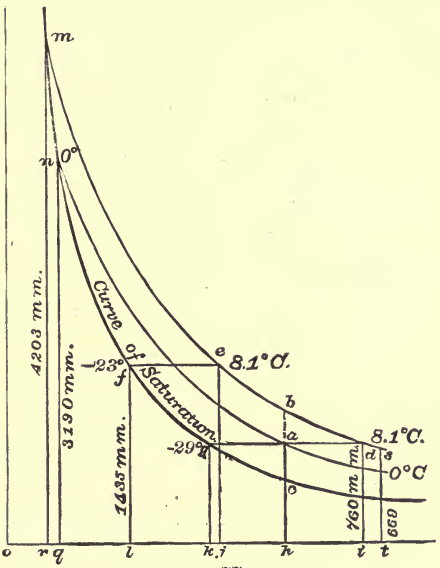

FIG. 77.

To find the latent heat of evaporation at c, having the same volume as at a, it is necessary to find the temperature at this point. Make

$$v = 20.7985$$

and equation (357) gives

$$p = \frac{91}{20.7985}\tau - \frac{16920}{(20.7985)^{1.97}}$$

$$= 4.3753\,\tau - 42.843\,;$$

which substituted in equation (350) gives

$$log\,(4.3753\,\tau - 42.843) = 8.4079 - \frac{2196}{\tau}\,;$$

$$\therefore \tau = 426.6\,; \quad \therefore T = -34.0°\text{ F.}$$

Eq. (350) $$\therefore p = 1823.7 \text{ lbs. per sq. foot.}$$

$$= 12.7 \text{ lbs. per sq. inch}\,;$$

and equation (358) gives

$$h_c = 578.96.$$

It was found above for this volume, 20.7985, that if $p + \tau$ were constant down to the point of saturation, the latent heat would be 580.66, a value exceeding that just found by only 1.70 thermal units—a difference in the right direction, and of reasonable amount. This test being satisfactory, we now apply it to other states.

For the state immediately below s, on the curve of saturation, we have

$$v = 24.372,$$

and with the same equations as in the preceding case, there results

$$\tau = 420.4\ ;\ \therefore\ T = -40°.26\ \text{F.},$$
$$p = 1531.1\ \text{lbs. per sq. ft.} = 10.6\ \text{per sq. inch},$$
$$h_e = 579.67\ \text{thermal units.}$$

For state m,

$$\tau = 507.24,\ \text{or}\ T = 46°.58\ \text{F.},$$

and from the same equations

$$p = 11988\ \text{per sq. ft.} = 83.25\ \text{per sq. inch},$$
$$v = 3.41\ \text{cubic feet},$$
$$h_e = 526.47\ \text{thermal units.}$$

For the state for which

$$v = 8,$$

we find

$$\tau = 468\ 7\ ;\ \therefore\ T = 8°.1\ \text{F.},$$
$$p = 5279\ \text{per ft.} = 36.8\ \text{per inch},$$
$$h_e = 550.52.$$

Assuming the form of expression adopted by Regnault for the latent heat of evaporation of all substances,

$$h_e = d - e\,T - f\,T^2, \tag{359}$$

and, using in it the data for the three last cases just given, we have

$$526.47 = d - 46.58\,e - 2169.7\,f,$$
$$550.52 = d - 8.1\,e - 6561\,f,$$
$$579.67 = d + 40\,e - 1600\,f.$$

These give

$$d = 555.50, \qquad e = 0.61302, \qquad f = 0.000219,$$

and equation (359) gives the following as

A more convenient formula for the latent heat of evaporation of ammonia:

$$h_e = 555.5 - 0.613\,T - 0.000219\,T^2. \tag{360}$$

The latent heat of ammonia vapor in the table at the end of the volume has been computed by means of this formula.

165. Specific volume of liquefied ammonia. If the volume of a pound of water be 0.016 of a cubic foot, then will the volume of a pound of liquid ammonia be, equation (351),

$$v_1 = \frac{0.016}{0.6502 - 0.0007\frac{7}{9}\,T}. \tag{361}$$

This formula is sufficiently accurate for temperatures between $-5°$ F. and 100° F. A mean value gives about 41 pounds per cubic foot.

166. Specific volume of ammonia gas. From equation (84), page 98,

$$v_2 - v_1 = \frac{778\,h_e}{\tau\,\frac{d\,p}{d\,\tau}}.$$

By the aid of equation (350), after omitting the subscript „, we have

$$v = \frac{h_e}{6.4993}\cdot\frac{\tau}{p} + v_1. \tag{362}$$

The volumes in the table of the Properties of Saturated Ammonia were computed from this equation. Since v_1 is small compared with v_2 it may generally be omitted.

167. Isothermals of ammonia vapor. If the vapor be saturated, the isothermal will be parallel to the axis of v, as $A\,B$, Fig. 74.

If the vapor be superheated, the equation will be (357), after making τ constant. It will be

$$p\,v = 91\,\tau_1 - \frac{16920}{v^{0.97}}. \tag{363}$$

The general equation of vapors in which the last term is a function of v only, will be

$$p\,v = a\,\tau - \frac{b}{v^n}. \tag{364}$$

168. Adiabatics of ammonia vapor. If the vapor be continually *saturated*, the equation of the adiabatic will be (*a*) or (*b*), page 184, or

$$u = x\,v = \left(c\,log_e\frac{\tau_1}{\tau} + \frac{x_1\,h_{e1}}{\tau_1}\right)\frac{\tau\,v}{h_e}, \tag{365}$$

in which u is the volume of the vapor and liquid when only the xth part of the liquid is vaporized; but as, in our analysis, the volume of the liquid compared with the vapor is neglected, it really represents the volume of the xth part of a pound of vapor; c is the specific heat of liquid ammonia, the experimental value of which is

$$c = 1.22876.$$

If the vapor be superheated, then the first of equations (*A*), page 48, and equation (364) give

$$d\,H = K_v\,d\,\tau + \tau\frac{a}{v}\,d\,v.$$

But for an adiabatic $d\,H = 0$;

$$\therefore K_v\frac{d\,\tau}{\tau} = -\,a\frac{d\,v}{v};$$

$$\therefore K_v\,log_e\frac{\tau}{\tau_2} = a\;log\frac{v_2}{v};$$

$$\therefore \frac{\tau}{\tau_2} = \left(\frac{v_2}{v}\right)^{\lambda}, \tag{366}$$

where v_2 and τ_2 are inferior limits, and

$$\lambda = \frac{a}{K_v} = \frac{91}{778 \times 0.3935} = 0.29725. \tag{367}$$

To obtain an equation between p and v eliminate τ between (366) and (364), giving

$$p\,v = a\,\tau_2\left(\frac{v_2}{v}\right)^{\lambda} - \frac{b}{v^n}. \tag{368}$$

For ammonia gas, these become

$$\frac{\tau}{\tau_2} = \left(\frac{v_2}{v}\right)^{0.29725} \tag{369}$$

$$p = 91\,\frac{\tau_2}{v_2}\left(\frac{v_2}{v}\right)^{1.29725} - \frac{16920}{v^{1.97}} \tag{370}$$

$$= 91\,\frac{\tau_2}{v_2}\left(\frac{\tau}{\tau_2}\right)^{4.3641} - \frac{16920}{v_2^{1.97}}\left(\frac{\tau}{\tau_2}\right)^{6.6274} \tag{371}$$

the last of which is in terms of p and τ as variables.

169. The specific heat *of the saturated vapor of ammonia of constant weight is negative.* Equation (139), page 147, gives, omitting terms containing τ^2,

$$s = c - \frac{837.5}{\tau}.$$

If $c = 1$, this will be negative for values of τ less than 837°, or 377° F.; hence, for the range of temperatures ordinarily used in engineering practice, the specific heat of saturated ammonia is negative, and the saturated vapor will condense with adiabatic expansion, and the liquid will evaporate with the compression of the vapor, and when all is vaporized will superheat.

Thus, in Fig. 84, if $B\,Cs$ be the curve of saturation, and the vapor be compressed adiabatically from any point, as C, on the curve of saturation, the adiabatic $C\,I$ will rise above $B\,C$, and if it be expanded from the same point it will fall below Cs. Equation (370) is the equation of $C\,I$, and (365) of $C\,K$, the part below C.

170. Specific heat of liquid ammonia. Assume the volume $m\,M$, Fig. 84, of the pound of liquid to be constant at all pressures, and let $M\,D$ be the absolute pres-

sure at the absolute temperature τ, $B\,C\,s$, the curve of saturation, $D\,H$, $A\,G$, $B\,F$, $I\,K$, adiabatics. Let the vapor be expanded from D at the pressure p and temperature τ until it is all evaporated at state C, thence compressed adiabatically to I, thence compressed at constant pressure to A, where it is liquefied, thence by the abstraction of heat let the pressure be reduced to D; then

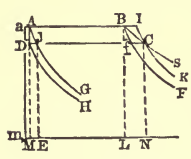

FIG. 84.

$$H\,D\,A\,G + G\,A\,I\,K = H\,D\,C\,K + D\,C\,I\,A.$$

Let the temperature of $A\,B$ be $\tau + d\,\tau$, and of I, $\tau + d\,\tau'$, for the vapor from B to I will be superheated, its temperature increasing with increase of volume; then, if c be the specific heat of the liquid,

$$\begin{aligned} H\,D\,A\,G &= J c\,d\,\tau, \\ D\,C\,I\,A &= v\,d\,p, \\ H\,D\,C\,K &= J h_{\mathrm{e}}, \\ G\,A\,I\,K &= G\,A\,B\,F + F\,B\,I\,K, \\ &= J h_{\mathrm{e}}' + J k_{\mathrm{p}}\,(d\,\tau' - d\,\tau), \\ h_{\mathrm{e}} - h_{\mathrm{e}}' &= -\,d\,h_{\mathrm{e}}; \end{aligned}$$

$$\therefore c = \frac{v}{J}\cdot\frac{d\,p}{d\,\tau} - \frac{d\,h_{\mathrm{e}}}{d\,\tau} - k_{\mathrm{p}}\left(\frac{d\,\tau'}{d\,\tau} - 1\right). \qquad (372)$$

Equation (360) gives, since $d\,\tau = d\,T$,

$$-\frac{d\,h_{\mathrm{e}}}{d\,\tau} = 0.6130 + 0.000438\,T.$$

From equation (350) find

$$\frac{v}{J}\frac{d\,p}{d\,\tau} = 6.49922\,\frac{p\,v}{\tau^2}.$$

Differentiating (371), after which change $d\,\tau$ to $d\,\tau'$ and drop all subscripts in the second member, and (372) becomes

$$c = 1.12136 + 0.000438\,T + \frac{p\,r}{\tau^2}\left[6.49922 - \frac{0.50836 \times 2.3026 \times 2196}{397.13 - \dfrac{112135}{\tau\,v^{0.97}}}\right] \qquad (373)$$

As this investigation depends upon a comparatively small range of volumes, from 11 to 24 (table on page 329), and assumptions in regard to the forms of the functions in equations (350) and the first equation on page 330, it may not be very reliable for a large range of temperatures. We will compute a value for a volume within the limits of the table, and will take values given on page 331, viz.:

$v = 20.7985$, $\tau = 426.6$, $T = -34°$ F., $p = 1823.7$ lbs., for which we find

$$c = 1.093.$$

The mean of eight determinations made by Dr. Hans von Strombeck, chemist, gave

$$c = 1.22876.$$

The experimental value was determined when the temperature of the liquid was about 80° F.

REFRIGERATING SYSTEM WITH VAPOR CONTINUALLY SATURATED.

171. Work of the compressor and condenser. In Fig. 85 let $E\ F$ be any adiabatic, p_1, τ_1 apply to $A\ B$, p_2, τ_2 to $D\ C$, x_1 the fraction of a pound of vapor at state E, x_2 at F, x_3 at A (when it is not zero), x_4 at J, v_1 the volume of a pound of vapor at B, v_2 the volume of a pound under the pressure p_2; the odd figures for subscripts belonging to the upper line, and the even to the lower.

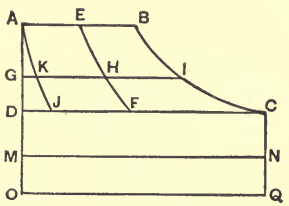

FIG. 85.

The work of compression from F to E, and of forcing it into the condenser from E to A, will be, equation (m), page 192,

$$U' = A\ E\ F\ D = J\left[c\,(\tau_1 - \tau_2) + x_1 h_{e1} - x_2 h_{e2}\right]. \quad (374)$$

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In the expansion chamber the vapor expands against a resistance, reducing the pressure from p_1 to p_2 and temperature from τ_1 to τ_2, doing the work $A\,D\,J$, and, assuming an adiabatic change, the expression for the work will be found by writing x_3 for x_1 and x_4 for x_2 in the preceding equation, since all the other quantities will remain the same;

$$\therefore\ U'' = A\,D\,J = J\left[c\,(\tau_1 - \tau_2) + x_3\,h_{e1} - x_4\,h_{e2}\right]. \quad (375)$$

After the vapor is forced into the condensing chamber, its specific volume is reduced by condensation from $A\,E$ to A (and for the sake of generalizing the analysis we assume for the present that it is not reduced entirely to a liquid at A, but that x_3 has a finite value), and the heat emitted from the circulating fluid—and absorbed by the condenser—will be the area between $E\,A$ and the indefinitely extended adiabatics $A\,J$ and $E\,F$, or,

$$H_1 = J\,(x_1 - x_3)\,h_{e1};$$

and the heat absorbed by the circulating fluid (ammonia or brine), *which is taken from the refrigerating room*, will be

$$-H_2 = -J\,(x_2 - x_4)\,h_{e2}; \quad (376)$$

then,

$$U' - U'' = H_1 - H_2.$$

Since the cycle is Carnot's, we have

$$\frac{(x_1 - x_3)\,h_{e1}}{\tau_1} = \frac{(x_2 - x_4)\,h_{e2}}{\tau_2}; \quad (377)$$

$$\therefore\ H_2 = \frac{\tau_2}{\tau_1}H_1;$$

$$\therefore\ U' - U'' = J\,(x_2 - x_4)\left(\frac{\tau_1}{\tau_2} - 1\right)h_{e2}.$$

The efficiency, referred to the work done by the compressor, will be

$$E_1 = \frac{H_2}{U' - U''} = \frac{\tau_2}{\tau_1 - \tau_2}, \quad (378)$$

which is independent of the amount of liquid evaporated, as it should be. More heat, however, will be removed by the work of the compressor if all the liquid be evaporated that is possible and leave it continually saturated; that is, if the fluid be vapor only at B; and also if it be entirely condensed at A. These conditions require that $x_1 = 1$ and $x_3 = 0$; and the equations for largest effect become

$$U' = J\left[c\,(\tau_1 - \tau_2) + h_{e1} - x_2\,h_{e2}\right]. \tag{379}$$

$$U'' = J\left[c\,(\tau_1 - \tau_2) - x_4\,h_{e2}\right]. \tag{380}$$

$$H_1 = J\,h_{e1}. \tag{381}$$

$$H_2 = J\,(x_2 - x_4)\,h_{e2}. \tag{382}$$

$$H_1 - H_2 = J\left[h_{e1} - (x_2 - x_4)\,h_{e2}\right] = U' - U''. \tag{383}$$

Eq. (365), $$x_2 = \left(c\,log_e\,\frac{\tau_1}{\tau_2} + \frac{x_1\,h_{e1}}{\tau_1}\right)\frac{\tau_2}{h_{e2}}. \tag{384}$$

For $x_1 = 0$, $$x_4 = c\,\frac{\tau_2}{h_{e2}}\,log_e\,\frac{\tau_1}{\tau_2}. \tag{385}$$

$$= 2.3026\,c\,\frac{\tau_2}{h_{e2}}\,log_{10}\,\frac{\tau_1}{\tau_2}.$$

These formulas solve the theoretical part of the problem. The temperature of the circulating fluid in the condenser will be several degrees above that of the condensing water, depending upon the amount of condensing surface—say about 10° F. above; and the temperature of the condensing water will depend upon external circumstances, and may be 40° F. in winter and 60° F. to 70° F. in summer. Also the temperature of the refrigerating room will be higher than that of the circulating fluid—say some 10° F. —depending upon the rapidity of the circulation and the amount of surface of the pipes. For the manufacture of ice, the refrigerating room is kept at a temperature of

about 15° F., and that of the brine may be between 0° F. and 5° F.

172. Volume of the compressor cylinder per n pounds of ammonia per stroke.

Let V be the required volume in cubic feet and v the volume of a pound of ammonia gas at the lower temperature, equation (362), then

$$V = n\,v. \tag{386}$$

173. Volume of the compressor cylinder *to produce a given refrigerating effect.*

Let h_2 be the thermal units abstracted per pound of ammonia,

$q = n_2 h_2$, the required number of thermal units to be abstracted—which will also be a measure of the refrigerating power,

V, the volume swept over by the piston or pistons—considered single-acting—per revolution,

N, the number of revolutions per minute,

v, the volume of a pound of ammonia gas at the lower temperature,

then

$$\frac{N\,V}{v}\,h_2 = q\,;$$

$$\therefore\ V = \frac{q\,v}{N h_2}. \tag{387}$$

174. Duty. The duty of a refrigerating plant may be referred to the number of thermal units required to melt one pound of ice. The latent heat of fusion of ice at the pressure of one atmosphere is 144 thermal units (page 89), and if h_2 be the thermal units abstracted by the circulating fluid per pound, then will the duty be

$$\textit{Ice-capacity, lbs.} = \frac{h_2}{144}. \tag{388}$$

If referred to one pound of coal fed to the furnace, then

$$\text{Ice-melting capacity per lb. of fuel, lbs.} = \frac{\left[\begin{array}{c}\text{Pounds of}\\ \text{circulating}\\ \text{fluid per}\\ \text{hour}\end{array}\right] \times \left[\begin{array}{c}\text{Specific}\\ \text{heat}\end{array}\right] \times \left[\begin{array}{c}\text{Range of}\\ \text{tempera-}\\ \text{ture}\end{array}\right]}{144 \times \text{pounds of fuel per hour.}} \qquad (389)$$

If w' pounds of coal evaporates W' pounds of steam per hour, then

$$\textit{Pounds of steam per lb. of fuel} = \frac{W'}{w'}, \qquad (390)$$

which may be used in equation (389).

EXERCISES

1. How many thermal units will be removed from the refrigerating room per pound of ammonia, the highest average temperature of the liquid being 62° F., and that after expansion — 1° F., the vapor being continually saturated, and the specific heat of the liquid 1.06 ?

Omit 0.66 in the absolute temperature.

From Eq. (360), $h_{c2} = 556.1.$

" " (384), $\left(1.06 \times 2.3026\, log_{10} \frac{522}{459} + \frac{517.5}{522}\right)\frac{459}{556.1}, \quad x_2 = 0.9308.$

" " (385), $1.06 \times 2.3026 \frac{459}{556.1}\, log_{10} \frac{522}{459}, \quad x_4 = 0.1125.$

" " (382), $h_2 = 455.16.$

That is, each pound of ammonia will carry away 455.06 thermal units. There must be taken into the compressor $x_2 = 93.08$ per cent of vapor, and therefore 6.92 per cent of liquid. There must be $x_4 = 11.25$ per cent of liquid vaporized during the rise of temperature from —1° F. to 62° F.

2. In the preceding exercise if the ammonia cools a brine whose specific heat is 0.8, the lowest temperature of the brine being 8° F., the highest 15° F., how many thermal units will be removed by the brine per pound, and how many pounds of brine will be required to absorb the heat of one pound of the ammonia ?

The heat imparted to the brine raises its temperature $15 - 8 = 7°$ F.; hence each pound will absorb $0.8 \times 7 = 5.6$ thermal units.

The ammonia imparts to the brine, equation (382),

$$h_2 = (x_2 - x_4)\ 556.1 = 454.06.$$

Let z be the pounds of brine necessary to absorb this heat in raising its temperature one degree, then

$$0.8 \times 7\ z = 454.06;$$
$$\therefore z = 81.17 \text{ pounds.}$$

When brine is used, larger pipes or greater velocity of the brine will be required than if ammonia only were used, and a lower temperature of the ammonia will be required.

3. What will be the efficiency in Exercise 1?

Equation (378) gives

$$E_1 = \frac{459}{63} = 7.28.$$

That is, *for every thermal unit of work done by the compressor*, 7.28 *thermal units will be removed from the cold room.* If the work done by the compressor be expressed in foot-pounds, and the heat removed be in thermal units, then

$$E_1 = \frac{7.28}{778} = 0.0093.$$

That is, for every foot-pound of work done by the compressor, 0.0093 of a thermal unit will be abstracted from the cold room.

4. How many pounds of water will be required by the condenser per pound of ammonia if condensed to a liquid at 63° F., the temperature of the water being increased from 50° F. to 64° F.?

The heat surrendered by the ammonia will be $h_1 = h_{e1}$, equation (381); and since the specific heat of water is unity, we have

$$y = \frac{h_{e1}}{14} = 36.9 \text{ pounds.}$$

5. How many cubic feet of water will be required in the preceding exercise, per pound of ammonia?

If less water is desirable, a greater range in its temperature must be allowed, and if a less range of temperature, then more water will be required.

6. What must be the volume of a single-acting compressor cylinder in Exercise 1 that it shall compress $\frac{1}{4}$ of a pound of ammonia gas with each stroke? Equation (386).

7. In Exercise 1 find the highest and lowest pressures in the compressor. Equation (350).

8. In Exercise 1 find the greatest and least *specific* volumes of the gas. Equation (362).

9. In Exercise 1 find the external latent heat of evaporation at the higher and lower temperatures. (This will be $p_1 v_1$ and $p_2 v_2$— neglecting the volume of the liquid.)

10. Find the negative work done by the compressor during one revolution and the positive work done by the one pound of ammonia in Exercise 1.

Negative work, Eq. (379),	$U' = 856577.$
Positive work, Eq. (380),	$U'' = 3282.$

11. If a compressor have two cylinders of equal size, each single-acting, diameter of each piston 18 inches, stroke 24 inches, ammonia gas entering at 0° F. and forced out at 62° F., each 32 revolutions per minute; how many thermal units will be removed from the cold room, no allowance being made for losses and the ammonia gas continually saturated; what the indicated horse-power of the compressor, the specific heat of liquid ammonia being 1.08?

Also, if 15 per cent be lost on account of clearance and imperfect working of the compressor and 50 per cent of the remainder due to losses in the refrigerating room; how many pounds of ice may be made from water at 62° F. cooled to 32° and frozen and the ice cooled to 24° F., the specific heat of the water being unity and of the ice 0.50?

Revolutions per hour	$n = 1920.$
Volume of each cylinder $\pi \times 81 \times 24 \div 1728$ cu. ft.	$V = 3.533.$
Working vol. displaced by pistons per hour, cu. ft.	$2\,n\,V = 13567.$
Specific vol. per lb. of gas at 0° F., Eq. (362), cu. ft.	$v = 9.0.$
Weight of gas compressed per hour, $2\,n\,V \div v$, lbs.	$W' = 1507.$
Vapor taken in per pound of fluid, Eq. (384)	$x_2 = 0.932.$

Liquid evaporated to reduce temperature, Eq. (385)...	x_4 =	0.111.
Latent heat of evaporation at 0° F.....	h_{e2} =	555.5.
Refrigeration per pound, Eq. (382), thermal units.....	h_2 =	456.1.
Condenser, heat removed, Eq. (381).....	h_1 =	517.5.
Work per pound, thermal units.....$(h_1 - h_2)$ =		61.4.
" " minute, thermal units.... .....$(h_1 - h_2)\ W' \div 60$ =		1542.3.
Horse-power, 1542.3 ÷ (33000 ÷ 778).....	$H.\ P.$ =	36.36.
Efficiency, Eq. (378).....	E_1 =	7.42.
Heat removed from cold room per hour..... .	$W'\ h_2$ =	687400.
Effectual heat removed, 0.85 × 0.50 × 687400.....	=	292145.
Heat expended in freezing, per lb., 30 + ½ of 8 + 144,	=	178.
Ice per hour, 292245 ÷ 178, lbs.....	=	1641.
Ice per horse-power per hour, lbs.....	=	45.1.
The IHP. of the steam-engine should be about 36.36 ÷ 0.6.....	$H.\ P.$ =	60.
Ice per IHP. would then be 45.1 × 0.6 lbs.....		27.
At 4 lbs. coal per IHP. per hour, lbs. of ice per lb. coal..... ...	=	6.8.

12. If 120000 pounds of brine passing through the cold room per hour has its temperature increased 5.2° F., specific heat 0.80, at an expenditure of 2000 pounds of steam generated by the burning of 200 pounds of coal; how many pounds of water may be frozen at and from 32° F. per pound of coal burned? Ans. 17.

(This data is almost exactly that of an actual case.)

CASE IN WHICH THE GAS IS SUPERHEATED BY COMPRESSION.

175. Superheating. It will be seen from the preceding exercise that the adjustment of liquid to vapor, in order to insure the largest result per pound of ammonia must be delicate, and can hardly be realized in practice; we therefore will now assume that; when the compressor cylinder is full, the fluid is all gas, with no liquid present, and is superheated at state C. In compressing it adiabatically, let the path be $C\ I$. If C were on the curve of saturation, the vapor would become superheated by adiabatic compression.

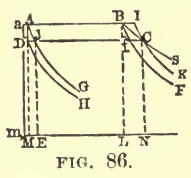

FIG. 86.

Let the state I be designated by accents, as v', p', τ'; and C by double accents, as v'', p'', τ''.

The work of compression and of forcing the gas out of the compressor will be, by the aid of equation (368),

$$U' = C\,I\,A\,D = \int_{v''}^{v'} v\,d\,p.$$

$$= a\,\tau_2 \frac{1+\lambda}{\lambda}\left[\left(\frac{v''}{v'}\right)^{\lambda} - 1\right] - b\frac{1+n}{n}\left[\frac{1}{v'^n} - \frac{1}{v''^n}\right] \tag{391}$$

$$= \int_{v''}^{v'} p\,d\,v + p'\,v' - p''\,v''$$

$$= p'\,v' - p''\,v'' + \frac{a\,\tau_2}{\lambda}\left[\left(\frac{v_2}{v''}\right)^{\lambda} - 1\right] - \frac{b}{n}\left[\frac{1}{v'^n} - \frac{1}{v''^n}\right] \tag{392}$$

$$= a\frac{1+\lambda}{\lambda}(\tau' - \tau'') - \frac{b}{v''^n}\cdot\frac{1+n}{n}\left[\left(\frac{\tau'}{\tau''}\right)^{\frac{n}{\lambda}} - 1\right]. \tag{393}$$

The last reduction may be effected by finding from (368) and (366)

$$p'\,v' - p''\,v'' = a\,(\tau' - \tau'') - b\left(\frac{1}{v'^n} - \frac{1}{v''^n}\right). \tag{394}$$

If the vapor be saturated at state f, and $f\,B$ be the adiabatic of superheated vapor, the value of the work $f\,B\,A\,D$ will be found by substituting v_2, τ_2, p_2, respectively, for v'', τ'', p'', giving

$$f\,B\,A\,D = a\frac{1+\lambda}{\lambda}(\tau' - \tau_2) - \frac{b}{v_2^n}\cdot\frac{1+n}{n}\left[\left(\frac{\tau'}{\tau_2}\right)^{\frac{n}{\lambda}} - 1\right]. \tag{395}$$

If the vapor be superheated at state C, its volume would be given by equation (364), p and τ being given. If it be saturated, v_2 will be given by equations (362) and (350) when either p or τ are given, this τ being τ_2 in (369), (370) and (371), when the initial state of the adiabatic of the superheated vapor is on the curve of saturation.

Since $C\,I$ is an adiabatic, equation (369) will give the volume v' at state I, and (371) the pressure, for the temper-

ature τ'. This pressure is assumed to be constant during condensation; during the first part, from I to B, the condensation is produced by a reduction of the temperature and volume, until the temperature is that of saturation, τ_1, under the pressure p_1, while from B to A the temperature and pressure are both constant, and the reduction of volume is effected by the condensation of vapor to a liquid. Assuming that the specific heat of the vapor is constant at constant pressure, then, for complete liquefaction, the heat abstracted in the operation $I\,A$ will be

$$h_1 = c_p\,(\tau' - \tau_1) + h_{e_1}. \tag{396}$$

Assume that heat is abstracted from the liquid at constant volume from state A, reducing the pressure from p_1 to p_2, and temperature from τ_1 to τ_2; the path of the fluid will be $A\,D$, and the heat so abstracted will be $H\,D\,A\,G$. If this heat be abstracted from the circulating fluid in the cold room, then will the room absorb that amount of heat, and in the evaporation and expansion afterward an equal amount of heat must be supplied from the cold room at the lower temperature before any useful amount can be absorbed by the circulating fluid; and if $H\,D\,j\,g$ be the amount so absorbed we have

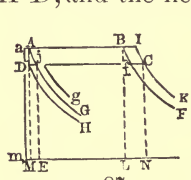

FIG. 87.

$$H\,D\,A\,G = H\,D\,j\,g.$$

The heat emitted will be

$$H\,D\,A\,G = c\,(\tau_1 - \tau_2).$$

Let $H\,D\,j\,g$ be the latent heat of evaporation in the x_6th part of a pound of vapor at the temperature τ_2; then

$$x_6\,h_{e_2} = c\,(\tau_1 - \tau_2). \tag{397}$$

(This value of x_6 is the same as x_4 in (380), when $u'' = 0$; but exceeds x_4 in Eq. (385).)

The refrigeration per pound per revolution will be

$$\begin{aligned} h_2 &= g\,j\,C\,K, \\ &= H\,D\,f\,F - H\,D\,j\,g + F\,f\,C\,K, \\ &= h_{e_2} - c\,(\tau_1 - \tau_2) + c_p\,(\tau'' - \tau_2), \qquad (398) \\ &= h_{e_2} + c\,(\tau_2 - \tau_0) - c\,(\tau_1 - \tau_0) + c_p\,(\tau'' - \tau_2), \\ &= \mathbf{h} - c\,(\tau_1 - \tau_0) + c_p\,(\tau'' - \tau_2), \qquad (399) \end{aligned}$$

in which τ_0 is the absolute temperature of melting ice, and $\mathbf{h}$ the total heat of vapor above τ_0, Article 85, Eq. (93).

Then equations (396) and (397),

$$\begin{aligned} h_1 - h_2 &= h_{e_1} + c_p(\tau' - \tau_1) - h_{e_2} - c_p\,(\tau'' - \tau_2) + c(\tau_1 - \tau_2) \\ &= H\,D\,A\,I\,K - h_{e_2} - c_p\ (\tau'' - \tau_2) \\ &= D\,A\,I\,C; \\ \therefore\ U'' &= J\,(h_1 - h_2), \qquad (400) \end{aligned}$$

which is the equivalent of equation (391) or of (395).

176. The efficiency will be

$$E_1 = \frac{J\,h_2}{U''} = \frac{h_2}{h_1 - h_2}. \qquad (401)$$

EXERCISE.

If the inferior absolute pressure of the ammonia be 29 pounds per square inch, temperature when it leaves the cooling coils and enters the compressor, 36° F., then compressed adiabatically to a temperature of 117° F., then condensed to saturation and to a liquid at the constant pressure corresponding to the 117° F., then admitted to the cooling coils and the temperature reduced to that corresponding to the initial absolute pressure, 29 pounds, then evaporated and heated to the initial state ; find the thermal units of refrigeration and of condensation, the specific heat of the liquid being 1.08 ; also the efficiency.

Retain results only to the nearest tenth.	
Inferior temperature, Eq. (350)	$\tau_2 = 458.7.$
Hence,	$T_2 = -1.9.$
Absolute temperature at state C. 460.6 + 36	$\tau'' = 496.6.$
Superheating	$\tau'' - \tau_2 = 37.9.$
Greatest volume of a pound, Eq. (357) for $p = 4176$,	$v'' = 9.8.$
At I, absolute temperature, 117 + 460.6	$\tau' = 577.6.$
" volume, Eq. (369)	$v' = 2.4.$
" pressure, Eq. (370)	$p' = 118.$
From B to A, absolute temperature, Eq. (350)	$\tau_1 = 527.8.$
Hence,	$T_1 = 67.2.$
Fall of temperature from A to D	$T_1 - T_2 = 69.1.$
Heat absorbed during this fall of temperature	$1.08 \times 69.1 = 74.6.$
Latent heat of evaporation at $-1°.9$ F., Eq. (360).	$h_{e2} = 557.$
" " " " " 66.2, " "	$h_{e1} = 515.$
Heat rejected during condensation, Eq. (396)	$h_1 = 540.8.$
Refrigeration per lb., Eq. (398)	$h_2 = 500.7.$
Work done by the compressor, per lb., Eq. (400).	$U = 31198.$
Efficiency, Eq. (401)	$E_1 = 12.5.$

177. Experimental Results. The following data and results are taken from the report of a test of a De La Vergne refrigerating plant by Messrs. R. M. Anderson and C. H. Page, Jr., having a nominal ice-melting capacity of about 110 tons in 24 hours. It was in its every-day working condition and was run at about two-thirds its ordinary capacity. Only the ice plant was involved in the experiment. It consisted of two single-acting vertical compressor cylinders, driven by a horizontal double-acting Corliss engine, as shown in Fig. 80, a feed pump and a condenser. The test was during 11 hours and 30 minutes. Two tests were made of the boilers, as the first indicated such high efficiency it was considered advisable to check the results. It will be seen that the second test also gave a high efficiency. The second test was during 12 hours. All the instruments used in the test were carefully standardized. (Thesis 1887.)

FUEL, FURNACE AND BOILERS.

There were two double return flue boilers arranged to run, automatically, between 60 lbs. and 70 lbs. pressure (gauge).

Fuel, Lehigh nut (anthracite), heat of combustion, B. T. U....	12229.6.
Coal for 12 hours, lbs	4422.
Wood for starting 377 lbs. 0.4 coal equivalent, lbs.....	150.8.
Coal, total equivalent, lbs.....	4572.8.
Unburnt coal, lbs.....	125.0.
Coal consumed, lbs.....	4447.8.
Combustible, total, lbs.....	3577.8.
Coal burned per hour, lbs.....	360.65. - 360.65
Heat in 360.65 lbs. of coal, 360.65 × 12229.6 B. T. U.....	4532901.
Coal per H. P. per h. (2d test, H. P. was 102.92).....	3.6013.
Furnace, grate area, sq. ft.....	39.
Ratio of heating surface to grate surface.....	35.63.
Boilers, heating surface, sq. ft.....	1389.
Water fed per hour, lbs.....	3559.7.
Evaporation per lb. coal fired, lbs.....	9.341.
" " " " " from and at 212°.....	9.957.
" " " " consumed.....	9.601.
" " " " " from and at 212°.....	10.226.
" " " combustible, from and at 212°.....	12.703.
Average gauge pressure, lbs.....	66.0.
Temperature of fire-room, deg. F.....	81.
Average temperature of flue boiler, deg. F.....	319.28.
" " " feed-water " ".....	180.2.
Total heat in 3559.7 lbs. steam above 180°.2 F., B. T. U...	3661463.
Efficiency of furnace and boiler, per cent, $\frac{366146300}{4532901}$ =	80.7.

ENGINE (CORLISS).

Piston, diameter of, inches.....	32.
" stroke " "	36.
" speed per minute, mean, ft.....	194.988.
Revolutions per minute, average.....	31.720.
Average indicated horse-power.....	91.13.
Water consumed per IHP. per hour, lbs.....	34.104.
Steam per IHP. per hour, lbs.....	25.79.
Steam condensed in the engine, per cent.....	24.35.
Average ratio of expansion	5.807.
Steam consumption by the feed-pump per hour, lbs.....	50.25.
" " " " engine per hour, lbs.....	3509.45.
Total heat in 3509.45 lbs. steam above 180°.2 F., B. T. U ...	3609479.
Heat changed to work per h., B. T. U., 102.92 × 1980000 ÷ 778 =	261930.
Efficiency of fluid..... $\frac{261930}{3609479}$ =	0.0721.

Coal per I.H.P. per hour for engine and pump	3.60.
Combustible for I.H.P. per hour, lbs.	2.91.

COMPRESSOR.

Number of cylinders, single-acting	2.
Length of stroke, inches.	36.
Diameter of pistons, each, inches	18.
Area head end of pistons, each, sq. in	254.47.
Average number of revolutions per m	31.720.
Piston displacement per stroke, cu. ft	5.301.
" " " hour, both, cu. ft	20179.
Volume of sealing oil per hour, cu. ft	143.8.
Volume filled with gas per hour, both, cu. ft	20036.
Indicated horse-power, mean	76.0892.
Heat eq. of work by compressor per hour, B. T. U	193645.
Efficiency of compressor from coal $\frac{193645}{4532901}$	= 0.0407.
" " mechanism	0.755.
Temperature of ammonia entering compressor, Deg. F	57.7.
" " " leaving " "	116.1.
Absolute pressure entering compressor, lbs	28.88.
" " leaving " "	132.01.

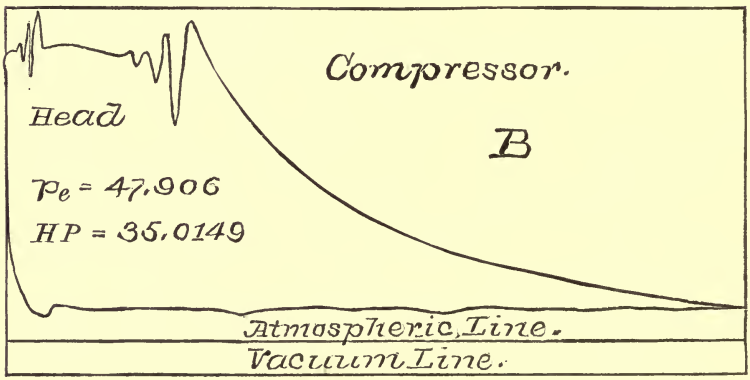

FIG. 88.

Fig. 88 is an exact copy of one of the indicator cards taken from one of the compressor cylinders. The lines nearly vertical at the upper part of the diagram are due to the oscillations of the indicator spring.

Refrigeration.

Pressure in cooling coil, lbs. absolute	28.88.
Hence, temp. of liquid, Deg. F	— 2.0.
Temp. compressed gas, Deg. F	116.1.
" of gas entering the compressor, Deg. F	57.7.
Rise of temperature due to compression, Deg. F	58.4.
Latent heat of evaporation of 1 lb. at — 2.0 F. Eq. (~~358~~ 360)	~~565.6.~~ 556.7
Superheating in cooling coils, 37.9° F., B. T. U	19.27.
Fall of temp. of liquid in cold room, Deg. F.	69.4.
Heat imparted to cold room by this fall, 1.08×69.1	74.95.
Heat removed from cold room per lb., $\frac{365.6}{556.7} + 19.27 - 74.95 =$	~~509.93.~~ 501.02
Ammonia evaporated per hour, lbs	1669.5.
" " " " B. T. U	851311.
"Ice-melting capacity" per hour, lbs	5995.
" " " for 24 hours, tons (each 2000 lbs.)	71.95.
" " " per IHP. per hour, lbs	65.79.
" " " " lb. of coal (3.6013 per HP.), lbs.	18.26.
" " " " 10 lbs. steam (182.6 ÷ 9.601) lbs.	19.02.
" " " " lb. combustible (65.79 ÷ 2.91), lbs.	22.61.
" " " " lb. ammonia evap., lbs	3.59.

Efficiencies.

Efficiency of furnace and boiler (see above)	0.807.
" " steam utilized by engine (see above)	0.0721.
" " engine referred to coal	0.0582.
" " compressor referred to engine (see above)	0.755.
" " " " " coal (775×0.0582)	0.0439.
" " refrigeration referred to compressor $\frac{851311}{193645} =$	4.39.
" " " " " IHP. of engine	3.31.
" " " " " boiler $3.67 \times 0.0643 =$	0.238.
" " " " " coal $0.236 \times 0.807 =$	0.192.

that is, for every thermal unit in the coal there was abstracted 0.19 of a thermal unit from the cold room.

In actual ice-making, only about 30 to 45 per cent of the pounds of ice-melting capacity can be produced as hard ice suitable for commercial purposes, giving, in this case, between 5.0 and 7.5 pounds of hard ice.

The experiments of Professor M. Schröter gave from 19.1 pounds to 37.4 pounds of ice (net) per hour per indicated

horse-power of the engine. (One result is given as 48.8 pounds, but the test was of too short duration to be reliable.)

If 4 pounds of coal were used per H. P. per hour, then there would be produced about 4.8 to 9.3 pounds, net, per pound of coal consumed.

Ledoux remarks that manufacturers estimate about 56 pounds per horse-power per hour measured on the driving-shaft; hence, if the delivered power be 0.80 of the indicated, this would be equivalent to about 45 pounds of ice per indicated horse-power. M. Schröter's tests, and the following, by Mr. Shreve, show that this is too high, if commercial ice is intended.

The amount of ice made depends upon many conditions: as, clearances in the cylinders, friction of mechanism, speed of engine, losses along the pipes, losses in opening the valves, leakage, losses by unavoidable radiation, losses at cans by water unfrozen and ice cleavages; and, these being considered, it seems advisable, in designing, to assume less than one-third the pounds of ice-melting capacity for the probable pounds of commercial ice to be produced.

178. Test of an ice-making plant. An ice-making plant of the Cincinnati Ice Manufacturing & Cold Storage Co. was tested in 1888, by Messrs. A. L. Shreve and L. W. Anderson, chiefly to determine the amount of solid ice which could be manufactured in 24 hours with the plant running under normal conditions. The plant consisted of two 25-ton (nominal) and one 50-ton ice-machines, boilers, pumps, etc., used in actual ice-making. While the machinery was doing its regular work, at a certain hour, the steam pressure was observed to be 75 pounds (gauge), and all the conditions of the furnace, engines, and plant generally were observed, and the conditions continued as nearly uniform as possible for 24 hours, during which time 108.87 tons (of 2000 lbs.) of ice were drawn, from which

should be deducted nearly a ton on account of the terminal condition being slightly different from the initial. The actual ice weighed out at 75 pounds' steam pressure, then, may be taken as 216,000 pounds.

The engines were not tested at the same time, so that the ice per horse-power was not definitely known; but an independent test of the boilers and engines was made prior to the "capacity" test given above, from which it appears that at 75 pounds' pressure, the three engines developed about 415 indicated horse-power; according to which 21 pounds of solid ice were weighed per indicated horse-power per hour. The actual amount may have been several per cent more or less. The machines were new and bearings large, and the engine and compressor absorbed an average of about 52 per cent of the indicated horse-power of the engine.

Absorption System.

179. The absorption system depends upon the fact that water will absorb many times its volume of ammonia gas; at 59° F. it will absorb 727 times its volume. This is a chemical action, and therefore generates heat. According to Favre and Silberman, 925.7 B. T. U. will be developed for each pound of gas absorbed. This action is substituted for the compressor in raising the temperature of the ammonia after leaving the cold room. According to Carius, the coefficient of solubility of ammonia gas in water is represented by the empirical formula (t being Deg. C.)

$$\beta = 1049.62 - 29.4963\ t + 0.676873\ t^2 - 0.0095621\ t^3;$$

according to which the solubility diminishes as the temperature increases and soon reaches a condition at which it ceases to act; and to insure continuous working the absorption chamber is cooled by water externally.

The process is illustrated in Fig. 89. A solution of aqua-ammonia being in the generator A and heated by means of steam passing through coils in this chamber, the vapor of

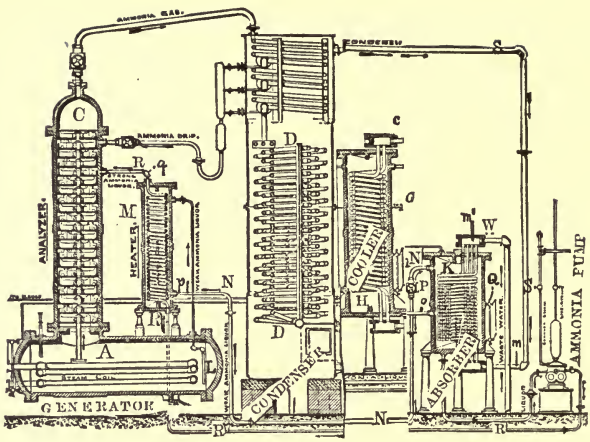

FIG. 89.

ammonia is generated and rises, passing through tortuous ways in the analyzer *C;* thence to the condenser *D.* The steam which rises in the analyzer *C* is partly condensed as it approaches the upper end of the vessel and falls back to the generator, and that which passes into the coils over *D* is led back by the ammonia drip, so that nearly pure ammonia gas enters the condenser *D.* Here the ammonia is at its highest pressure and temperature, and its state may be represented by *B*, Fig. 90, or by the upper right-hand corner of the indicator diagram of Fig. 88. The ammonia gas passes through coils of pipes in the condenser, about which circulates water; the ammonia being condensed to a liquid under a constant pressure, the path of the fluid being represented by *B A*, Fig. 90. The liquid passes to the lower part of the coils, or to a receiver especially provided, and thence through a cock, by which the reduction of pressure is regulated as

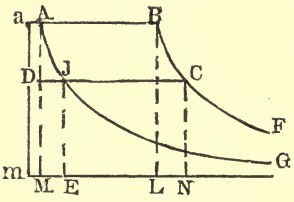

FIG. 90.

it passes into the cooler *H*. The reduction of pressure is represented by *A D*, and of evaporation during the expansion by *D J*. The liquid and a small amount of saturated vapor is now in the cooler *H*, where the liquid vaporizes, absorbing heat, the operation being represented by *J C*, Fig. 90, or the corresponding line in Fig. 88. The brine, which circulates in the cold room, is cooled by the cold ammonia as it passes through the cooler.

During the process of cooling the brine, the liquid ammonia becomes a gas. The gas passes from the cooler *H* to the absorber *K*, the pressure in *K* being kept a little lower than in *H* by the ammonia pump, shown at the right, which draws its supply from *K* and forces it into the generator *A*. Into the absorber *K*, water is forced and absorbs a large volume of the gas, as stated above, generating heat, the operation for which being represented by *C B*, Fig. 90, or the compression line in Fig. 88. In this system pure water is not used in the absorber, but instead thereof, water is drawn from the lower part of the generator *A* by the pipe *L*, containing but little ammonia, the mixture being called "weak ammoniacal liquor." By the absorption of the gas, strong aqua-ammonia is formed, which is pumped back into the generator, and the operation repeated.

It will be seen that the operation completes a cycle, and that the changes in the states of the ammonia are similar to those in which a compressor is used; hence, if there were no losses of heat, except those described, the efficiency would be the same.

The vessel *K* is kept sufficiently cool to facilitate the chemical action by means of water flowing over it.

180. Test of absorption plant. Professor J. E. Denton made a seven days' continuous test of an absorption plant with the following results.* Every element en-

* *Trans. Am. Soc. Mech. Eng.*, Vol. X, May, 1889.

tering into the problem was, *as far as practicable, directly measured.*

Average pressures, above atmosphere, generator, lbs. per. sq. in.	150.77.
" " " " steam, " " "	47.70.
" " " " cooler, " " "	23.69.
" " " " absorber, " " "	23.4.
Average temperatures, Deg. F., Generator	272.
" " " Condenser, inlet	54½.
" " " " outlet	80.
" " " " range	25½.
" " " Brine, inlet	21.20.
" " " " outlet	16.14.
" " " " range	5 06.
" " " Absorber, inlet	80.
" " " " outlet	111.
" " " " range	31.
" " " Heater, upper outlet to generator	212.
" " " " lower " absorber.	178.
" " " " inlet from absorber	132.
" " " Inlet from generator	272.
" " " Water returned to main boilers	260.
Steam per hour for boiler and ammonia pump, lbs	1986.
Brine circulated per hour, cu. ft	1633.7.
" " " " pounds	119260.
" Specific heat	0.800.
" Heat eliminated per pound, B. T. U	4.104.
" Cooling capacity per 24 hours, tons of melting ice	40.67.
" " " per pound of steam, B. T. U	243.
" Ice-melting capacity per 10 lbs. of steam, lbs	17.1.

Calorics, refrigerating effect per kilo. of steam consumed	135.
Heat rejected at condenser per hour, B. T. U	918000.
" " " absorber " " "	1116000.
" consumed by generator per lb. of steam condensed, B. T. U.	932.
Condensing water per hour, lbs	36000.
Condensing coil, approx. sq. ft. of surface	870.
Absorber " " " " " "	350.
Steam " " " " " "	200.
Pump, Ammonia, dia. steam cyl., in	9.
" " " ammonia cyl., in	3⅜.

Pump, Ammonia, stroke, in	10.
" " revolutions per minute	22.
" Brine, steam cyl., diam., in	9½.
" " brine cyl., "	8.
" stroke, in	10
" revolutions per m	70.

Effective stroke of pumps 0.8 of full stroke.

180*a*. Sulptur Dioxide, SO_2 (or Sulphurous Acid). The following relations have been found for this acid:

Specific heat of the gas, 0.15483.
" " " " liquid, 0.4.

Relation between the pressure and temperature of the saturated vapor,

$$log\, p = 5.2330 - \frac{1439.0}{\tau} - \frac{235629}{\tau^2}. \qquad (a)$$

Equation of the gas, or superheated vapor,

$$p\, v = 23.87\, \tau - \frac{2457.45}{v^{1.59}}. \qquad (b)$$

Latent heat of vaporization,

$$h_e = 171.26 - 0.25605\, T - 0.0013795\, T^2. \qquad (c)$$

Volume of a pound of the saturated vapor,

$$v = \frac{778}{2.3026} \cdot \frac{h_e}{1439 + \frac{465318}{\tau}} \cdot \frac{\tau}{p} + v_1. \qquad (d)$$

Volume of a pound of the liquid,

$$v_1 = \frac{0.016}{1.484 - 0.0015659\, T}. \qquad (e)$$

(*Trans. Soc. Mech. Eng.*, 1890.)

CHAPTER VI.

COMBUSTION

181. Essential principle. *Combustion*, chemically speaking, is the combination of chemical elements producing heat. *Burning*, popularly speaking, is the result of a rapid combination of oxygen with other elements. Carbon and hydrogen are the chief elements of the fuel used for engineering purposes. Sulphur, another element, is frequently present, but is comparatively of little value.

When two substances unite chemically, forming a substance different from either, it is said that a *chemical affinity* exists between them. The difference between a mechanical mixture and a chemical combination may be illustrated in the case of gunpowder. The process of manufacture makes a mechanical mixture of charcoal, sulphur, and nitre, but if the gunpowder be fired a chemical combination results and a large volume of gas is produced, generating a large amount of heat and developing a strong elastic force; and the original substance entirely disappears and new substances composed of different combinations of the original elements are formed.

Definite proportions. In every chemical compound a definite and unvarying proportion of its elements exists among themselves.

For instance, in water there is always 8 times as much oxygen by weight as there is of hydrogen, so that in 100

pounds of water there is 88.8 pounds of oxygen and 11.1 pounds of hydrogen. Any chemical compound of oxygen and hydrogen in other proportions would be a substance entirely different from water; or, if in the 100 parts there were some other element, as carbon, the substance would also be different from water.

The *chemical equivalent* or *atomic weight* is expressed by a definite number, and the chemical principle of definite proportions may be expressed in the form of the two following laws:

1. The proportions by weight in which substances combine chemically can all be expressed by their chemical equivalents, or by simple multiples of their chemical equivalents.

2. The chemical equivalent of a compound is the sum of the chemical equivalents of its constituents.

Perfect gases at a given pressure and temperature combine in proportion to their *volume*.

Neglecting fractions the following are the chemical *equivalents* for the principal elementary constituents with which we have to deal in fuel and air:

TABLE I.

Name.	Symbol.	Chemical equivalent.	
		By weight.	By volume.
Oxygen	O	16	1
Nitrogen	N	14	1
Hydrogen	H	1	1
Carbon	C	12	?
Sulphur	S	32	

The composition of a compound substance is indicated by writing the symbol of the elements one after the other, and

affixing to each symbol, in the form of a subscript, the number of its equivalents which enter into one equivalent of the compound. Thus, water contains two chemical equivalents of hydrogen to one of oxygen, and is indicated by the expression $H_2 O$; and the constituents by weight will be 2 H + 16 O. Similarly, $C O_2$, carbonic acid, contains one equivalent of carbon and two of oxygen, and by weight 12 C + 32 O.

The following table gives the composition of several substances:

TABLE II.

Name.	Symbol of chemical composition.	Proportions of elements by weight.	Chemical equivalent by weight.	Proportions of elements by volume.	Chemical equivalent by volume.
Air		N 77 + O 23	100	N 79 O 21	100
Water	$H_2 O$	H 2 + O 16	18	H 2 + O	2
Ammonia	$N H_3$	H 3 + N 14	17	H 3 + N	2
Carbonic oxide	C O	C 12 + O 16	28	C + O	2
Carbonic acid	$C O_2$	C 12 + O 32	44	C + O2	2
Olefiant gas	$C_2 H_4$	C 12 + H 2	14	C + H2	2
Marsh gas or fire-damp.	$C H_4$	C 12 + H 4	16	C + H4	2
Sulphurous acid	$S O_2$	S 32 + O 32	64		2
Sulphuretted hydrogen.	$S H_2$	S 32 + H 2	34		2
Bisulphuret of carbon.	$S_2 C$	S 64 + C 12	76		2

Air is not a chemical compound, but a mechanical mixture of nitrogen and oxygen.

182. The heat of combustion of one pound of a substance combining with sufficient oxygen to completely burn it has been found by experiment. The usual process is to surround a small furnace with a quantity of water so arranged as to prevent the escape of heat; the increased

temperature produced in the water by the burning of each pound of the fuel in the furnace being a measure of the "heat of combustion." The results of the experiments of Favre and Silberman are given in the following table:

TABLE III

SHOWING THE TOTAL QUANTITIES OF HEAT EVOLVED BY THE COMPLETE COMBUSTION OF ONE POUND OF COMBUSTIBLE WITH OXYGEN; ADAPTED FROM THE RESULTS OBTAINED BY FAVRE AND SILBERMAN. THE UNIT OF WEIGHT IN THIS TABLE BEING ONE POUND, AND THE UNIT OF TEMPERATURE ONE DEGREE FAHR. (FROM 39° TO 40°).

Substance.	Formula.	Product.	Units of heat.
GASES.			
Hydrogen	H	H_2O	62,032
Carbonic oxide	C O	CO_2	4,325
Marsh gas	CH_4	CO_2 & H_2O	23,513
Olefiant gas	C_2H_4	CO_2 & H_2O	21,343
LIQUIDS.			
Oil of turpentine	$C_{10}H_{16}$	CO_2 & H_2O	19,533
Alcohol	C_2H_6O	CO_2 & H_2O	12,931
Spermaceti (solid)	$C_{32}H_{64}O_2$	CO_2 & H_2O	18,616
~~Sulphate~~ Bisulphide of carbon	CS_2	CO_2 & SO_2	6,122
SOLIDS.			
Carbon (wood charcoal)	C	C O	4,451
		CO_2	14,544
Gas coke			14,485
Graphite from blast furnaces			13,972
Native Graphite			14,035
Sulphur (native)	S	SO_2	4,048
Phosphorus (observed by Andrews.)	P	P_2O_5	10,715

The heat units in a pound of fuel will be nearly the sum of the heat units of combustion of its constituents. Take, for example, olefiant gas: According to Table II. the chemical equivalents by weight are 14, of which 12 are carbon

and 2, hydrogen; or, $\frac{6}{7}$ of the compound is carbon and $\frac{1}{7}$ hydrogen. Then, from Table III., we find

$\frac{1}{7}$ lb. H gives $\frac{1}{7}$ of 62032	=	8862 B. T. U.
$\frac{6}{7}$ lb. carbon gives $\frac{6}{7}$ of 14554	=	12475 " " "
Total.....	=	21337 " " "

which is only 6 thermal units less than the value given in the table, as deduced from experiment.

If the principal constituents are carbon, hydrogen, and oxygen, it is found that the total heat of combustion in B. T. U. will be given nearly by the following formula:

$$h = 14500\ (C + 4.28\ (H - \tfrac{1}{8}\ O)\ ),$$

in which $4.28 = \frac{62032}{14500}$, reduces the hydrogen to an equivalent of carbon, and $\frac{1}{8}\ O$ is deducted from the hydrogen, for it is assumed that the oxygen present unites with the hydrogen, forming water. Such substances are called hydrocarbons.

The total heat of combustion is usually computed from its chemical analysis, as shown on page 261.

The following table gives the total heat of combustion of certain fuels (see *Journal of United Service Institution*, Eng., Vol. XI., 1867; Box *On Heat*, p. 60). The specimens were of the best quality, and are too high for ordinary practice. Commercial coal of similar grade would be about 0.7 to 0.8 of these values. Commercial Lehigh (anthracite), analyzed at the Institute, gave 12229 B. T. U.

TABLE IV.

TOTAL HEAT OF COMBUSTION OF FUEL.

Fuel.	Carbon.	Hydrogen.	Oxygen.	Equivalent to pure carbon.	Pounds of water evaporated at 212°.	Total heat of combustion.
	C.	H.	O.	C′.	E	*h.*
I. Charcoal—from wood	0.93			0.93	14	13500
" from peat				0.80	12	11600
II. Coke—good	0.94			0.94	14	13620
" middling	0.88			0.88	13.2	12760
" bad	0.82			0.82	12 3	11890
III. Coal—						
1. Anthracite	0.915	0.035	0.026	1 05	15.75	15225
2. Dry bituminous	0.90	0.04	0.02	1.06	15.9	15370
3. " "	0.87	0.04	0.03	1.025	15.4	14860
4. " "	0.80	0.054	0.016	1 02	15 3	14790
5. " "	0.77	0.05	0.06	0.95	14.25	13775
6. Caking	0.88	0.052	0.054	1.075	16.0	15837
7. "	0.81	0.052	0,04	1.01	15.15	14645
8. Cannel	0.84	0.056	0.08	1.04	15.6	15080
9. Dry long flaming	0.77	0.052	0.15	0.91	13.65	13195
10. Lignite	0.70	0.05	0.20	0.81	12.15	11745
IV. Peat—dry	0.58	0 06	0.31	0.66	10.0	9660
" containing 25% moisture					7.25	7000
V. Wood—Dry	0.50			0.50	7.5	7245
" containing 20% moisture					5.8	5600
VI. Mineral Oil—						
from	0.84	0.16	0	1.52	22.7	21930
to	0.85	0.15	0	1.49	22.5	21735

183. The incombustible matter is called *ash*. The principal ingredients of ash are shown in the following analysis, which is from the geological survey of Ohio:

Bituminous coal. Percentage of ash, 5.15.

Silica	58.75
Alumina	35.30
Sesquioxide of iron	2.09
Lime	1.20

Magnesia	0.68
Potash and soda	1.08
Phosphoric acid	0.13
Sulphuric acid	0.24
Sulphur, combined	0.41
	99.88

The proportions vary greatly with different fuels.

184. Air required for combustion. Consider pure carbon. The chemical equivalent of oxygen is, according to Table I., $\frac{16}{12}$ of that of carbon. If the carbon be completely burned, $C\,O_2$ is formed, so that the proportion by weight will be $\frac{32}{12}$ of oxygen to 1 of carbon. According to Table II., 0.23 of the air by weight is oxygen; hence

$$\textit{Weight of air per lb. of carbon} = \tfrac{32}{12} \div 0.23 = 12 \text{ lbs., nearly.}$$

If the compound contains carbon, hydrogen and oxygen, we will have, nearly,

$$\textit{Weight of air per lb. fuel} = A = 12\,C + 36\,(H - \tfrac{1}{8}\,O).$$

The following table, computed from this formula, is given by Rankine.

TABLE V.

Fuel.	C.	H.	O.	A.
I. Charcoal—from wood	0.93			11.16
" from peat	0.80			9.6
II. Coke—good	0.94			11.28
III. Coal—anthracite	0.915	0.035	0.026	12.13
" dry bituminous	0.87	0.05	0.04	12.06
" caking	0.85	0.05	0.06	11.73
" "	0.75	0.05	0.05	10.58
" cannel	0.84	0.06	0.08	11.88
" dry long flaming	0.77	0.05	0.15	10.22
" lignite	0.70	0.05	0.20	9.30
IV. Peat—dry	0.58	0.06	0.31	7.68
V. Wood—dry	0.50			6.00
VI. Mineral Oil	0.85	0.15	0	15.65

Besides the air necessary to furnish the oxygen, some is required for dilution, so as to secure a more free access of air to the fuel. The above table shows that about 12 pounds of air is required per pound of fuel, and experiment indicates that from 1½ to 2 times this amount is required in the furnace for combustion and dilution. An excess of air causes a waste of heat by transporting an unnecessary amount of heat up the chimney, and a deficiency causes imperfect combustion. Forced draft requires less air than a natural draft.

185. Forced draft. Most American sea-going steamers have boilers designed to burn anthracite coal with natural draft, and 5 to 5½ pounds of coal is burned per hour per square foot of grate. Torpedo-boats, with bituminous coal and forced draft of 6 inches of water, may burn 96 pounds per square foot of grate per hour.

In a furnace in which 19 pounds of anthracite coal were burned, a forced draft by means of a screw revolving in the chimney was introduced, causing a burning of 38½ pounds of coal, and a production of 80 per cent more steam than in the former case. In 1847 Robert L. Stevens introduced the plan of air-tight fire-rooms, by which means a forced draft was produced by forcing air into the room occupied by the firemen.

186. Temperature of fire. If the volume be constant, as in case of an explosion in a closed vessel, then will the rise of temperature be given by equation (37), page 55; but if the pressure be constant, as in the ordinary furnace, then will it be given by equation (38).

Take the case of pure carbon burned with 24 pounds of air. The total heat of combustion will be, Table III., 14544 thermal units. This heat is expended in heating 25 pounds of matter, of which 24 pounds is air, whose specific heat is 0.238, and one pound of carbonic acid gas, whose specific heat is 0.217. Call the specific heat of the mixture 0.237; then

$$\tau_1 - \tau_2 = \frac{14544}{25 \times 0.237} = 2454^\circ \text{ F.}$$

The only measurement with a pyrometer which has come to my notice gives a much lower temperature than is found by this formula.

187. Height of chimney. The height of the chimney to produce a natural draft must be such that the difference between the weight of a column of the hot gases, having one square foot for its base and height equal to the height of the chimney, and that of a column of equal height of external air, shall produce the required velocity of air in the chimney.

Let w_0 be the weight of fuel burned in the furnace per second,

V_0, the volume of the air at 32° supplied per pound of fuel burned,

τ_0, the absolute temperature at 32°,

A, the area of the cross-section of the chimney,

m, $A \div$ perimeter of chimney,

τ_1, the absolute temperature of the gases discharged from the chimney,

w, the weight of a cubic foot of the hot gases,

l, the length from the furnace to the top of chimney,

u, the velocity of the current in the chimney per second;

then, if 24 pounds of air be supplied per pound of fuel,

$$V_0 = 25 \times 12 = 300 \text{ cu. ft.}$$

$$u\,A = w_0\,V_0\frac{\tau_1}{\tau_0} = \text{volume of gases per second;}$$

$$\therefore u = \frac{w_0\,V_0\,\tau_1}{A\,\tau_0}.$$

$$w = \frac{\tau_0}{\tau_1}\left(0.0807 + \frac{1}{V_0}\right).$$

According to Peclet, the head to produce the velocity u for 20 lbs. coal burned per sq. ft. of grate per hour will be

$$h = \frac{u^2}{2g}\left(13 + \frac{0.012\,l}{m}\right).$$

Having h, the height of the chimney may be found.

Let H be the height of chimney,

τ_2, the absolute temperature of the external air; then

Weight of the column of air $= 0.0807 \cdot \frac{\tau_0}{\tau_2} H.$

" " " " " gases $= \left(0.0807 + \frac{1}{V_0}\right)\frac{\tau_0}{\tau_1} \cdot H.$

" " a column of gases of length h

$$= \left(0.0807 + \frac{1}{V_0}\right)\frac{\tau_0}{\tau_1} h;$$

$$\therefore \left(0.0807 + \frac{1}{300}\right)\frac{\tau_0}{\tau_1}(H + h) = 0.0807\,\frac{\tau_0}{\tau_2} H;$$

$$\therefore H = \frac{h}{0.96\,\frac{\tau_1}{\tau_2} - 1}.$$

The weight discharged per second will be

$$w u = \frac{\tau_0}{\tau_1}\left(0.0807 + \frac{1}{V_0}\right)\sqrt{h}\sqrt{\frac{2g}{13 + \frac{0.012\,l}{m}}}$$

$$= constant \frac{\sqrt{0.96\,\frac{\tau_1}{\tau_2} - 1}}{\tau_1}.$$

This will be a maximum for

$$\tau_1 = \frac{\tau_2}{0.48} = 2\frac{1}{12}\tau_2;$$

that is, *the best chimney draft takes place when the absolute temperature of the gas in the chimney is to that of the external air as* 25 *to* 12.

This in the preceding equation gives

$$h = H.$$

In the solution for a maximum, l is considered constant, an assumption which affects the result by only a small amount.

The height of a chimney is often determined by surrounding circumstances, and sometimes by imagined conditions of future use; and in such cases are not subject to computation.

In ordinary practice chimneys are from forty to one hundred and twenty feet. Above one hundred feet the effect of additional height is comparatively small.

The tallest chimney of which we have knowledge is 441.6 feet high, eleven feet and a half in diameter at the base, and ten feet at the top. It was built by the Mechernich Lead Mining Co. (Van Nostrand's *Eng. Mag.*, 1886, page 264). For dimensions of large chimneys, see Van Nostrand's *Eng. Mag.*, September, 1883, page 216; also *Trans. Am. Soc. Civ. Engineers*, 1885; also No. 1, *Science Series*, by D. Van Nostrand.

APPENDIX I.

(Extract from an article by the author in the *Philosophical Magazine* for November, 1885.*)

THE LUMINIFEROUS ETHER.

Two properties of the luminiferous ether appear to be known and measurable with a high degree of accuracy. One is its ability to transmit light at the rate of 186300 miles per second,† and the other its ability to transmit from the sun to the earth a definite amount of heat energy.

In regard to the latter, Herschel found, from a series of experiments, that the direct heat of the sun, received on a body at the earth capable of absorbing and retaining it, is competent to melt an inch in thickness of ice every two hours and thirteen minutes. This is equivalent to nearly 71 foot-pounds of energy per second.

In 1838 M. Pouillet found that the heat energy transmitted from the sun to the earth would, if none were absorbed by our atmosphere, raise 1.76 grammes of water 1° C. in one minute on each square centimeter of the earth normally exposed to the rays of the sun.‡

This is equivalent to 83.5 foot-pounds of energy per second, and is the value used by Sir William Thomson in determining the probable density of the ether.§ Later determinations of the value of the solar constant by MM. Sorret, Crova, and Violle have made it as high as 2.2 to 2.5 calories. But the most recent, as well as the most reliable, determination is by Professor S. P. Langley, who brought to his service the most refined apparatus yet used for this purpose, and secured his data under favorable conditions; from which the value is found to be 2.8 ± calories‖ with some uncertainty still remaining in regard to the first figure of the deci-

* Published in Van Nostrand's *Engineering Magazine*, January, 1886. Also *Science Series*, No. 85.

† Professor Michelson found the velocity of light to be 299740 kilometers per second in air, and 299828 kilometers in a vacuum, giving an index of refraction of 1 000265. (*Journal of Arts and Science*, 1879, Vol. XVIII., p. 390.)

‡ *Comptes Rendus*, 1838, Tom. VII., pp. 24–26.

§ *Trans. Roy. Soc. of Edinburgh*, Vol. XXI., Part I.

‖ *Am. Journ. of Arts and Science*, March, 1883, p. 195. Also *Comptes Rendus*.

mal. We will consider it as exactly 2.8 in this analysis, according to which, there being 7000 grains in a pound and 15.432 grains in a gramme, we have for the equivalent energy

$$\frac{2.8 \times 15.432}{7000} \times \frac{9}{5} \times \frac{772 \times 144}{0.155 \times 60} = 133 \text{ foot-pounds}$$

per second for each square foot of surface normally exposed to the sun's rays, which value we will use. Beyond these facts, no progress can be made without an assumption. Computations have been made of the density, and also of the elasticity, of the ether founded on the most arbitrary, and in some cases the most extravagant, hypotheses. Thus, Herschel estimated the stress (elasticity) to exceed

$$17 \times 10^9 = (17{,}000{,}000{,}000) \text{ pounds per square inch ;}*$$

and this high authority has doubtless caused it to be widely accepted as approximately correct. But his analysis was founded upon the *assumption* that the density of the ether was the same as that of air at sea-level, which is not only arbitrary, but so contrary to what we should expect from its non-resisting qualities as to leave his conclusion of no value. That author also erred in assuming that the tensions of gases were as the wave-velocities in each, instead of the mean square of the velocity of the molecules of a self-agitated gas ; but this is unimportant, as it happens to be a matter of quality rather than of quantity. Herschel adds, "Considered according to any hypothesis, it is impossible to escape the conclusion that the ether is under great stress." We hope to show that this conclusion is not warranted ; that a great stress necessitates a great density ; but that both may be exceedingly small. A great density of the ether not only presents great physical difficulties, but, as we hope to show, is inconsistent with the uniform elasticity and density of the ether which it is believed to possess ; and every consideration would lead one to accept the lowest density consistent with those qualities which would enable it to perform functions producing known results.

In a work on the *Physics of Ether*, by S. Tolver Preston, it is estimated that the probable inferior limit of the tension of the ether is 500 tons per square inch, a very small value compared with that of Herschel's. But the hypothesis upon which this author founded his analysis was—The tension of the ether exceeds the force necessary to separate the atoms of oxygen and hydrogen in a molecule of water ; as if the atoms were forced together by the pressure of the ether, as two Magdeburg hemispheres are forced together by the external air when there is a vacuum between them. This assumption is also gratuitous, and is rejected for want of a rational foundation.

Young remarks : "The luminiferous ether pervading all space is not

* *Familiar Lectures*, p. 282.

only highly elastic, but absolutely solid." * We are not certain in what sense this author considered it as solid ; but if it be in the sense that the particles retain their relative positions, and do not perform excursions as they do in liquids, it is a mere hypothesis, which may or may not have a real existence. If it be in the sense that the particles suffer less resistance to a transverse than to a longitudinal movement, there are some grounds for the statement, as shown in circularly-polarized light. Bars of solids are more easily twisted than elongated, and, generally, the shearing resistance is less than for a direct stress. It certainly cannot be claimed that the compressibility of the ether (in case we could capture a quantity of it) is less than that of solids.

Sir William Thomson made a more plausible hypothesis, by assuming that "the maximum displacement of the molecules of the ether in the transmission of heat energy was $\frac{1}{50}$ of a wave length of light, the average of which may be taken as $\frac{1}{5000}$ of an inch." Hence the displacement was assumed to be $\frac{1}{250000}$ of an inch ; by means of which he found the weight of a cubic foot to be $\frac{2}{3} \times 10^{-20}$ of a pound.† We also notice that Hr. Belli estimated the density of the ether to be $\frac{1}{2} \times 10^{-15}$ of a pound ; ‡ but M. Herwitz, assuming this value to be too small and Thomson's as too large, arbitrarily assumed it as 10^{-18} of a pound per cubic foot ; but arbitrary values are of small account unless checked by actual results.

We propose to treat the ether as if it conformed to the Kinetic Theory of Gases, and determine its several properties on the conditions that it shall transmit a wave with the velocity of 186300 miles per second, and also transmit 133 foot-pounds of energy per second per square foot. This is equivalent to considering it as gaseous in its nature, and at once compels us to consider it as molecular ; and, indeed, it is difficult to conceive of a medium transmitting light and energy without being molecular. The Electromagnetic Theory of Light suggested by Maxwell, as well as the views of Newton, Thomson, Herschel, Preston, and others, are all in keeping with the molecular hypothesis. If the properties which we find by this analysis are not those of the ether, we shall at least have determined the properties of a substance which might be substituted for the ether, and secure the two results already named. It may be asked, Can the Kinetic theory, which is applicable to gases in which waves are propagated by a to-and-fro motion of the particles, be applicable to a medium in which the particles have a transverse movement, whether rectilinear, circular, elliptical or irregular ? In favor of such an application, it may be stated that the general formulæ of analysis by which wave motion in general, and refraction, reflection and polarization in particular, are discussed, are fundamentally the same ; and in the establishment of the

* *Young's Works*, Vol. I., p. 415. † *Phil. Mag.*, 1855 [4] IX., p. 39.
‡ Cf. *Fortschritte der Physik*, 1859.

equations the only hypothesis in regard to the path of a particle is—It will move along the path of least resistance. The expression $V^2 \propto e \div \delta$ is generally true for all elastic media, regardless of the path of the individual molecules. Indeed, granting the molecular constitution of the ether, is it not probable that the Kinetic theory applies more rigidly to it than to the most perfect of the known gases? *

The 133 foot-pounds of energy per second is the solar heat energy in a prism whose base is 1 square foot and altitude 186300 miles, the distance passed over by a ray in one second; hence the energy in 1 cubic foot will be

$$\frac{133}{186300 \times 5280} = \frac{4}{3 \times 10^7} \text{ foot-pounds.} \qquad (1)$$

Where results are given in tenth-units of high order, as in the last expression, it seems an unnecessary refinement to retain more than two or three figures to the left hand of the *tens*; and we will write such expressions as if they were the exact results of the computations.

If V be the velocity of a wave in an elastic medium whose coefficient of elasticity, or in other words, its tension, is e and density δ, both for the same unit, we have the well-known relation

$$V = \sqrt{\frac{d\,e}{d\,\delta}}.$$

And for gases we have

$$e = \delta^{\gamma},$$

where $\gamma = 1.4$; and the differential of the latter substituted in the former gives

$$V = \sqrt{\frac{\gamma\, e}{\delta}}. \qquad (2)$$

The tension of a gas varies directly as the kinetic energy of its molecules per unit of volume. If v^2 be the mean square of the velocities of the molecules of a self-agitated gas, we have

$$e \propto \delta\, v^2, \text{ or } v^2 = x\,\frac{e}{\delta}, \qquad (3)$$

where x is a factor to be determined. Equations (2) and (3) give

$$v^2 = \frac{x}{\gamma}\, V^2. \qquad (4)$$

Assuming, with Clausius, that the heat energy of a molecule due to the action of its constituent atoms, whether of rotation or otherwise, is a multiple of its energy of translation, we have for the energy in a unit of volume producing heat,

* See also remarks by G. J. Stoney, *Phil. Mag.*, 1868 [4] XXXVI., pp. 132, 133.

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$$\tfrac{1}{2}\, y\, \delta\, v^2,$$

where y is a factor to be determined. If c be the specific heat of a gas, w its weight per cubic foot at the place where $g = 32.2$, J. Joule's mechanical equivalent, τ its absolute temperature; then the essential energy of a cubic foot of the medium will be $c\, w\, \tau\, J$; and observing that $w = g\, \delta$, we have

$$\tfrac{1}{2}\, y\, \delta\, v^2 = c\, g\, \delta\, \tau\, J, \tag{5}$$

which, reduced by (4), gives

$$x\, y = \frac{2\, c\, g\, \gamma\, \tau\, J}{V^2}, \tag{6}$$

the second member of which is constant for a given gas. To find its value we have

	Hydrogen.	Air.	Oxygen.
Specific heat,*	3.4093	0.2375	0.2175
Velocity of sound, feet per second, at $\tau = 493.2°$, ...	4163	1090	1040

and $g = 32.2$, $\gamma = 1.4$, $J = 772$. These, substituted in the second member of (6), give

$x\, y$ for hydrogen, ...	6.599
" air, ...	6.706
" oxygen, ...	6.596
	3)19.901
Mean, ...	6.63

This value, which is nearly constant for the more perfect gases, we propose to call *the modulus of the gas*, and represent it by μ; and for the purposes of this paper we will use

$$\mu = 6.6.$$

This relation of the product $x\, y$ being a constant, has, so far as we are informed, been overlooked by physicists, and is worthy of special notice, since it determines the value of one of the factors when the other has been found. Krönig, Clausius,† and Maxwell give for x the constant number 3, but variable values for y.‡

We are confident that the value of x is not strictly constant; or if it is, it exceeds 3, since the effect of the viscosity of a gas would necessitate a larger velocity to produce a given tension than if it were perfectly free

* Stewart on *Heat*, p. 229.

† *Phil. Mag.*, 1857 [4] XIV., p. 123.

‡ *Theory of Heat*, pp. 314 and 317. Maxwell states that the value for y is probably equal to 1.634 for air and several of the perfect gases. This would make $x = 4$ nearly.

from internal friction. For our purpose, it will be unnecessary to find the separate values of x and y; but if we have occasion to use the former in making general illustrations, we will call it 3, as others have done heretofore. If the correct value of x exceeds 3, it will follow that the velocity of the molecules exceeds the values heretofore computed.* According to Thomson, Stokes showed that in the case of circularly polarized light the energy was half potential and half kinetic ;† in which case $y = 2$, and therefore $x = 3.3$.

The energy in a cubic foot of the ether at the earth being given by (1) and (5), we have, by the aid of (4),

$$\tfrac{1}{2}\, y\, \delta\, v^2 = \tfrac{1}{2}\, \mu\, \frac{\delta}{\gamma}\, V^2 = \frac{4}{3 \times 10^7}; \tag{8}$$

$$\therefore \delta = \frac{4 \times 1.4 \times 2}{3 \times 10^7 \times 6.6 \times (186300 \times 5280)^2} = \frac{2}{35 \times 10^{24}} \text{ lb.}, \tag{9}$$

which is the mass of a cubic foot of the ether at the earth, and which would weigh at the place where $g = 32.2$ about

$$w = \frac{2}{10^{24}} \text{ of a pound}, \tag{10}$$

compared with which Thomson's value is less than 4000 times this value. Thomson remarked that the density could hardly be 100,000 times as small—a limit so generous as to include far within it the value given in (9). According to equation (10), a quantity of the ether whose volume equals that of the earth, would weigh about $\frac{1}{20}$ of a pound. If a particle describes the circumference of a circle in the same time that a ray passes over a wave-length λ, the radius of the circle will be, using equation (4),

$$r = \frac{v\, t}{2\, \pi} = \sqrt{\frac{x}{\gamma}} \cdot V \cdot \frac{\lambda}{2\, \pi\, V} = \tfrac{10}{43}\, \lambda,$$

or the displacement from its normal position will be about $\frac{10}{43}$ of a wave-length, or about $\frac{1}{2150000}$ of an inch at the earth.

Eliminating V between (2) and (8) gives

$$e = \frac{8}{3\, \mu \times 10^7} = \frac{4}{10^8} \tag{11}$$

for the tension of the ether per square foot at the earth, and is equivalent to about 1.1 of a pound on a square mile. The tension of the atmosphere at sea-level is more than 30,000,000,000 times this value. It some-

* Maxwell gives for the mean square of the velocities, or, in other words, the velocity whose square is the mean of the squares of the actual velocities of the molecules, in feet per second at 493.2° F. above absolute zero, hydrogen 6232, oxygen 1572, carbonic oxide 1276, carbonic acid 1570. *Phil. Mag.*, 1873, p. 68. Our equation (4) gives for air 1593.

† *Phil. Mag.*, 1855 [4] IX., p. 37.

what exceeds the tension of the most perfect vacuum yet produced by artificial means, so far as we are informed. Crookes produced a vacuum of .02 millionth of an atmosphere * without reaching the limit of the capacity of the pumps ; and Professor Rood produced one of $\frac{1}{3900000000}$ of an atmosphere † without passing the limit of action of his apparatus. The latter gives a pressure per square foot of

$\frac{14.7 \times 144}{390000000} = \frac{1}{180000}$ of a pound. This, in round numbers, is 140 times the value given in equation (11). Even at this great rarity of the atmosphere, the quantity of matter in a cubic foot of the air would be some 200 million million times the quantity in a cubic foot of the ether—such is the exceeding levity of the ether.

Admitting that the ether is subject to attraction according to the Newtonian law, and of compression according to the law of Mariotte, we propose to find *the relation between the density of the ether at the surface of an attracting sphere and that at any other point in space*, providing that the sphere be cold and the only attracting body, and the gas considered the only one involved.

Let δ_0, e_0, w_0 be respectively the density, elasticity and weight of a unit of the medium, whether ether, air, or any other gas at the surface of the sphere ; δ, e, w, the corresponding quantities at a distance z from the surface of the sphere ; r the radius of the sphere, g_0 the acceleration due to gravity at its surface, and g that at distance $r + z$ from the centre of the sphere. Then

$$\frac{\delta}{\delta_0} = \frac{e}{e_0} = \frac{w}{g} \div \frac{w_0}{g_0}$$

and

$$g = g_0 \frac{r^2}{(r+z)^2};$$

$$\therefore e = \frac{e_0}{w_0} \cdot \frac{g_0}{g} w = \frac{e_0}{w_0} \frac{(r+z)^2}{r^2} w. \tag{12}$$

But

$$d e = - w \, d z = - g \, \delta \, d z \tag{13}$$

$$\therefore \frac{d e}{e} = - \frac{g_0 \, \delta_0}{e_0} \cdot \frac{r^2}{(r+z)^2} \, d z.$$

Integrating between e and e_0, z and o we have

* *On the Viscosity of Gases at High Exhaustion*, by William Crookes, F.R.S., *Phil. Trans. Roy. Soc.*, Part II. (1881), p. 400 : "Going up to an exhaustion of .02 millionth of an atmosphere, the highest point to which I have carried the measurements, although by no means the highest exhaustion of which the pump is capable."

† *Journ. of Arts and Science*, 1881, Vol. XXII., p. 90.

$$e = e_0\, \varepsilon^{-\frac{g_0\, \delta_0}{e_0} \cdot \frac{r\, z}{r+z}}, \tag{14}$$

$$\delta = \delta_0\, \varepsilon^{-\frac{g_0\, \delta_0}{e_0} \cdot \frac{r\, z}{r+z}}. \tag{15}$$

Neglecting the attraction of the earth for the ether, and considering the sun as the only attracting body, we have g_0 at the sun 28.6×32.2, and at the earth, $z = 210\, r$, $r = 441{,}000$ miles, the sun's radius; $\delta = \frac{2}{3^5} \times 10^{-24}$, equation (9), and $e = \frac{4}{3^5} \times 10^{-6}$; and these, in (14) and (15), give

$$e = e_0\, \varepsilon^{\frac{28.6 \times 32.2 \times 2 \times 33 \times 10^6}{4 \times 35 \times 10^{24}} \times \frac{210}{211} \times 441000 \times 5280}$$

$$= e_0\, \varepsilon^{\frac{1}{1000000}} \text{ nearly}, \tag{16}$$

and

$$\delta = \delta_0\, \varepsilon^{\frac{1}{1000000}} \text{ nearly}, \tag{16'}$$

for the tension and density of the ether at the surface of the sun under the conditions imposed. But the millionth root of ε is practically unity; hence the elasticity and density at the sun is practically the same as at the earth.

Now, starting at the sun with this result, and finding the density at a distance z from it, then making z infinite, we shall get about the 995,000 root of ε, the value of which is also sensibly equal to unity; hence the density at infinity would be sensibly the same as at the surface of the sun, the difference in the densities at the sun and at infinity being less than $\frac{1}{1000000}$ part of that at the sun. In order to make the density vary sensibly with the distance, the attraction of the central body must be something like a million times as great as that of the sun, or have a diameter a million times as large; but there is no such known body, therefore *the density and tension of the ether may be considered uniform throughout space.* Such has been our conception of it, and it is an agreeable surprise to find it so fully confirmed by analysis.

If the density were uniform, the weight of a given volume of it would vary as the force of gravity. At the surface of the sun a cubic foot would weigh [equation (10) multiplied by 28.6, or] 57×10^{-24}; hence, for a height h it would weigh

$$\frac{57}{10^{24}} \int_0^h \frac{r^2}{(r+z)^2}\, d\, z = \frac{57}{10^{24}} \cdot \frac{r\, h}{r+h}, \tag{17}$$

which for $h = \infty$ becomes $\frac{13}{10^{14}}$ of a pound, which is the pressure upon a square foot of the sun of a column of infinite height under the conditions

imposed. This would compress the first foot of the column about $\frac{1}{1000000}$ of its length, and would cause a corresponding increase in the density, the value of which, after this compression, will be found by multiplying the value given in equation (9) by $\frac{9999999}{10000000}$, which will leave the result sensibly the same as before. Hence, from this standpoint, we again conclude that the density of the ether may be considered as sensibly uniform throughout space, providing its temperature be essentially uniform.

If we assume that the law of the resistance by which the ether opposes the motion of a body varies as the square of the velocity of the body, we are still unable to assign the coefficient which will give the numerical value; but it is safe to assume that the entire mass of the ether occupying the path of a body moving through it, will not have a velocity imparted to it exceeding that of the body; but, to be on the safe side, we will assume that it imparts a velocity equal to itself. The energy thus imparted will be lost to the body. To simplify the case, consider a planet moving in a circular orbit: r the radius of the planet, d its distance from the sun, D its specific gravity compared with water as unity, v_1 the velocity in its orbit; then the mass of ether occupying the place of the planet during one revolution about the sun will be, using equation (9),

$$\frac{2}{35 \times 10^{24}} \pi r^2 \times 2 \pi d,$$

which, multiplied by $\frac{1}{2} v_1^2$, will give the energy imparted to it. The kinetic energy of a planet, neglecting its rotation, will be

$$\tfrac{4}{3} \pi r^3 \times 62\tfrac{1}{2} D \times \frac{{v_1}^2}{2 g}.$$

Dividing the former, after multiplying it by $\frac{1}{2} {v_1}^2$, by the latter, gives

$$\frac{1}{7 \times 10^{24}} \cdot \frac{d}{r D} \tag{18}$$

for the fraction of the energy lost during one revolution about the sun. Applying this to the earth, we have

$$d \div r D = 93000000 \div 3912 \times 5\tfrac{1}{2} = 43000,$$

and (18) becomes

$$\frac{6}{10^{29}} \text{ nearly,} \tag{19}$$

for the fraction of the energy lost in one year; and hence *at this rate* would require more than 1,666,000 trillion (1,666,000,000,000,000,000,000) years to bring it to rest.

Equation (18) is not applicable to the resistance offered to a comet, on account of the elongated orbit of the latter; but some idea of the effect of the resistance of the ether to the movement of a comet may be found

by considering what it would be if the orbit were circular, having for its radius the perihelion distance. According to Professor Morrison, the perihelion distance of the great comet (6), 1882,* was 716200 miles, its aphelion distance will be 5,000,000,000 miles, the diameter of its nucleus shortly before disappearing on the solar disk was 7600 miles, the velocity at perihelion 295 miles per second, and at aphelion 75 feet per second. But little is known in regard to the density of comets ; but, to be on the safe side, we will assume it as $\frac{1}{1000}$ that of water. This data will reduce (18) to 13×10^{-18} for the fraction of energy lost during one of its revolutions about the sun ; and as it would make a revolution in, say, 20 hours, it would lose in one of our years about 57×10^{-16} of its energy, *at which rate* it would go on for 170 trillions of years. Similarly, at its aphelion its *rate* of loss would be less than $\frac{1}{6} \times 10^{-15}$ of its energy in more than 2000 years—the time of one revolution in its orbit.

The most careful observations and calculations have failed to detect any effect due to the resistance of matter in space ; and the above analysis shows that, within historic times, it has in any case scarcely amounted to an infinitesimal, certainly not sufficient to be measured. And when we consider that our assumptions have been very largely on the unfavorable side, and, further, that the energy imparted to the ether may partly, at least, be restored to the body, we assume that its resistance never can be measured. Laplace, when he found that the force of gravitation, if propagated by an elastic medium, must have a velocity exceeding 100 million times that of light, concluded that astronomers might continue to consider its action as instantaneous (*Mécanique Céleste*, B. X., ch. 8, p. 22, 9035) ; so may we, with nearly as much confidence, continue to consider the resistance of the ether as *nil*.

Equation (6) gives

$$c\,\tau = \frac{6.6\ (186300 \times 5280)^2}{2 \times 32.2 \times 1.4 \times 772} = 92 \times 10^{12} \qquad (20)$$

from which the specific heat of the ether may be found if its temperature were known. M. Fourier, the first to assign a value to *the temperature of space*, assumed it to be somewhat inferior to the temperature at the poles of the earth or about 50° C. to 60° C. below zero.† M. Pouillet, considering the atmosphere as a diathermanous medium, capable of absorbing in different degrees the radiant heat from the sun and the dark heat from the earth, deduced for the heat of space—or, as he and Fourier called it, the stellar heat—approximately,—142° C.‡ (— 287° F.), which

* *Monthly Notices of the Royal Astronomical Society*, Vol. XLIV., 2, p. 54.

† *Ann. der Chemie*, Tome XVII., p. 155.

‡ *Comptes Rendus*, 1838, Vol. VII., p. 61. Pouillet's formula is

$$a' = 1.235\,\frac{2-b}{2-b'} - 0.489,$$

is about 174° F. above absolute zero. It is well known that Pouillet's data were imperfect, several important elements being neglected, notably that of the humidity of the air ; still, it is not only the first, but, so far as we know, the only attempt to formulate this relation. It served to show what has since been indicated by more direct experiments, that the temperature of space is very low. The delicate experiments of Professor Langley, before referred to, show a great difference in the degree of absorption by our atmosphere of different wave-lengths. The mean of the values for nine different wave-lengths, treated by M. Pouillet's formula, gives 139° F. above absolute zero, and the smallest value of absorption, which was for the infra-red, gives only 71° F. above absolute zero for the heat of space.

The heat of space may be considered as composed of three parts . (1) stellar heat, (2) the heat contained in the dark matter of space, (3) the essential heat of the ether.

1. By the stellar heat we mean the heat received directly from the stars. It is a matter of easy calculation that, if the 50,000,000 of stars supposed to be visible with the most powerful telescopes were all at the distance of the nearest fixed star (α Centauri), or 221,000 astronomical units from the earth, and if each radiated the same amount of heat as our sun, the intensity varying as the inverse squares of the distances, the earth would receive from them all less than $\frac{1}{10\,00}$ as much heat as it now receives from the sun. And when we consider that only a very few stars are within measurable distances, and that the remote ones may be, when compared with these, well-nigh infinitely distant, it is evident that the amount of heat received from the stars is insignificant, and may be discarded at the earth.

2. It is certain that there is a large amount of dark matter in space, since the meteoric dust and meteorites must come from beyond our atmosphere. The zodiacal light is supposed to be an evidence of meteoric matter between the earth and sun. The tails of comets are visible by some action of light upon some kind of matter. Matter in space not exposed to the rays of the sun will be at about the same temperature as the ether ; but if in the rays of the sun and destitute of an atmosphere at the distance of the earth from the sun, its temperature would be very low. If present laws can be extended so far, and the earth were without an atmosphere, and the heat received were not conducted away, it has been computed that the mean temperature at the equator would be about

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in which b' = the absorptive power by the atmosphere of the sun's heat,
b = the absorptive power of terrestrial heat,
t' = the temperature of the stellar heat,
a = 1.0077.

If $b = 1$, its maximum, $b' = 0.2$, we find $t' = -235°$ C. (− 391° F.), or 71° F. above absolute zero.

— 70° C. (— 94° F.) ; and at the poles — 221° C.,* or 114° F. above absolute zero. The last result is obtained on the supposition that the poles receive heat directly from the sun a part of the year ; it is further shown that if the poles were never exposed to the rays of the sun, the temperature would fall to that of the ether of space. But the data are not uniform, and there is too large an extension of empirical formula to satisfy one that the above numerical results are reliable : still they point more and more strongly to a temperature not many degrees above absolute zero.

3. By the essential heat of the ether we mean the temperature which would be indicated by a thermometer graduated from absolute zero in a room located in space beyond our atmosphere, whose walls were impervious to the passage of external heat. It is the heat due to the self-agitated ether, just as air has a temperature when not exposed to the rays of the sun. If the ether be perfectly diathermanous to the sun's rays, it will receive no heat, on account of the heat of the sun flowing through it, though it may be heated from other sources. As direct evidence of an extremely low temperature of space, we cite the facts in regard to the meteorite which fell at Dharmsalla, India, July 14th, 1860.† "The most remarkable thing about it was, while the mass had been inflamed and melted at the surface, the fragments gathered immediately after the fall and held for an instant were *so cold that the fingers were chilled.* This extraordinary assertion, which is contained in the report with no expression of doubt, indicates that the mass of the meteorite retained in its interior the intense cold of the interplanetary space, while the surface was ignited in passing through the terrestrial atmosphere." Since this body had been exposed to the rays of the sun, its temperature must have exceeded that of the space through which it passed, as well as been warmed by the heat developed at its surface, from which it may be inferred that it had been *intensely* cold. Direct investigations, given above, indicate that this temperature is less than 200° F. above absolute zero ; and we cannot assert that it is not less than 100° F. above, or even much less.

But, however low be the temperature of the ether, it cannot be absolutely cold, or, in other words, it must have a temperature above absolute zero, for otherwise it would be destitute of elasticity, and hence incapable of transmitting a wave. This is shown by eliminating V between equations (2) and (6), giving

$$c\,\tau = \frac{\mu}{2\,g\,\delta\,J}\,e, \tag{21}$$

* *Professional Papers of the Signal Service U. S. A.*, Washington, D. C., 1884, No. XII., p. 54.

† *Comptes Rendus*, 1861, Tome LIII., p. 1018.

in which if $\tau = 0$, e will be zero, all the other factors being finite, and if $e = 0$, then $V = 0$ in (2). Indeed, this principle is so well recognized in physics, that a proof in this place seems superfluous. Being unable, in the present state of our knowledge, to do more than assign the probable superior limit of the temperature, we will, for the purposes of this analysis, assume $\tau = 20°$ F., absolute, being confident that the actual value is between $\frac{1}{10}$ of and 10 times this value. This value in equation (20) gives

$$c = 46 \times 10^{11} = 4{,}600{,}000{,}000{,}000 \qquad (22)$$

for the specific heat of the ether, that of water being unity. This number so vastly—we might say infinitely—exceeds that for any known gas, as to justify one, at first thought, in looking with suspicion upon the applicability of the above analysis to this medium. Assumptions in regard to the absolute temperature will scarcely improve the appearance of this number. If it be assumed that the absolute temperature be only one degree, the number in equation (22) would be only twenty times as large; and if the absolute temperature be assumed at 1000000° F., the resulting specific heat would still be more than a million times as large as for hydrogen. A few considerations of other properties of the ether may aid one in being reconciled to this seemingly paradoxical result. Is the result any more incredible than the fact, generally admitted, that every particle of the ether, in transmitting a wave of light, continually makes 590000000000000000 (6×10^{14} nearly) complete cycles of movements every second, for a wave-length of $\frac{1}{50000}$ of an inch? The number of such complete movements in air for the fundamental c is only 264; and hence the ratio of the former to the latter of these numbers is nearly 2×10^{12}. The ratio of the specific heat given in (22) to that of hydrogen is nearly $1\frac{1}{2} \times 10^{12}$, which is not so different from that just given for the ratio of cyclical movements in a second of the ether and air. The velocity of sound in air at 493° F. above absolute zero is about 1090 feet per second; but if the temperature could be reduced to 20° F., absolute, the law being extended so far, the velocity would be only

$$V = 1090 \sqrt{\frac{20}{493}} = 217 \text{ feet};$$

but the velocity of light is 982,000,000 feet per second, a number about $4\frac{1}{2}$ million times the former, and near a million times that of the velocity in air under ordinary conditions. The ratio of the mass of air in a cubic foot at sea-level to that of a cubic foot of the ether as computed, far exceeds any of these ratios. The fact is, the known qualities of the ether in transmitting light and heat so far transcend those of any known terrestrial substance, that we might anticipate the fact that, in regard to magnitude, all its properties will be extremely exceptional when compared with such substances. We must accept substantially the number

in equation (22), or subject this medium to different laws than those of gases.

We may deduce this result by another process ; thus, since the specific heats of different gases are as the squares of the wave-velocities in the respective substances, the other elements being the same, if the specific heat of air be 0.23, we should have for the specific heat of the ether

$$c = 0.23 \left(\frac{186300 \times 5280}{217} \right)^2 = 46 \times 10^{11},$$

as before. The correct value of the specific heat of air, 0.2375, would give over 47×10^{11}, and nearly 48×10^{11} ; but these differences are quite immaterial in this connection, the object being to check the former result.

On the other hand, in order that common air might be able to transmit a wave with the known velocity of light, its specific heat being taken constantly at 0.23, its temperature would be, according to equation (20),

$$\tau = \frac{92 \times 10^{12}}{0.23} = 4 \times 10^{14} \text{ degrees F. } (= 400{,}000{,}000{,}000^\circ \text{ F.}).$$

If the sun were composed of a substance having such specific heat, it could radiate heat at its present rate for more than a hundred millions of centuries without its temperature being reduced 1° F., exclusive of any supply from external sources, or from a contraction of its volume. We know only such substances in the sun as we are able to experiment with in the laboratory; and if there be an exceptional substance in it, we have no means at present of determining its physical properties. It is, moreover, a question whether the ether constitutes an essential part of bodies. We conceive of it only as the great agent for transmitting light and heat throughout the universe.

On account of the enormous value of the specific heat, it will require an inconceivably large amount of heat (mechanically measured) to increase the temperature of one pound of it perceptibly. Thus, if heat from the sun, by passing through a pound of water at the earth, would raise the temperature 100° F. and maintain it at, say, 600° F., absolute, it would, under similar conditions, raise the temperature of one pound of the ether, if its power of absorption be the same as that of water, $\frac{1}{1600000000}$ of a degree.

The distance of the earth from the sun being 210 times the radius of the latter, the amount of heat passing a square foot of spherical surface at the sun will be about 45000 times the heat received on a square foot at the earth normally exposed to its rays, so that, under the conditions imposed, the temperature would not be a billionth of a degree F. higher at the sun than at the earth. This, then, is a condition favorable to a sensibly uniform temperature, even if heated by the sun's rays. We are now inclined to admit that the ether is not perfectly diathermanous to

the sun's rays, but that its temperature, however small, may be due directly to the absorption of the heat of central suns; for we begin to realize the fact that the ether may possess many of the qualities of gases, such as a molecular constitution, and hence also mass, elasticity, specific heat, compressibility, and expansibility, although the magnitude of these properties is anomalous. We have already considered its compressibility at the surface of the sun, due to the weight of an infinite column, and found it to be exceedingly small; now, it may be possible that the expansion due to the excess of temperature of a small fraction of one degree at the surface of the sun over that at remote distances will diminish the density as much, or about as much, as pressure increased it, thereby making the density even more exactly uniform than it otherwise would be. According to what we know of refraction, it is impossible for a ray of light to be refracted in passing through the ether only—at least, not by a measurable amount; for not only are the density and elasticity practically uniform, but their ratio is, if possible, even more constant as shown by equations (16) and (16′). But the freedom of the ether molecules may be constrained, or their velocity impeded, by their entanglement with gross matter, such as the gases and transparent solids; in which case refraction may be produced in a ray passing obliquely through strata of varying densities.* Neither is it believed that the ether does, or can, reflect light; for if it did, the entire sky would be more nearly luminous. The rays in free space move in right lines.

The masses of the molecules in different gases being inversely as their specific heats, and as the specific heat of hydrogen is 3.4, and the computed mass of one of its molecules $\frac{11}{18} \times 10^{-29}$ † of a pound, we have for

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* Professor Michaelson concludes from his experiments that the luminiferous ether has no perceptible motion in reference to the earth, in other words, it is at the surface of the earth carried along with the earth the same as the atmosphere. (Paper read at the meeting of the American Association for the Advancement of Science, 1887.)

† Stoney concludes that "it is therefore probable that there are not fewer than something like a unit eighteen (10^{18}) of molecules in a cubic millimeter of a gas at ordinary temperature and pressure" (*Phil. Mag.*, 1868 [4] XXXVI., p. 141). According to the Kinetic theory, the number of molecules in a given volume under the same pressure and temperature is the same for all gases. The weight of a cubic foot of hydrogen at the temperature of melting ice and under constant pressure being 0.005592 of a pound, and as a cubic foot equals 28,315,000 cubic millimeters, the probable mass of a molecule of hydrogen will be

$$\frac{0.005592}{32.2 \times 28315000 \times 10^{25}} = \frac{11}{18 \times 10^{29}} \text{ lb.}$$

Maxwell gives $\frac{46}{10^{25}}$ of a gramme $= \frac{3}{7 \times 10^{28}}$ lb., which is about 3/5 the value given above (*Phil. Mag.*, 1873 [4], XLVI., p. 468).

The difference in these results arises chiefly from the calculated number of molecules in a cubic foot of gas under ordinary conditions. Thomson gives as the approximate

the computed mass of a molecule of the luminiferous ether,

$$m = \frac{11}{18 \times 10^{29}} \times \frac{3.4}{46 \times 10^{11}} = \frac{1}{22 \times 10^{40}} \text{ lb.} \tag{23}$$

The mass of a cubic foot of the ether, equation (9), divided by the mass of a molecule, gives the number of molecules in a cubic foot, which will be

$$n = \frac{2}{35 \times 10^{24}} \times \frac{22 \times 10^{40}}{1} = \tfrac{44}{35} \times 10^{16}, \tag{24}$$

which call 10^{16}. This number, though large, is greatly exceeded by the estimated number of molecules in a cubic foot of air under standard conditions, which, according to Thomson, does not exceed 17×10^{25}, a number nearly 17,000,000,000 times as large as that in equation (24); and yet, at moderate heights, the number of molecules in a given volume of air will be less than that of the ether.

Assuming that air is compressed according to Boyle's law, and is subjected to the attraction of the earth, equation (15) will give the law of the decrease of the density. Taking the density of air at sea-level at $\frac{1}{400}$ of a pound per cubic foot, $e_0 = 14.7$ lbs. per square inch, $r = 20687000$ feet, equation (15) becomes

$$\delta = \tfrac{1}{400} \times 10^{-345 \frac{z}{r+z}}. \tag{25}$$

If $z = \infty$, $\delta = \frac{1}{400} \times 10^{-345}$, which would be the limit of the density, and it is a novel coincidence that this limit is nearly identical with the value found for the density at the height of one radius of the earth according to the ordinary exponential law, wherein gravity is considered uniform.*

If the number of the molecules in a cubic foot follows the same law, then at the height z there will be

$$17 \times 10^{-345 \frac{z}{r+z} + 25} \tag{26}$$

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probable number 17×10^{25}, which is about 3/5 the value given by Stoney. Thomson's value would make the mass of a molecule of ether about $\frac{1}{13} \times 10^{-40}$ of a pound, which is not much different from that found above.

* The ordinary exponential law results from dropping $\frac{z}{r}$ compared with unity in equation (15), giving

$$\delta = \delta_0 \, \varepsilon^{-\frac{z}{26221}} = \delta_0 \, 10^{-\frac{z \text{ ft.}}{60387}} = \tfrac{1}{400} \times 10^{-\frac{z \text{ miles}}{11.44}},$$

in the last of which, if $z = 3956$, the exponent becomes 345.

molecules per cubic foot. Similarly, the value of the length of the mean free path would be *

$$2 \times 10^{345 \frac{z}{r+z} - 6} \text{ inches.} \qquad (27)$$

By means of these values, the following table may be formed.

Height.		Density or tension, that at the earth being unity.	Number of molecules in a cubic foot.	Length of the mean free path.
Fractional parts of earth's radius.	Approximate in miles.			
0	0	1	17×10^{25}	2×10^{-6} inch.
$\frac{1}{79}$	50	$10^{-4.3}$	$17 \times 10^{20.7}$	$2 \times 10^{-1.7}$ "
$\frac{1}{39}$	100	$10^{-8.4}$	$17 \times 10^{16.6}$	$2 \times 10^{2.4}$ "
$\frac{1}{20}$	200	$10^{-16.4}$	$17 \times 10^{8.6}$	792000 miles.
$\frac{1}{14}$	282	10^{-23}	17×10^{2}	31×10^{11} "
$\frac{1}{10}$	395	10^{-31}	17×10^{-6}	31×10^{19} "
$\frac{1}{5}$	800	10^{-57}	17×10^{-32}	31×10^{45} "
1	3956	10^{-172}	17×10^{-147}	31×10^{160} "
2	7912	10^{-230}	17×10^{-205}	31×1^{218} "
∞	∞	10^{-345}	17×10^{-320}	31×10^{333} "

The numbers in the third column multiplied by $\frac{1}{4000}$ will give the density (or mass per cubic foot) at the respective altitudes ; and the same numbers multiplied by 15 (or, more accurately, 14.7) will give the tension per square inch. According to this law, at an elevation of 300 miles the density of the atmosphere will be somewhat less than the density of the ether as given by equation (9).

To find the height at which the tension of the atmosphere, according to the above law, will be the same as that of the ether, we have, by means of equations (11) and (25), substituting in the latter 2116 for $\frac{1}{4000}$,

$$2116 \times 10^{-345 \frac{z}{r+z}} = \frac{4}{10^8},$$

which solved gives

$$z = \frac{r}{31.24} = 126.6 \text{ miles.}$$

* *Phil. Mag.*, 1873 [4], XLVI., p. 468.

so that at the height of 127 miles the tension would be less than that of the ether, the temperature being uniform.

The mean free path, according to the above law, in which gravity varies as the inverse squares is less, and for great heights much less, than would be found according to the ordinary exponential law. Thus Crookes states that the mean free path of a molecule at the height of 200 miles is about 10000000 miles ; * but according to the above law it becomes about 792000 miles.

If a cubic inch of air at sea-level were carried to the height of $\frac{1}{7}$ the radius of the earth, and then allowed to expand freely, so as to become of the computed density of the atmosphere at that point, it would fill a space of $4 \times 10^{28 \cdot 12}$ cubic miles, or a sphere whose radius is 2,398,000,000 miles, which is nearly equal to the distance of the planet Neptune from the sun ; and there would be less than one molecule to the mile. Such are some of the results of extending a law to extreme cases regardless of physical limitations or of the imperfection of the data on which it is founded. For instance, a uniform temperature is assumed, and, impliedly, an unlimited divisibility of the molecules. The latter is necessary in order to maintain a law of continuity. But modern investigations show that not only air, but all the gases, are composed of molecules of definite magnitudes whose dimensions can be approximately determined ; and hence if there be only a few molecules in a cubic foot, and much less if there be but one molecule in a cubic mile, it cannot be claimed that the gas will be governed by the same laws as at the surface of the earth.

We conclude, then, that a medium whose density is such that a volume of it equal to about twenty volumes of the earth would weigh one pound, and whose tension is such that the pressure on a square mile would be about one pound, and whose specific heat is such that it would require as much heat to raise the temperature of one pound of it 1° F. as it would to raise about 2,300,000,000 tons of water the same amount, will satisfy the requirements of nature in being able to transmit a wave of light or heat 186300 miles per second, and transmit 133 foot-pounds of heat-energy from the sun to the earth, each second per square foot of surface normally exposed, and also be everywhere practically non-resisting and sensibly uniform in temperature, density and elasticity. This medium we call the *Luminiferous Ether.*

ADDENDA.

Granting that the temperature of the ether, however low, is produced by the heat from central suns passing through it, we may determine the effect upon it of a change of temperature of the source of heat.

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* *Phil. Trans. Roy. Soc.*, London, 1881, Part II., p. 389.

The law for perfect gases is—continuing our notation—

$$e\,v = R\,\tau \tag{36}$$

where R is $e_0\,v_0 \div \tau_0$, these values being initial. Since v will necessarily be constant we see that e will vary as τ, where τ is the temperature of the ether, and equation (21) becomes

$$\delta = \frac{u}{2\,g\,J\,c} \cdot \frac{e}{\tau} = \text{constant},$$

as it should, since the mean density cannot change, the volume being constant. This equation reveals no new truth, but is consistent with the conditions which we anticipate in nature. The only way in which the density can change by a diminution of elasticity of the ether, is to cause it to be more dense near the attractive bodies, and more rare in space more remote from them ; or, in other words, the ether would not be so nearly uniform as at present.

Assuming the density as uniform while the elasticity changes, it appears from equation (2) that the velocity of light through it will vary as the square root of the elasticity. Thus, if the heat of our sun diminishes so as to become one fourth as intense as at present, and if the elasticity of the ether also becomes one fourth as much as at present, then will the velocity of light be one half as great as at present.

We may find the conditions which would cause a gas of the pressure of our atmosphere at sea-level and of the same specific heat, to be as nearly uniform throughout space as is the ether. This will be found with sufficient accuracy for our purpose by finding such a value for δ as will make the numerator in equation (15), $\frac{1}{1000000}$, the same as given in (16), where $e_0 = 2116$ the tension of the air per square foot. We will find

$$\delta_0 = \frac{1}{10^{15}}.$$

The volumes being inversely as the densities, the last result combined with equation (36) shows that the required rarity (or density) may be secured by a temperature 10^{15} times that of the present temperature. If the absolute temperature be 500° when the pressure of the air per square foot is 2000 pounds, then if it be heated to something like

$$500{,}000{,}000{,}000{,}000°\ \text{F.},$$

the tension would be nearly uniform throughout space. A volume of such air of the size of the earth would weigh less than $\frac{1}{50}$ of a pound at a place where $g = 32.2$.

APPENDIX II.

SECOND LAW OF THERMODYNAMICS.

The second law of thermodynamics has, by different writers, been stated in a variety of ways, and, apparently, with ideas so diverse as not to cover a common principle. For the convenience of the student in considering this subject, we here quote some of the expressions which have been given by certain authors.

Maxwell, in his *Theory of Heat*, p. 153, says, "Admitting heat to be a form of energy, the second law asserts that it is impossible, by the unaided action of natural processes, to transform any part of the heat of a body into mechanical work, except by allowing heat to pass from that body into another at a lower temperature. Clausius, who first stated the principle of Carnot in a manner consistent with the true theory of heat, expresses this law as follows :

"'*It is impossible for a self-acting machine, unaided by any external agency, to convert heat from one body to another at a higher temperature.*'

"Thomson gives it a slightly different form :

"'*It is impossible, by means of inanimate material agency, to derive mechanical effect from any portion of matter by cooling it below the temperature of the coldest of surrounding objects.*'" The last quotation may be found in *Phil. Mag.*, 1852, IV. ; Thomson's *Mathematical and Physical Papers*, p. 179 ; and Clausius's statement on p. 181.

Clausius considers this principle as "a new fundamental principle," and states it thus : "Heat cannot pass from a colder to a hotter body without compensation." (*Mechanical Theory of Heat*, Browne's translation, p. 78.)

It appears, so far as we can judge, that Maxwell has, gratuitously, claimed for these writers the above statement for the second law ; for not only they, but Rankine included, consider those statements as axioms. In regard to Rankine's views, see *Miscellaneous Scientific Papers*, p. 449 ; *Steam-Engine*, p. 224.

There would be a certain propriety in calling this the second law, and if necessary establish a third, for it is the first principle in the order of development involved in the physical operation of realizing Carnot's cycle, in which the expansion being isothermal requires a supply of heat from

a source, and experience shows that the temperature of the source must at least equal that of the working substance, and in reality be infinitesimally higher, since heat from a colder body will not make a hot body hotter. But the question is not what might have been the second law, but—what is it? We quote from Rankine:

"The internal work is incapable of direct measurement. Here it is that the second law becomes useful; for it informs us how to deduce the whole amount of work done—internal and external—from the knowledge which we have of the external work. That law is capable of being stated in a variety of forms, expressed in different ways, although virtually equivalent to each other. The most convenient form for the present purpose appears to be the following:

To find the whole work, internal and external, multiply the absolute temperature at which the change of dimensions takes place by the rate per degree at which the external work is varied by a small variation of temperature." (Rankine's *Miscellaneous Scientific Papers*, p. 434; *The Engineer*, June 28, 1867.)

This is substantially the statement of the second law in first ed., p. 33, since the italicized extract just given is an expression for the heat absorbed during an isothermal expansion. The form in the text was not given because it was considered the ideally best statement of this law, but because it had proved to be the most useful form for class-room instruction which the author had tried, and had the above sanction of Rankine.

Rankine gives substantially the same statement in different places. (*Papers*, pp. 309, 418, 427; *Steam-Engine*, p. 308, Art. 244; p. 309, Art. 245.)

That Rankine recognized Carnot's principle of the elementary reversible engine as the second law is shown from the following extract:

"*The law of efficiency of a perfect heat engine may be stated thus: If the substance (for example, air or water) which does the work in a perfect heat engine receives all the heat expended at one fixed temperature, and gives out all the heat which remains unconverted into work at a lower fixed temperature, the fraction of the whole heat expended which is converted into external work is expressed by dividing the difference between those temperatures by the higher of them, reckoned from the absolute zero. Now, this is, in fact, the second law of thermodynamics expressed in other words.*" (*Miscellaneous Sc. Papers*, p. 436; *The Engineer*, June, 1867.) Such being Rankine's explicit statement, we may expect to find this principle implied, if not expressed, in all his other statements.

One of the most condensed and obscure statements of this law by this author is in his work on the *Steam-Engine*, p. 306, which is,

"*If the total actual heat of a homogeneous and uniformly hot substance be conceived to be divided into any number of equal parts, the effects of those parts in causing work to be performed are equal.*"

The obscurity exists chiefly in the fact that several principles involved in the practical application of the law are not stated in the immediate context, but are left for the reader to infer. It would be difficult, if not impossible, to apply it, had not the author given a symbolic representation of it. He says: "Let unity of weight of a homogeneous substance, possessing the actual heat Q, undergo any indefinitely small change, so as to perform the indefinitely small work $d\ U$. It is required to find how much of this work is performed by the disappearance of heat. Conceive Q to be divided into an indefinite number of indefinitely small parts, each of which is $\delta\ Q$. (In his original paper he used $d\ Q$.) Each of these parts will cause to be performed the quantity of work represented by

$$\delta Q \cdot \frac{d\,(d\ U)}{d\ Q},$$

consequently, the work performed by the disappearance of heat will be

$$Q\ \frac{d\,(d\ U)}{d\ Q}."$$

The reduction from the former expression to the latter is equivalent to an integration considering the fractional part as constant during the integration. This is vital, and it is accomplished, physically, by connecting the working substance with a *source* possessing constantly the actual heat Q. Possibly this is implied in the expression "*uniformly hot substance*" in the law stated above; but if not, the law seems to be defective in this particular, unless we resort to the only other alternative of considering the "homogeneous substance" as a perfect gas. Heat is absorbed in doing work, and it is this heat, as heat, independent of any particular substance, that is to be divided into equal parts; and, having this conception, it is apparent that each part of the heat will do the same amount of work. It is difficult to determine the exact meaning of the expression—"It is required to find how much of this work is done by the disappearance of heat;" for, with isothermal expansion, not only all the external work is done by the disappearance of heat, but the internal work also, according to which the words "of this" should be expunged. Another view, and probably the correct one, is,—it is required to find how much of this work is done by the disappearance of an equal amount of heat. If this be the intended meaning, then by referring to Fig. 13, p. 33, it will be seen that the area represented by the upper strip $A\ B\ c\ d$ is common both to that part of the external work $v_1\ A\ B\ v_2$, and to the area $\phi_1\ A\ B\ \phi_2$, representing the actual heat absorbed; and, hence, in per-

forming this element of external work, an equal amount of heat will disappear. $A\,B\,c\,d = d\,U = \int d\,p\,d\,v$; and $d\,(d\,U) = d\,p\,d\,v = A\,b\,g\,d$. But $d\,p$ is a direct function of τ, being limited by the consecutive isothermals $A\,B$ and $d\,c$; and this fact is, by the calculus, indicated in the following manner,

$$\left(\frac{d\,p}{d\,\tau}\right) d\,\tau\,;$$

and we have

$$\therefore A\,b\,g\,d = d\,\tau\,\left(\frac{d\,p}{d\,\tau}\right)\,d\,v,$$

or,

$$= d\,Q\,\frac{d\,(d\,U)}{d\,Q}.$$

Then the second law above declares that if the increment of heat $d\,Q$ causes the work $A\,b\,g\,d$ to be performed by the disappearance of an equal amount of heat, then will the total heat absorbed, $\varphi_1\,A\,b\,n$, be $\frac{Q}{d\,Q}$ times as much, giving

$$Q\,\frac{d\,(d\,U)}{d\,Q}.$$

The "actual heat is divided into equal parts" by the successive isothermals of the substance.

The explanation by Rankine of this operation is more satisfactory in his original paper than in his *Steam-Engine*. (*Misc. Sc. Papers*, p. 312.)

His general law of the transformation of energy—"*The effect of the presence in a substance of a quantity of actual energy, in causing transformation of energy, is the sum of the effects of all its parts*" (*Steam Engine*, p. 309)—implies that the office of the working substance is simply to transfer actual energy from a source to a receiver of a potential form, as when the heat of a furnace is transferred to work; under which conditions the actual energy of the working substance must be maintained constant.

Sir William Thomson thus stated the second law:

"Prop. II. (Carnot and Clausius).—*If an engine be such that, when it is worked backward, the physical and mechanical agencies in every part of its motions are all reversed, it produces as much mechanical effect as can be produced by any thermodynamic engine, with the same source and refrigerator, from a given quantity of heat.*" (Thomson's *Papers*, p. 178.)

Credit is here given to Clausius for a part, at least, of the fundamental principle involved, and hence it is unnecessary to consider his views.

We find, then, that three of the principal founders of the science of

thermodynamics—Clausius, Rankine, Thomson—give as the SECOND LAW *the principle of Carnot's ideal elementary reversible engine.*

Unless the axioms of these writers, which are by Maxwell stated as the second law, be considered as including the reversible engine, it appears to be improper to consider them as the second law. These writers stated them as axioms, and not as the second law.

The two laws of thermodynamics are the result of experience, guided by scientific investigation. Rankine says: "The laws of thermodynamics, as here stated, are simply the condensed expression of the facts of experiment." (*Misc. Sc. Papers*, p. 427; *Phil. Mag.*, Oct., 1865.)

The statement of the second law referred to in this quotation is simply the expression $\tau \frac{d\,p}{d\,\tau}\, d\,v$ written out in words; and hence is equivalent to one of the preceding quotations, and also to the one on page 33 of the text.

Another statement:

The first law asserts a fixed, unvarying relation between heat energy and the mechanical energy into which it is transmuted; but in a working engine all the heat absorbed cannot be transmuted into work, and the second law asserts that a certain fractional part of the heat absorbed may be transmuted into mechanical energy when the substance is worked in Carnot's cycle.

ADDENDA.

[The articles in this Addenda are numbered the same as those in the body of the work to which the subject-matter pertains.]

14. 16. Since air is not a perfect gas, the divisions on an air thermometer will not be equal for equal increments of actual heat absorbed by the air. The relation between p, v, τ, as used by Thomson and Joule, is given in equation (7), page 13. The experiments of Regnault enable one to determine the constants. The results of these experiments, generalized, enabled Sir William Thomson to construct the following table, in which the degrees are for the centigrade scale from 0 to 300, and d is the relative density of the air in the thermometer. If at 0° C. it be under the pressure of one atmosphere, 760 mm., then will $d = 1$, and the factors of d will be the fraction of one degree on the centigrade scale by which the readings differ from what they would if air were a perfect gas.

Temperature by absolute scale measured from 0° C. $t = \tau - 273.7$	Temperature centigrade on air thermometer. $\theta = 100 \frac{p_\tau - p_{273\cdot7}}{p_{373\cdot7} - p_{273\cdot7}}$	Temperature by absolute scale measured from 0° C. $t = \tau - 273.7$	Temperature centigrade on air thermometer. $\theta = 100 \frac{p_\tau - p_{273\cdot7}}{p_{373\cdot7} - p_{273\cdot7}}$
t	°	t	°
0	0	160	$160 - 0.0970 \times d$
20	$20 + 0.0294 \times d$	180	$180 - 0.1363$ "
40	$40 + 0.0398$ "	200	$200 - 0.1772$ "
60	$60 + 0.0361$ "	220	$220 - 0.2202$ "
80	$80 + 0.0220$ "	240	$240 - 0.2627$ "
100	100	260	$260 - 0.3099$ "
120	$120 - 0\ 0280$ "	280	$280 - 0.3562$ "
140	$140 - 0.0607$ "	300	$300 - 0.4030$ "

(*Thomson's Papers, p.* 100.)

22. Numerous equations have been proposed to represent the results of experiments upon gaseous substances. Rankine's equation, (4), p. 13, is the most general, and is sufficiently accurate for all substances used in engineering practice. It seems a useless labor to construct equations that will represent with extreme accuracy the experiments made by any person, for the results of different experimenters will differ, and a formula that will agree nearly with one set will not agree with others. If the experiments are reliable, like those of Regnault, the formula, when plotted, should exhibit the *law* indicated by the experiments, and give approximately the values found by experiment. The formulæ pertaining to steam will be given in Article 78. The following are equations for *carbonic acid gas.*

Rankine—also Thomson and Joule—gave an equation of the form

$$p\,v = R\,\tau - \frac{b}{\tau\,v}.$$

(*Phil. Trans.*, 1854, p. 336 ; 1862, p. 579.)

Hirn gave

$$(p + r)\,(v - x) = R\,\tau;$$

where $x =$ " la somme de volumes des atomes ;"

$r =$ " la somme des action internes."

(*Théorie Mécanique de la Chaleur*, 2e éd., i., p. 195 ; 3e éd., ii., p. 211.)

Racknel, in 1871 and 1872, gave the formula

$$p\,v = R\,\tau\left(1 - \frac{B_1}{v}\right) = R\;\tau\left(1 - \frac{a\,\tau}{p}\right)$$

where a is a constant to be determined by experiment.

J. D. Van der Waals gave

$$p = R\,\frac{\tau}{v - b} - \frac{a}{v^2},$$

in which if the unit of pressure is one atmosphere, and the unit of volume that which a kilo. of carbonic acid occupies under the pressure of one atmosphere at the melting point of ice, then

$$R = 0.003673,$$
$$a = 0.00874,$$
$$b = 0.0023.$$

Over de Continuiteit van den Gas en Vloeistoestand, Leinden, 1873, p. 76, Op. cit., p. 76.

Clausius, in 1880, gave

$$p = R\,\frac{\tau}{v - \alpha} - \frac{c}{\tau\,(v + \beta)^2},$$

in which if the pressure be in kilogrammes per square metre, and volume in cubic metres, we have per kilogramme of carbonic acid.

$$R = 19.273,$$
$$c = 5533,$$
$$\alpha = 0.000426,$$
$$\beta = 0.000494.$$

This formula gave results agreeing remarkably well with those of observation. (*Phil. Mag.*, 1880, (1), 401.)

23, 24, 25. Thermal lines. The more common thermal lines are defined in the body of the book; but the following are sometimes used:

Isopiestic, or *Isobar* lines are lines of equal pressure, and, therefore, on the plane $p\ v$, are parallel to the axis of v.

Isometric lines are lines of equal volume, and their projections on the plane $p\ v$ are parallel to the axis of p.

Isengeric, or *Isodynamic* lines are lines of equal energy. In this case the internal energy remains constant, and all the heat absorbed during the change of state is transmuted into external work. See top of page 129. If the gas be perfect, the *isengeric* coincides with an *isothermal*.

Isentropic lines are lines of equal entropy, and hence coincide with *adiabatics*.

40. Page 33. In order that the algebraic expressions may be serviceable in numerical problems, the volume $O\ v_1$, Fig. 13, must represent a definite *mass* of the working substance; and we assume a *unit-mass;* and in English measures let it be *one pound*. Clausius, Zeuner, and others, in some cases, include the internal work in the expression—*internal energy;* but we prefer to apply the term *work* to all that part of the heat absorbed which is destroyed—put out of existence for the time being—transmuted into another form of energy; and if any part remains, call it a *change of internal energy*.

The second law is sometimes called, briefly, *the reversible engine;* or, more fully, *an expression of the facts involved in the simple reversible engine*. Isothermal expan-

sion and compression are the first fundamental principles of this law; the second being the axiom of Thomson that—no engine can be worked with mechanical profit at a lower temperature than that of the coldest of surrounding objects; and since absolute cold cannot be produced in surrounding objects, it follows that only a fractional part of the heat absorbed can be transmuted into external work.

It is worthy of remark that during isothermal expansion the heat of the working fluid does no work; it is merely an agent for transmuting actual heat energy into work. If the working fluid be a perfect gas, it will transfer the heat directly from the source to the piston of the engine. If the working fluid be an imperfect gas, a part of the heat from the source will be transferred from the source to the piston of the engine and transmuted into external work, and the remaining part will be transmuted into internal work, being the work necessary to overcome the resistance of the particles in being separated during expansion. The actual, or kinetic, energy of the working fluid remains constant during isothermal expansion.

Page 34. Sir William Thomson has proposed two scales of absolute temperatures. In the *first scale* it was proposed to consider *the difference of the temperatures of the source and refrigerator as constant when the work done by a perfect engine on abstracting a unit of heat from the source is constant, whatever be the temperature of the heat absorbed.*

To get an idea of this principle, observe that in Fig. *a*, the successive divisions represent equal works done in successive elementary engines; but the heat absorbed, $\varphi_1 A B \varphi_2$, in doing the work $A B c d$, is more than $\varphi_1 d c \varphi_2$ in doing the equal work $d c i j$. But the preceding principle requires that the heat absorbed along $y z$ must equal that along $A B$, while the elementary works done in the cycles must be the same: therefore, to represent this case $y z$ must

be prolonged so that the area under it and between the two adiabatics through its extremities shall equal $\varphi_1\ A\ B\ \varphi_2$, and the elementary strip under $y\ z$ must be so narrow that its area shall equal $A\ B\ c\ d$.

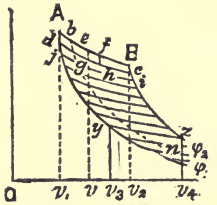

FIG. *a*.

To find the symbolic expression, let

$H = \varphi_1\ A\ B\ \varphi_2$ = the heat absorbed, in foot-pounds,

$d\ H = A\ B\ c\ d$ = the heat transmuted into work,

$d\ t$ = the difference of absolute temperatures between the source and refrigerator,

$\frac{1}{\mu}$ = the fractional part of the heat absorbed that is transmuted into work per *unit of temperature*;

then

$$\textit{the work} = \mu\ H\ d\ t = d\ H\ ;\ \therefore\ \frac{d\ H}{H\ d\ \tau} = \mu.$$

But according to the principle above stated, μ is not only independent of the temperature, but the ratio of the left member is to be constant;* hence μ is constant, and we have by integrating between the limits H_1 and H_2 for heats, and t_1 and t_2 for temperatures,

$$\frac{H_1}{H_2} = e^{\mu\ (t_1 - t_2)}\ ;$$

—that is, if the differences on the scale of the thermometer

---

* Or integrating, we have:

$$\textit{Work} = H_1 - H_2 = H_1\ (1 - e^{-\int_{t_2}^{t_1} \mu\ d\ t}).$$

But, in this scale, the work is a function of the difference of temperatures, whatever be the temperature of H_1, and this condition requires that μ should be constant. This relation is clearly shown in equation (*a*).

increase arithmetically, the *ratio of heats* of the source and refrigerator will increase geometrically.

The efficiency would be

$$\frac{H_1 - H_2}{H_1} = 1 - e^{-\mu (t_1 - t_2)};$$

and the work done

$$H_1 - H_2 = H_1 (1 - e^{-\mu (t_1 - t_2)}); \qquad (a)$$

or, the work will be the same for each degree on the scale for the same amount of heat absorbed, regardless of its temperature.

Let this scale have 180 divisions between the melting point of ice and the boiling point of water; then, since the efficiency of the perfect elementary engine worked between these temperatures is

$$\frac{H_1 - H_2}{H_1} = \frac{180}{460.66 + 212} = \frac{180}{672.66} = 0.2684 +;$$

$$\therefore 1 - e^{-180\mu} = \frac{180}{672.66};$$

$$\therefore \mu = 0.00173.$$

According to the absolute scale in ordinary use,

$$\frac{H_1 - H_2}{H_1} = \frac{\tau_1 - \tau_2}{\tau_1} = \frac{T_1 - T_2}{T_1 + 460.66},$$

the last fraction being applicable to the Fahrenheit scale. Let this scale and the former absolute scale above considered both be numbered 212 at the boiling point of water, then

$$T_1 = 212 = t_1,$$

and dropping the subscript 2 we have for the relation between t and T,

$$\frac{212 - T}{672.66} = 1 - \frac{1}{e^{\mu (212 - t)}},$$

from which we find

$$t = 212 - \frac{2.30258\ log_{10}\ \frac{672.66}{460.66 + T}}{\mu = 0.00173};$$

by means of which, if any degree T be given on the Fahrenheit scale, the corresponding number t may be found on Thomson's first absolute scale. If $T = -460.66$, $t = -\infty$; that is, *the absolute zero of this scale is minus infinity.* The higher the temperature T, more and more degrees will be required to make a degree on the absolute scale.

This scale is not practical. It was devised while Thomson held to the old theory of heat (or *caloric*), which maintained that heat was material, and hence could not be converted into work. He said: "The conversion of heat (or *caloric*) into mechanical effect is probably impossible, certainly undiscovered." (*Phil. Mag.*, 1848, (2), 315.) Work was then supposed to be derived from heat by *letting it down* from a hot body to a cold one without diminishing the quantity of heat; just as work is obtained from water by *letting it down* from one level to a lower one while passing through a motor without diminishing the quantity of water.

(*Phil. Mag.*, 1848, (2), 313; *Trans. R. S. E.*, XX. (1851), 273; *Phil. Mag.*, 1852, (2), 106; *Thomson's Papers*, *Vol. I.*, p. 139.)

Making $\mu = \frac{1}{\tau}$ gives, by integration,

$$\frac{H_1 - H_2}{H_1} = \frac{U}{H_1} = \frac{\tau_1 - \tau_2}{\tau_1},$$

or Thomson's *second scale.* (*Trans. R. S. E.*, 1854, p. 125; *Phil. Mag.*, 1856, (1), 216.) A comparison of this scale with the air thermometer is given in Article 14 of this Addenda.

Page 38. In the third line from the top it will be observed that the argument depends upon an inference, "It is *inferred*," &c. No *absolute* proof of equation (21) has been made. Rankine deduced it in a short way, founding it upon the hypothesis that—*in a given mass of a substance the quantities of sensible heat are proportional to their absolute temperatures.* (*Misc. Sc. Papers*, pp. 50, 56, 376, 377, 409; *Phil. Trans.*, 1854.) Thomson and Joule established it by a long and very delicate series of experiments, determining that $\mu = \frac{1}{\tau}$, establishing this value within $\frac{1}{400}$ of its actual value. Clausius, in his later work, established it by a process of analysis, but it is somewhat obscured by the more general equations in which it is involved. Yet there is no question as to the correctness of this equation; for not only do Thomson and Joule's experiments prove it to be as nearly mathematically exact as it is possible by means of physical experiments, but it produces substantially correct results when applied to various problems in this science; and this is the best test of a physical law. The Newtonian law of universal gravitation is accepted as mathematically exact for all problems to which it is applied involving finite distances, although the law does not admit of an *absolute* proof.

The hypothesis of Rankine, referred to above, was criticised by Clausius, and Rankine modified it thus: "A change of real specific heat, sometimes considerable, often accompanies the change between any two of those conditions" of solid, liquid, or gaseous. This definition was not accepted by Clausius as correct. (Clausius *On Heat*, 1879, pp. 345–348; Rankine's *Prime Movers*, p. 307; *Phil. Mag.*, Ser. 4, Vol. VII., p. 10; *Pogg. Ann.*, Vol. CXX., p. 426; *Phil. Mag.*, Ser. 4, Vol. XXX., p. 410.) If this principle be not rigorously and universally exact, it is, so far as known, correct within the limits of error of observa-

tion for imperfect gases; and these are the substances to which this science is very largely applied.

42. Page 43. During isothermal expansion at a given temperature, τ, a certain amount of external work will be done, as $v_1\ d\ e\ v_2$, Fig. *a*, and a certain amount of internal work which we assume is not yet known. If the same amount of expansion be performed at a temperature $\tau + d\tau$, the external work $v_1\ A\ B\ v_2$ will be done, and a certain amount of internal work, which, as before, may be unknown. One of the brilliant points in Rankine's establishment of equation (21) was his conception that the difference between the external works for two equal isothermal expansions in which the temperatures of the source differed infinitesimally, being τ in one case and $\tau + d\tau$ in the other, equalled the work done by an elementary engine for the same isothermal expansion, the temperature of the source being that of the higher temperature, $\tau + d\tau$, and of refrigerator that of the lower, or τ. In other words, the increment of external work due to an elementary increase of the temperature of the source equalled the increased increment of the heat absorbed. Or the increment $A\ B\ c\ d$ of the heat $\varphi_1\ b\ c\ \varphi_2$ equals the increment $A\ B\ c\ d$ of the external work $v_1\ A\ B\ v_2$. This is equivalent to asserting that the difference of the internal works done during the performance of the external works $v_1\ d\ c\ v_2$ and $v_1\ A\ B\ v_2$ is zero; or, at most, a difference of the second order compared with the difference of external works.

As a verification of this principle for a particular case, let the equation of the gas be $p = R\frac{\tau}{v} - \frac{b}{\tau\, v^2}$; then will the area of one of any one of the strips in $\varphi_1\ A\ B\ \varphi_2$ be

$$d\ c\ i\ j = d\tau \int \left(\frac{dp}{d\tau}\right) dv = \left[R\ log\frac{v_2}{v_1} + \frac{b}{\tau^2}\left(\frac{1}{v_1} - \frac{1}{v_2}\right)\right] d\tau,$$

as already given in Exercise 2, page 44. The external work for isothermal expansion will be

$$v_1 A B v_2 = \int p\, d v = R \tau \log \frac{v_2}{v_1} - \frac{b}{\tau}\left(\frac{1}{v_1} - \frac{1}{v_2}\right),$$

as in Exercise 4, page 45. The differential of the latter in regard to τ gives

$$A B c d = \left[R \log \frac{v_2}{v_1} + \frac{b}{\tau^2}\left(\frac{1}{v_1} - \frac{1}{v_2}\right)\right] d \tau,$$

which is the same as found above for $d c i j$.

43. In the Exercises on page 44, it must be understood that they refer to a *unit-mass*, and when numbers are given, they refer to one pound of the gas.

48. By consulting the records of Regnault's experiments, one becomes impressed with the large amount of work done by him and the extreme accuracy with which his experiments were made.

51. By a purely analytical relation between the physical properties of a perfect gas, it has been shown that

$$\gamma = 1 + \frac{4}{\pi^2} = 1.405285.$$

(*Phil. Mag.*, 1885, (1), 520.)

58. Page 63. Air has been compressed to 500 pounds per square inch for use in the Vincennes-Villa tramway. (*La Nature Sc. Am. Sup.*, 1888, Mar. 17, p. 10167.) Air has been compressed to 1000 pounds per square inch for use in dynamite guns.

58. Exercise 15, page 71.

$$H = a\, v^n.$$

Equations (B) give

$$v\, d p + \gamma\, p\, d v = a\, n\, (\gamma - 1)\, v^{n-1}\, d v.$$

Let

$$v = x, \quad p = y, \quad P = \frac{\gamma}{x}, \quad Q = a\, n\, (\gamma - 1)\, x^{n-2} = B\, x^{n-2}$$

and the equation becomes

$$\frac{dy}{dx} + Py = Q,$$

where P and Q are functions of x. This is a differential equation of the first order and first degree.

To find the integral, first let $Q = 0$, then

$$y = e^{-\int P\,dx} C_x,$$

where C_x is a function of x instead of a *constant* of integration. Differentiating,

$$\frac{dy}{dx} = -PC_x e^{-\int P\,dx} + e^{-\int P\,dx}\frac{dC_x}{dx} = -Py + e^{-\int P\,dx}\frac{dC_x}{dx}$$

$$\therefore Q\,dx = e^{-\int P\,dx}\,.\,dC_x;\quad \therefore C_x = \int Q e^{\int P\,dx}\,dx;$$

$$\therefore y = e^{-\int P\,dx}\int Q e^{\int P\,dx}\,dx;$$

$$\therefore p = \frac{a\,n\,(\gamma-1)}{n-1+\gamma} v^{n-1} + C v^{-\gamma}$$

Exercise 16 may also be reduced to the linear form, giving

$$v = \frac{a\,n\,(\gamma-1)}{\gamma\,(n-1)+1} p^{n-1} + C p^{-\frac{1}{\gamma}}$$

Exercise 17 gives the differential equation

$$\frac{dp}{dv} = \frac{n\,a\,(\gamma-1)\,v^{n-1} - \gamma p}{n\,b\,(1-\gamma)\,p^{n-1} - v}.$$

71. Page 89. The melting point—or freezing point—of

liquid carbon disulphide — 116° C.
" absolute alcohol — 130.5° C.
Alcohol becomes viscid at — 129° C.
(*Phil. Mag.*, 1884, (1), 490.)

72. Page 90. By experiment it has been found that the melting point of ice is raised 0.0066° C. by a reduction of pressure from 760 mm. to 5 mm. (*Phil. Mag.*, 1887, (2), 295.)

74. Of liquids and saturated vapor. Regnault found *the latent heat of evaporation* by determining the *total heat* and *the heat of the liquids* independently, and taking their difference.

Total heat of liquids, being the *number* of thermal units necessary to raise the temperature of a unit-mass from that of melting ice to t degrees centigrade, as determined by Regnault, at atmospheric pressure.

Substance.	Number of Thermal Units.	
Water	$q = t + 0.00002\ t^2 + 0.0000003\ t^3.$	(1)
Alcohol	$q = 0.54754\ t + 0.001122\ t^2 + 0.000002\ t^3.$	
Ether	$q = 0.52901\ t + 0.0002959\ t^2.$	
Chloroform	$q = 0.23235\ t + 0.0000507\ t^2.$	
Chloride of carbon	$q = 0.19788\ t + 0.0000906\ t^2.$	
Acetic acid	$q = 0.506403\ t + 0.000397\ t^2.$	
Bisulphide of carbon	$q = 0.23523\ t + 0.000082\ t^2.$	

The general law of these equations may be represented by the empirical equation

$$q = a_1\ t + b_1\ t^2 + c_1\ t^3. \qquad (2)$$

To reduce these to English units, we observe that q, the *number* of thermal units, is independent of the unit of mass, or weight, since the ratio between the quantities of heat, in this case, and the respective quantities of liquid experimented upon will be constant, but will be dependent upon the thermometric scale.

Since the degree in the British thermal unit is $\frac{5}{9}$ that in the French, the *number* of British thermal units will be $\frac{9}{5}$ times the number of French units for the same temperature; and if T be the temperature on the Fahrenheit scale we have

$$t = \tfrac{5}{9}\,(T - 32);$$

hence, for water we would have, q denoting the *number* of B. T. U.,

$$q = \tfrac{9}{5}\,q = \tfrac{9}{5}\,[\tfrac{5}{9}\,(T-32) + 0.00002\ [\tfrac{5}{9}\,(T-32)]^2 + 0.0000003\ [\tfrac{5}{9}\,(T-32)]^3];$$

and similarly for the other liquids. Substituting and reducing, we have

Substance.	No. of B. T. U. at temp. T.
Water, $q =$	$-31.991656 + 0.99957333\ T + 0.000002222\ T^2 + 0.0000000926\ T^3$.
Alcohol, $q =$	$-16.903214 + 0.509543\ T + 0.00056407\ T^2 + 0.000000617284\ T^3$.
Ether, $q =$	$-16.759986 + 0.518489\ T + 0.00016439\ T^2$.
Chloroform, $q =$	$-7.406358 + 0.230547\ T + 0.00002817\ T^2$.
Chloride of carbon, $q =$	$-6.280619 + 0.194659\ T + 0.00005033\ T^2$.
Acetic acid, $q =$	$-15.979005 + 0\ 492287\ T + 0.00022055\ T^2$.
Bisulphide of carbon, $q =$	$-7.480711 + 0.232314\ T + 0.00004555\ T^2$.

(3)

The general law will be

$$q = a_2 + b_2\ T + c_2\ T^2 + d_2\ T^3. \qquad (4)$$

The figures in equations (3) were obtained by carrying out the decimals to many more places, and then retaining the above to the nearest unit for the right-hand figure.

The specific heat *at* any temperature will be the differential coefficients of the preceding expressions, which, for water, will be

$$\frac{d\,q}{d\,t} = 1 + 0.00004\ t + 0.0000009\ t^2, \qquad (5)$$

per degree centigrade, and

$$\frac{d\,q}{d\,T} = 0.999573 + 0.000004444\ T + 0.00000027768\ T^2, \qquad (6)$$

per degree Fahrenheit.

These results for water are not exactly the same as Rankine's or Bosscha's, given in Article 95, but any one of them is sufficiently exact for ordinary practice.

74, 85. Total heat of vapor. This expression includes the heat imparted to the liquid in raising its temperature from that of the melting point of ice to that at which the vapor is generated added to the heat necessary to evaporate the liquid at the higher temperature. The latent

heat of evaporation includes both internal and external work; the external being the work of enlarging the volume at the pressure corresponding to the higher temperature, and may be represented by the work done by a pound of saturated steam in pushing a piston against a constant resistance up to the point of cut-off in an engine, and the internal, that of overcoming the mutual attractions between the molecules. Let h be the number of heat units necessary to raise one kilogram of a liquid from 0° C. to a temperature t, and vaporize it at that temperature; then Regnault's experiments may be represented by the following empirical formulæ:

Substance.	Number of heat units in the "total heat of vapor" in French thermal units.	
Water	$h = 606.50 + 0.305\ t.$	(7)
Ether	$h = 94.00 + 0.45000\ t - 0.00055556\ t^2.$	
Acetic acid	$h = 140.50 + 0.36644\ t - 0.000516\ t^2.$	
Chloroform	$h = 67.00 + 0.1375\ t.$	
Chloride of carbon	$h = 52.00 + 0.14625\ t - 0.000172\ t^2.$	
Bisulphide of carbon.	$h = 90.00 + 0.14601\ t - 0.0004123\ t^2.$	

For English units, if h be the number of heat units in one pound of the substance on the Fahr. scale, then for water we would have

$$\begin{aligned} h = \tfrac{9}{5}\,\mathrm{h} &= \tfrac{9}{5}\,[606.5 + 0.305 \times \tfrac{5}{9}\,(T - 32)] \\ &= 1091.7 + 0.305\,(T - 32) \\ &= 1081.94 + 0.305\ T; \end{aligned}$$

$$\therefore H = 841829 + 237.29\ T,$$

which differs slightly from Eq. (95), page 111, because some fractions were omitted in determining the latter. In this manner we find the following results:

Substance.	Number of B. T. U. in the "total heat of vapor" of 1 lb., Fahr. scale, above 0° F.	
Water	$h = 1081.94 + 0.305\ T.$	(7)
Ether	$h = 154.4839481 + 0\ 46974324\ T - 0.000308633\ T^2.$	
Acetic acid.	$h = 240.880363 + 0.3847866\ T - 0.000286666\ T^2.$	
Chloroform.	$h = 116.2 + 0.1375\ T.$	
Ch. of carbon	$h = 88.82215 + 0.1523655\ T - 0.00009555\ T^2.$	
B. of carbon.	$h = 157.093127 + 0.16066955\ T - 0.000229055\ T^2.$	

General equation

$$h = a_3 + b_3\ T - c_3\ T^2. \qquad (8)$$

74. *Latent heat of evaporation.* Subtracting the "heat of the liquid" from the "total heat of the vapor" gives the latent heat of evaporation; hence

$$h_e = h - q;$$

and making the substitutions from above, we have the following results:

Substance.	Latent heat of evaporation, being the No. of French heat units in one kilo. of the vapor at the boiling point.	
Water	$h_e = 606.5 - 0.695\ t - 0.00002\ t^2 - 0.0000003\ t^3.$	(9)
Ether	$h_e = 94.0 - 0.07901\ t - 0.0008514\ t^2.$	
Acetic acid	$h_e = 140.0 - 0.13999\ t - 0.0009125\ t^2.$	
Chloroform	$h_e = 67.0 - 0.09485\ t - 0.0000507\ t^2.$	
Chlo. of carbon.	$h_e = 52.0 - 0.05173\ t - 0.0002526\ t^2.$	
Bisulp. of carbon	$h_e = 90.0 - 0.08922\ t - 0.0004938\ t^2.$	

In English units these become:

Substance.	Latent heat of evaporation, being the heat necessary to evaporate one pound of the substance at the boiling point, in B. T. U.	
Water	$h_e = 1121.7 - 0.6946\ T - 0.000002222\ T^2 - 0.0000000926\ T^3.$	(10)
Ether	$h_e = 171.24 - 0.0487\ T - 0.000473\ T^2.$	
Acetic acid	$h_e = 256.86 - 0.1075\ T - 0.000507\ T^2.$	
Chloroform	$h_e = 123.60 - 0.0930\ T - 0.000282\ T^2.$	
Chlo. carbon.	$h_e = 95.103 - 0.0423\ T - 0.0001403\ T^2.$	
Bisulph. carbon.	$h_e = 164.57 - 0.0716\ T - 0.0002746\ T^2.$	
Alcohol	$h_e = 527.04 - 0.92211\ T - 0.000679\ T^2.$	

General equation:

$$h_e = a_4 - b_4\ T - c_4\ T^2 - d_4\ T^3. \qquad (11)$$

These for English units and absolute temperature on the Fahrenheit scale become:

Substance.	Latent heat of evaporation in B. T. U., absolute temperature, τ.	
Water	$h_e = 1442.474 - 0.751472\ \tau + 0.0012538\ \tau^2 - 0.0000000926\ \tau^3.$	(12)
Ether	$h_e = 93.3214 + 0.3870\ \tau - 0.000473\ \tau^2.$	
Acetic acid	$h_e = 197.925 + 0.3595\ \tau - 0.0005070\ \tau^2.$	
Chloroform	$h_e = 160.4924 - 0.0371\ \tau - 0.0000282\ \tau^2.$	
Chlo. of carbon.	$h_e = 85.0245 + 0.0865\ \tau - 0.0001403\ \tau^2.$	
Bisulp. carbon.	$h_e = 140.1806 + 0.1810\ \tau - 0.0002743\ \tau^2.$	

General equation:

$$h_e = a_5 + b_5\ \tau + c_5\ \tau^2 + d_5\ \tau^3. \qquad (13)$$

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The effect of retaining the smaller decimals will be apparent by comparing the above results for water with the corresponding ones on page 95. Those on the latter page are considered sufficiently accurate for practice. None of them can be relied upon for temperatures much outside of those in the experiments upon which they are founded. The two equations are not very different within the range of temperatures ordinarily used in practice.

To find the heat which does the disgregation work, we must find the external work done during evaporation. This may be done as follows: The pressure for the absolute temperature of the vapor is, equation (80), page 97,

$$\log p = A - \frac{B}{\tau} - \frac{C}{\tau^2}.$$

We have computed the following constants by means of Regnault's experiments. They are for degrees Fahrenheit and pounds per square foot:

Fluid.	*A.*	*log B.*	*log C.*
Steam	8.28203	3.441474	5.583973
Ether	7.5641	3.3134249	5.2173549
Alcohol	8.6817	3.4721707	5.4354446
Bisulph. Carbon.	7.4263	3.3274293	5.1344146
Chloroform	4.3807	B is 3.288394 negative.	6.1899631
Sulp. Dioxide	7.3914	3.1580608	5.3667327
Naphtha	6.4618	2.949092	5.796469
Ammonia	8.4079	3.34154	
Mercury	7.9711	3.74293	

For steam.	*For ether.*
$B = 2763.59,$	$B = 2057.8,$
$C = 383683.$	$C = 164950.$

Now find the volume of a pound of the vapor by means of equation (84), or the approximate one, (86). Thus,

$$v_2 = v_1 + \frac{H_e}{\tau \frac{dp}{d\tau}} = v_1 + \frac{Jh_e}{\tau \frac{dp}{d\tau}}.$$

The value of h_e is given by equations (10) and (11), and substituting, gives

$$p\,v_2 - p\,v_1 = \frac{a - b\,T - c\,T^2}{\left(\frac{B}{\tau} + \frac{2\,C}{\tau^2}\right) 2.3026} J.$$

This is the *outer* work. The *disgregation* work will be

$$\rho = H_e - p\,(v_2 - v_1). \qquad (14)$$

Zeuner, by this laborious process, computed the disgregation work for a range of temperatures, and for various substances, and assumed, arbitrarily, that they followed the law

$$\rho = a_e - b_e\,t - c_e\,t^2,$$

and determined the constants by means of his previous computations, and obtained the following results :

For *French units.*

Ether..... $\rho = 86.54 - 0.10648\,t - 0.0007160\,t^2$.
Acetic acid..... $\rho = 131.63 - 0.20184\,t - 0.0006280\,t^2$.
Chloroform.... $\rho = 62.44 - 0.11282\,t - 0.0000140\,t^2$.
Chlo. of carbon $\rho = 48.57 - 0.06844\,t - 0.0002080\,t^2$.
Bisulp. of carbon $\rho = 82.79 - 0.11446\,t - 0.0004020\,t^2$.

For saturated steam, the *outer* work may be found very nearly from a table of the properties of saturated steam, by multiplying together the corresponding pressures and volumes. The product will be $p\,v_2$, Eq. (14). The temperatures being given in such a table, we may find the *total latent heat of evaporation* by means of equation (78), page 95.

Reduce the pressures to pounds per square foot, if necessary; and substitute in equation (14), to find the disgregation work.

The latent heat of evaporation, as commonly used, might be called *the* APPARENT *latent heat of evaporation ;* and *the disgregation work*, the REAL *latent heat of evaporation.*

Page 96. The latent heat of evaporation is reported differently by different authors. Thus I find that one author gives for oil of turpentine, 123, another, 133, and still another, 184; and I have not ascertained which is correct. For *ether*, 164.0, 162.8. Alcohol, 304.8, 372.7, 385. Naphtha, boiling point, 306, 141° F. ; latent heat of evaporation, 184, 236.

Densities of some vapors compared with that of air when near their boiling points :

Atmospheric air	1.000
Steam	0.6235
Alcohol vapor	1.6138
Sulphuric ether vapor	2.5860
Vapor of oil of turpentine	3.0130
Vapor of mercury	6.976

The densities of vapors at the boiling points of the liquids are approximately inversely as their latent heats of evaporation.

Thus,

$$\frac{\textit{Density of vapor of alcohol} = 1.6138}{\textit{Density of steam} = 0.6235} = 2.58.$$

$$\frac{\textit{Latent heat of evaporation of steam} = 966.1}{\textit{Latent heat of evaporation of alcohol} = 372.7} = 2.59.$$

76. Rankine, in his article *On the Centrifugal Theory of Gases*,* deduces an equation of the form,

$$\log p = a - \frac{b}{\tau},$$

for the relation between the pressure p and absolute temperature τ of saturated vapor. It was found, according to

* *Mis. Sc. Papers*, p. 43 ; *Phil. Mag.*, Dec., 1851.

Regnault's experiments and others, to be accurate for a limited range of temperatures only. Rankine then proceeded to find an *empirical formula* that would represent more accurately a greater range of temperatures, and was led, by analogy, to try a third term containing the inverse square of τ, thus giving

$$log\, p = A - \frac{B}{\tau} - \frac{C}{\tau^2},$$

which was found to represent, quite satisfactorily, the results of experiments upon steam, mercury, alcohol, ether, turpentine, and petroleum.

Some fifty, or more, formulas have been devised to express the relation between the pressure and temperature of saturated steam; all of which are sufficiently accurate for certain small ranges of temperature and pressure. Rankine's, given above, is the most accurate for a large range. Some of the most celebrated of the other formulas are:

Dulong and Arago's, for pressures above four atmospheres

$$p = (0.4873 + 0.012244\, t)^5 \text{ lbs. per sq. in.},$$

t being the temperature centigrade.

Mallet's, from 1 to 4 atmospheres,

$$p = \left(\frac{75 + t}{111.78}\right)^6 \text{ lbs. per sq. in.} \tag{15}$$

Tredgold's is the same, except that 175 is substituted for 111.78.

Pambour's, from 1 to 4 atmospheres, t ° C.

$$p = \left(\frac{72.67 + t}{110.93}\right)^6 \text{ lbs. per sq. in.} \tag{16}$$

Roche's

$$p = a\, b^{\frac{1}{m + n\, t}} \tag{17}$$

Regnault made three equations; the first applying from − 30° C. to 0° C.

$$log\, p = a + b\, \alpha^n, \text{ millimetres,} \tag{18}$$

in which $a = -0.08038$; $log\, b = 9.6024724 - 10$; $log\, \alpha = 0.0333980$; $n = 32° + t$.

From 0° to 100° C.

$$log\ p = a - b\ \alpha^t + c\ \beta^t, \text{ millimetres,} \tag{19}$$

in which $a = 4.7384380$; $log\ b = 0.6116485$; $log\ c = 8.1340339 - 10$; $log\ \alpha = 9.9967449 - 10$; $log\ \beta = 0.0068650$.

From 100° to 220° C.

$$log\ p = a - b\ \alpha^n + c\ \beta^n, \tag{20}$$

in which $a = 5.4583895$; $log\ b = 0.4121470$; $log\ c = 7.7448901 - 10$; $log\ \alpha = 9.99741212 - 10$; $log\ \beta = 0.007590697$.

Zeuner, by a special investigation, deduced a formula applicable both to saturated and superheated steam, which is

$$p\ v = R\ \tau - C\ p^n, \tag{21}$$

in which $R = \frac{p_o\ v_o}{\tau_o}$, although it is better to consider R, C, n, as constants to be found from experiment. For steam this becomes

$$p\ v = 50.933\ \tau - 192.50\ p^{\frac{7}{8}}, \tag{22}$$

in which p is the pressure in kilograms per square metre, and v the specific volume in kilograms per cubic metre. This formula is sufficiently accurate from 0.2 of an atmosphere to 15 atmospheres and, so far as tested, gives good results for superheated steam.

Hr. Ritter, from a discussion of experiments by Hirn, proposed the equation *

$$t = \frac{p\ v}{R} + \frac{S}{p\ v^{\frac{1}{2}}}, \tag{23}$$

in which

$$R = 4.653,\ S = 1043800;$$

which gives values for v agreeing almost exactly with the results found by equation (84), page 98. But it is too complex for analytical discussions.

Unwin proposed the equation †

$$log_{10} p = a - \frac{b}{\tau^n} \tag{24}$$

$$= 7.5030 - \frac{7579}{\tau^{1.25}}$$

for saturated steam; in which p is in millimetres of mercury and $\tau_o = -273$, on the centigrade scale. This equation is nearly, but not quite, as accurate as Rankine's; and possesses some advantages for analytical purposes. Unwin finds, for the latent heat of evaporation,

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* *Pogg. Ann.* (2), iii. (1878), 447.

† On the relations of temperature, pressure and volume of saturated steam.—*Phil. Mag.*, 1886, (1), 299–308.

$$h_e = 799 - \frac{894}{(7.503 - log_{10}\, p)}, \tag{25}$$

and for the specific volume of saturated steam,

$$v_2 = v_1 + \frac{10821\, h_e}{p\,(a - log_{10}\, p)}. \tag{26}$$

Also, for alcohol ;

$$log\, p = 7.448 - \frac{874}{t^{1.29}}. \tag{27}$$

For ether ;

$$log\, p = 6.9968 - \frac{3047}{t^{1.153}}. \tag{28}$$

For carbonic acid ;

$$log\, p = 8.4625 - \frac{302.8}{t^{0.77}}. \tag{29}$$

The author, considering steam as an imperfect fluid—or gas—both when saturated and superheated, applied Rankine's general equation for imperfect gases with the following results :—

The general equation being

$$p\, v = R\, \tau - a_0 - \frac{a_1}{\tau} - \frac{a_2}{\tau^2} - \text{&c.}, \tag{30}$$

and first assuming that the resulting equation might, possibly, be of the same form as his for carbonic acid gas, I made $a_0 = 0$, $a_1 = \frac{b}{v}$, and all the remaining terms zero, giving

$$p\, v = R\, \tau - \frac{b}{v\, \tau}. \tag{31}$$

But this gave no satisfactory result. I then assumed

$$a_0 = \frac{b}{v^n}$$

and all succeeding terms zero, as Rankine considered that a_0, a_1, &c., were inverse functions of v, thus giving

$$p\, v = R\, \tau - \frac{b}{v^n}. \tag{32}$$

To find the constants R, b, n, requires three contemporaneous values of the variables p, v, τ. I determined values of v by assuming several values for p and τ by equation (85), page 98, and compared the results with the recent tables of Professor Peabody, and in no case did they differ by more than .02 and in most cases they agreed exactly to the second decimal figure—and this too notwithstanding he used Regnault's equations for the relation between pressures and temperatures, while I used Rankine's, being equation (80), page 97; but using for constants my own values on page 98, instead of those computed by Rankine. So close an agreement was not anticipated under these conditions; and where the difference was greatest it might possibly have been less had I used more decimals. I therefore use Peabody's tables with confidence. I have, however, used $v_2 = 26.58$ for the specific volume of saturated steam at 212°, and pressure of 14.7 lbs. per square inch as computed on page 102; but all other values I have taken from the tables.

Using as arguments the three sets of values:

$$\begin{array}{lll} p_1 = 14.7 \times 144, & p_2 = 100 \times 144, & p_3 = 160 \times 144, \\ v_1 = 26.58, & v_2 = 4.38, & v_3 = 2.83, \\ \tau_1 = 672.66; & \tau_2 = 788.16; & \tau_3 = 824.06; \end{array}$$

I find

$$R = 96.95, \qquad b = 18473, \qquad n = 0.22;$$

giving the equation

$$p\,v = 96.95\,\tau - \frac{18473}{v^{0.22}} \tag{33}$$

More decimals in the value of n would make v^n slightly larger, and as only two are retained, a comparative compensation would be produced by increasing the value of the numerator, and the result of a few trials justified this modification, and we finally have

For saturated and superheated steam

$$p_f v = 96.95\ \tau - \frac{18500}{v^{0.22}}, \tag{34}$$

in which p_f will be in pounds per square foot, and v the cubic feet per pound. If p be in pounds per square inch and v the cubic feet per pound; then

$$p_1 v = 0.6732\ \tau - \frac{128.5}{v^{0.22}}. \tag{35}$$

In order to test this equation, I made the following table for saturated steam; taking from the table the values of v and T, and comparing the computed and tabular values of p.

TEST OF EQUATION (34).

Specific Volume cu. feet.	Temperature.		Pressure.			Fraction of error. Columns 5 and 6.
				Pounds per sq. foot.		
	Degrees F.	Absolute.	Pounds per sq. in.	Calculated. Eq. (34).	Tabular value.	
v.	T.	τ.	p.	p_f		
90.31	153.1	613.76	4	583	576	$+\frac{1}{83}$
53.37	176.9	617.56	7	1013	1008	$+\frac{1}{201}$
26.58	212.0	672.66	14.7	2116	2116.2	0
7.10	292.5	753.16	60	8597	8640	$-\frac{1}{200}$
5.42	311.8	772.46	80	11462	11520	$-\frac{1}{199}$
4.40	327.6	778.26	100	14550	14400	$+\frac{1}{96}$
2.83	363.4	824.06	160	23030	23040	$-\frac{1}{2304}$
2.294	381.7	842.36	200	28882	28800	$+\frac{1}{350}$
1	2	3	4	5	6	7

The agreement is good, and if the pressures were expressed in pounds per square inch, the whole numbers would agree to within one unit.

The measurements of superheated steam are few in number; but using those given by Hirn, we have the following:

SUPERHEATED STEAM.

Specific volumes.		Temperatures.			Pressures, lbs. per square foot.		Errors.
cu. metre per kilo.	cu. ft. per lb.	C.	T Fah.	τ.	Computed Eq. (24).	Tabular.	
1.85	29.635	141	285.8	746.46	2146	2116.2	$+\frac{1}{70}$
0.697	11.165	200	392.0	852.66	6435	6348.6	$+\frac{1}{72}$
0.5752	9.214	246	474.8	935.46	8721	8464.8	$+\frac{1}{33}$
0.414	6.631	205	401.0	861.66	10759	10581.0	$+\frac{1}{59}$

These results do not agree so nearly as for saturated steam, and the errors are all in one sense, the computed values being all too large. This we would anticipate since the more the steam is superheated, the more nearly will it behave like a perfect gas, and conform more nearly to equation (162), page 172; or to one of that form.

In a paper read before the Society of Mechanical Engineers in May, 1889, I discussed several forms of equations for saturated steam. Among them was the following, proposed by Mr. E. R. Dawson, a graduate at the Institute:

$$p = 0.6734\,\frac{\tau}{v} - \frac{1636.88}{v^{\frac{5}{4}}} - 0.16.$$

The close agreement of Zeuner's equation, (22) above, with experiment, is more apparent than real; since if our values of $J = 427$ instead of 424, and $\tau_0 = -273.7$ instead of -273 be used, the computed results would be more in error than his appeared to be. I would use the preceding equation, (34), with as much confidence for superheated steam as I would Zeuner's.

78a. *The critical temperature* is the state at which no external work is done when a liquid changes to a vapor. Or, in a series of isothermal changes in which the liquid is changed to a vapor, it is the state at which the external work vanishes. Or, again, it is the state at which the *real* latent heat of evaporation is zero.

Page 104, the external work done during the evaporation of one pound of the liquid will be $p \times B\ C$, at the temperature τ. When the abscissa between $F\ E$ and $E\ C$ is zero, as at E, the corresponding temperature will be the critical temperature for that substance.

Neglecting the external work due to the enlargement of the volume of the liquid, the critical temperature will be that temperature which will reduce the *apparent* as well as the *real* latent heat of evaporation to zero. For steam, the first two terms of equation (12) of this Addenda gives

$$\tau = 1919\ ;\ \text{or}\ T = 1459^\circ\ \text{F.}$$

The critical temperature of a few substances has been found by experiment. Thus, it is for

	Deg. C.
Carbon tetrachloride	292.5
Carbon disulphide	276.1
Acetone	246.1

(*Pogg. Ann.*, cli., (1874), 303.)

Theory gives higher values than these. The critical temperatures and pressures for twenty-one substances is given in the *Philosophical Magazine*, 1884, (2), page 214.

Avenarius showed by experiment that over a certain temperature fixed for each substance there is no distinction between the liquid and vapor states, so that *pressure alone will not cause a gas to liquefy.*

79. In Rankine's tables the absolute zero was assumed at 461.2 below the zero of Fahrenheit's scale, while those at the end of this work are computed with 460.66.

85. See Article 75 of this Addenda.

96. Clausius claims to be the first to announce that internal work is a function of the initial and terminal states only. (Clausius *On Heat*, page 35.)

97. It will be a good exercise for the student to give geometrical interpretations of the equations on page 132.

97a. "*On the dimensions of temperature* in length, mass and time; and on the absolute C. G. S. unit of temperature" (*Phil. Mag.*, 1887, (2), 96). It is shown that, in accordance with Thomson's absolute scale, *the unit temperature would be that of a perfect gas whose mean kinetic energy per molecule was one erg.*

If E = the mean kinetic energy of a molecule of the gas,
τ = absolute temperature of a perfect gas,
k = a constant,
p = pressure per unit,
v = volume of a pound of the gas,
n = the number of molecules in a pound;

then,

$$E = k\tau,$$

and making $k = 1$,

$$E = \tau.$$

But, equation (2) of Appendix, gives, if $v_1 = \frac{1}{\delta}$ for 1 pound,

$$x\, p\, v_1 = v^2 = 2\, n\, E;$$

hence, making $x = 3$, and omitting the subscript, we have

$$p\, v = \tfrac{2}{3}\, n\, E;$$

$$\therefore E = \tfrac{3}{2}\,\frac{10^6 \times 1}{6 \times 10^{21}} = 2.5 \times 10^{-16} = \tau$$

for the value of the temperature at 0° C., or − 273° C. absolute, C. G. S., and

the absolute unit = $273 \div 2.5 \times 10^{-16}$ = about 10^{18} C. degrees.

"Having seen that *temperature* is of the same dimensions as *energy*, and knowing that the same is true of *heat*, it follows that *entropy*, whose dimensions are *heat* ÷ *temperature*, is a purely numerical quantity; and the unit of entropy is therefore independent of all other physical units. In fact, *the entropy of a perfect gas increases by unity, when (without altering its temperature) it receives by conduction a quantity of heat equal to the mean energy of one of its molecules.*"

98. Priming or superheating. Equation (135), page 144, may be put in a more customary form as follows:

Let

T_1 = the temperature of the water at the boiling point under the given pressure in degrees Fahr.,

T_2 = the temperature of the feed water, which is assumed to be the same as that of the higher temperature of the water supplying the calorimeter.

T_3 = the initial temperature of the water supplied to the calorimeter.

$\mathbf{h}$ = the *total heat of steam*, being the heat units necessary to raise the temperature of one pound of water from 32° F. to the boiling point and evaporating it at that point. Its value may be found in special tables, or from equation (134) after making $x = 1$.

h_1 = the heat units in one pound of the water at the boiling point above 32° F., which will be $T_1 - 32$, nearly, but may be found more accurately from Article 95, or more directly from suitable tables:

h_2 = the heat units in one pound of the feed water above 32° F.,

h = the heat units in one pound of the steam above the temperature of the feed water as determined from a calorimeter,

w = the weight of the steam condensed in the calorimeter,

W = the weight of the water supplied to the calorimeter;

then

$$h = \frac{W}{w}(T_2 - T_3), \tag{1}$$

$$\tau_1 - \tau_2 = T_1 - T_2 = h_1 - h_2, \tag{2}$$

$$h_e = \mathbf{h} - h_1 = \mathbf{h} - (T_1 - 32). \tag{3}$$

Dividing both numerator and denominator of equation (135) by $J = C$, and substituting the above values gives

$$\textit{Per cent of priming} = 100\ \frac{h_e + \tau_1 - \tau_2 - h}{h_e} \qquad (4)$$

$$= 100\ \frac{\mathbf{h} - h_2 - \frac{W}{w}(T_2 - T_3)}{\mathbf{h} - h_1} \qquad (5)$$

If this equation becomes negative there will have been superheating.

In equation (5), $\mathbf{h} - h_2$ is the heat supplied above the temperature of the feed water to produce *one pound* of saturated steam, and $\frac{W}{w}(T_2 - T_3)$ is the heat supplied to one pound of steam and water combined. If the entire pound be steam at the point of saturation, this quantity would equal the former and the expression would reduce to zero. But if it exceeded the former there would have been superheating, and the expression becomes negative. The numerical difference between the two terms would be the number of heat units of superheating, and this divided by the specific heat of steam (0.48) gives the number of degrees of superheating; or

$$\textit{Deg. superheating} = \frac{\frac{w}{W}(T_2 - T_3) - (\mathbf{h} - h_2)}{0.48}. \qquad (6)$$

These two expressions may be partly combined in one, thus; if

$$q = \mathbf{h} - h_2 - \frac{w}{W}(T_2 - T_3); \qquad (7)$$

then, if q be positive, we have

$$\textit{Per cent priming} = 100\ \frac{q}{h_e};$$

and if q be negative,

$$\textit{Degrees of superheating} = \frac{q}{c} = \frac{q}{0.48},$$

disregarding the sign of q in the result.

Accuracy in the calorimeter test requires a correction for the heat transferred to the vessel and the vapor escaping at the surface. The corrections will depend upon the size of the vessel, its material and the manner of conducting the experiments, and do not admit of a general practical expression.

99. Specific heat of saturated vapor. To explain more fully the fact that the specific heat of saturated steam is negative, let $c\ A\ b$, Fig. c, be *the curve of saturation*, and let A, on this curve, be the state of the steam; then it is found that if it expands without transmission of heat the curve of pressures will fall below $A\ b$ and follow some line as $A\ d$ (equations (150) and (151), page 154, and below equation (185a), page 178). Some of the steam will have been condensed, so that there will be a less *mass of vapor* than at the state A. In order, then, that the initial amount, one pound, shall remain as steam during the expansion, heat must be supplied to the steam. On the other hand, if the steam be compressed without transmission of heat it will be superheated and follow some line as $A\ e$; and to make its path coincide with $A\ c$, heat must be abstracted. Rankine found, from an inspection of actual curves of expansion as shown on indicator diagrams, that the equation of the curve of expansion was

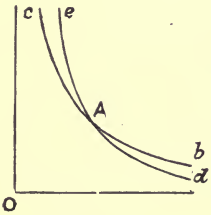


FIG. c.

$$p\ v^{1.0} = \textit{constant}, \text{ nearly.}$$

But we have previously found that the equation of the curve of saturation is

$$p\ v^{1.3} = \textit{constant}, \text{ nearly;}$$

which equations show that, if they have one common state

the curve of expansion will fall under the curve of saturation, and the curve of compression rise above it, since it is a steeper curve at the common point. This is true for all temperatures of steam less than about 976° F.

Ether engine. The specific heat of the saturated vapor of ether will be equation (139), page 147,

$$s = c - \frac{h_e}{\tau} + \frac{d\,h_e}{d\,\tau};$$

and retaining only two terms of equation $(12)_2$, of this Addenda, we have

$$h_e = 93.32 + 0.3870\,\tau;$$

$$\therefore \frac{h_e}{\tau} = \frac{93.32}{\tau} + 0.3870;$$

$$\frac{d\,h_e}{d\,\tau} = 0.3870.$$

We also have

$$c = 0.517;$$

$$\therefore s = 0.517 - \frac{93.32}{\tau},$$

which is zero for

$$\tau = 180.5°;$$

or,

$$T = -\,280°\ \text{F};$$

hence, for all temperatures above −280° F., s will be positive; that is, *the specific heat of the saturated vapor of ether is positive for all temperatures occurring in practice.*

This reverses certain conditions of the steam-engine. Referring to Fig. *c*, now let $e\ A\ d$ be the curve of saturation of the vapor of ether; then if the vapor be *compressed* from the state A the path of the fluid will fall *below* $A\ e$, and heat must be *supplied* in order that the path shall be $A\ e$, that of saturation. On the contrary, if it be expanded without transmission of heat, the vapor will be superheated and its path will rise above $A\ d$; and in order that its path shall coincide with $A\ d$, heat must be abstracted.

In the ether engine, if v_2 be the volume of a pound of the saturated vapor at a temperature τ_2, and it be *compressed* until its temperature is τ, then will the volume of the vapor be, equation (150), page 154,

$$u = \frac{\tau\, v}{H_e}\left(J\, c\, log\frac{\tau_2}{\tau} + v_2 \frac{d\, p_2}{d\, \tau_2}\right);$$

and the volume of the vapor condensed will be

$$v - u.$$

Other vapors may be examined in the same manner. (See *Vapor Engines* in the following Article.)

Temperature of inversion. The temperature at which the specific heat of the vapor becomes zero is called *the temperature of inversion*; below which expansion will cause condensation, and above which expansion will cause vaporization, and the contrary, the specific heat being negative.

"M. Hirn* gave an experimental demonstration in 1862 of the condensation accompanying a sudden adiabatic expansion of dry saturated steam, and thus proved the negative sign of s' at low temperatures; he allowed steam to pass gently from a boiler, where it was generated under 5 atmospheres pressure, through a copper cylinder, 200 c. long and 15 c. in diameter, the ends of which were closed by plates of glass, until all air and condensed water had been driven out and the sides had attained the temperature of the steam; the exit-stopcock of the cylinder was then shut, and, the cylinder being full of dry saturated steam, the connection with the boiler was cut off and the exit-stopcock suddenly opened; the pressure at once fell, and the cylinder, which had previously appeared perfectly transparent to an observer looking along its axis, became perfectly opaque from the formation of a cloud; this cloud, however, soon disappeared, heat being supplied by the vessel as it cooled from 152° C. (the temperature of saturated steam under 5 ordinary atmospheres) to 100° C. (the temperature under 1 atmosphere).

M. Cazin† improved this apparatus by connecting the cylinder with another in which a piston was movable, and placing the whole in an oil-bath the temperature of which could be varied at will; saturated vapor in the one could then be allowed to suddenly expand into the other, or

* *Bulletin de la Soc. industr. de Mulhouse*, cxxxiii. (1862), 129; *Cosmos*, xxii. (1863), 413.

† *C. R.* lxii. (1866), 56.

when filling both could be suddenly compressed by the motion of the piston. A cloud was always formed when steam expanded, but never on its compression, and with carbon bisulphide the same occurred; on the contrary, as noticed earlier by M. Hirn, ether vapor never condensed during expansion, but always on compression, showing that its temperature of inversion is below ordinary temperatures; further, the temperature of inversion appeared to be between 125° and 129° C. for chloroform, and for benzine between 115° and 130° C. These results are in keeping with theory, M. Regnault's formulæ giving 790.2°, — 113°, 123.5°, 100° C. for the temperatures of inversion of carbon bisulphide, ether, chloroform, and benzine respectively."

112. The results of Articles 110, 111, 112, illustrate the fact that the efficiency, when the operation is in the cycles there assumed, is less than that for Carnot's cycle when worked between the same limits of temperature. The initial pressures are the same in each of the three cases, being $p_1 = 14400$ lbs. The range of temperatures in the Exercise on page 174, for superheated steam, expanded isothermally, is

$$T_1 - T_4 = 450 - 110 = 340.$$

Initial absolute temperature $= 910.66$;

hence, if the heat absorbed had been worked in a Carnot's cycle the efficiency would have been

$$E = \frac{340}{910.66} = 0.373,$$

instead of 0.207 as given in equation (171a).

In the Exercises on pages 176 and 178 for saturated steam expanded adiabatically, the initial and final temperatures are the same, being

$$T_1 = 327.6,\ T_4 = 110;$$
$$\therefore\ T_1 - T_4 = 217.6,$$
$$\tau_1 = 460.66 + 327.6 = 778.66;$$
$$\therefore\ \frac{T_1 - T_4}{\tau_1} = 0.267,$$

instead of 0.204 or simply 0.200. The latter is 75 per cent of the former.

If the feed water were of the same temperature as the exhaust steam, we would have

$$\frac{T_1 - T_3}{\tau_1} = \frac{327.6 - 134}{778.26} = 0.245,$$

which also exceeds the efficiencies actually found.

Vapor engines. *To find the maximum efficiency of the ideal vapor engine.*

Let $M\,O = v_A$, Fig. *d*, be the volume of one pound of the liquid, which assume to be constant at all pressures, $A\,O = p_A$ = the absolute pressure at the temperature τ_1. Let the vapor expand, the temperature being maintained constant; then, since it is in contact with the liquid, the vapor will be saturated and the pressure will be constant, equation (80), and the line $A\,B$, parallel to $O\,H$, will be an isothermal. Continue the expansion until the liquid is completely evaporated, and represent the volume $M\,G$ by v_B. From B let it be expanded without transmission of heat to a temperature τ_2 and volume $M\,H = v_0$; then compressed, *abstracting heat so as to maintain the constant temperature* τ_2, and pressure p_2, to such a point J that when compressed without transmission of heat it will be entirely liquefied when it reaches the initial state A. The adiabatics $B\,C$ and $A\,J$ will be represented by equation (149). The line $B\,N$ represents the curve of saturation, and if the specific heat of the vapor be negative, as steam, the adiabatic $B\,C$ will fall under the line $B\,N$ as before shown. But if the specific heat be positive, like the vapor of ether, then may $B\,C$ represent the curve of saturation and $B\,N$ an adiabatic, provided the vapor is saturated at state N; or, in other words, if the vapor is to remain saturated, the expansion $A\,B$ will

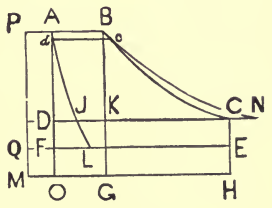


FIG. *d*.

extend to such a point B that, when expanded adiabatically, reducing the temperature from τ_1 to τ_2, the liquid will not be completely evaporated before the lower temperature is reached. If the ether be completely evaporated at state B it will be superheated when expanded adiabatically as shown in Article 99 of this Addenda. and $B\,N$ will be the adiabatic of an imperfect gas. Similarly, the point of compression J will be further to the right for ether, and all vapors whose specific heat is positive, than for those which are negative.

The heat absorbed will be, equations (21) and (74),

$$H_1 = \tau_1 \frac{d\,p_1}{d\,\tau_1} \int_{v_A}^{v_B} d\,v = \tau_1 \frac{d\,p_1}{d\,\tau_1} (v_B - v_A). \qquad (1)$$

The heat emitted from C to J will be

$$H_2 = \tau_2 \frac{d\,p_2}{d\,\tau_2} (v_C - v_J). \qquad (2)$$

Making $d\,\tau_1 = d\,\tau_2 = d\,\tau$, we have,

$$A\,B\,c\,d = \frac{d\,p_1}{d\,\tau} (v_B - v_A) = \frac{d\,p_2}{d\,\tau} (v_C - v_J), \qquad (3)$$

since the areas of all the strips formed by equidistant isothermals between adiabatics will be equal, as shown in Article 40, page 32.

The efficiency will be

$$\frac{H_1 - H_2}{H_1} = \frac{(\tau_1 - \tau_2) \frac{d\,p_1}{d\,\tau} (v_B - v_A)}{\tau_1 \frac{d\,p_1}{d\,\tau} (v_B - v_A)} = \frac{\tau_1 - \tau_2}{\tau_1}, \qquad (4)$$

which is the same as for the perfect elementary engine, equation (159), page 161. This result might have been anticipated, since the cycle is Carnot's.

97*a*. **To represent geometrically certain relations.**

Equations (*A*), page 48, or (123), page 135, give

$$d\varphi = \frac{d\,H}{\tau} = \left(\frac{d\,p}{d\,\tau}\right)_v d\,v + C_v \frac{d\,\tau}{\tau}.$$

If τ be constant during the expansion $d\,v$, we have $d\,\tau = 0$, and

$$(d\,\varphi)_\tau = \left(\frac{d\,p}{d\,\tau}\right)_v (d\,v)_\tau.$$

Dividing both sides by $d\,v$ gives

$$\left(\frac{d\,\varphi}{d\,v}\right)_\tau = \left(\frac{d\,p}{d\,\tau}\right)_v . 1.$$

The factor $\left(\frac{d\,p}{d\,\tau}\right)_v$ is the *rate* of change of pressure per unit of temperature, and therefore if the rate were uniform during the change of unity of temperature from τ to $\tau + 1$, it would be the increase of pressure due to an increase of one degree of temperature. Draw two parallel lines to represent two isothermals differing by unity. As in the calculus, these lines may be tangents to actual isothermals. Let $a\,w$ and $g\,p$ be the isothermals, differing by one degree of temperature. At a let the pressure be p; then will

$$\left(\frac{d\,p}{d\,\tau}\right)_v = a\,c.$$

If the abscissa of b in reference to a be $a\,e = d\,v$, then $a\,c\,f\,b = a\,c \cdot a\,e = \left(\frac{d\,p}{d\,\tau}\right)_v d\,v$.

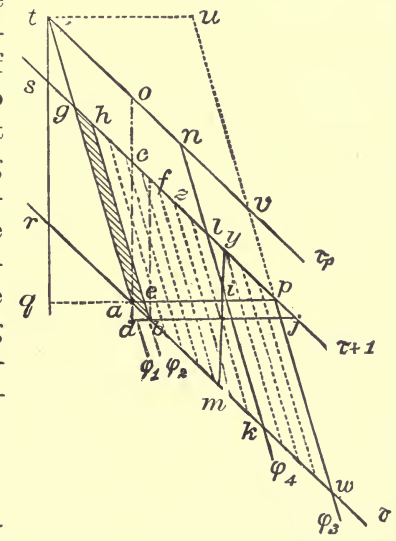


FIG. e.

Let the straight line $a\,g$ through a be an adiabatic (tangent to an actual adiabatic). Divide $a\,p$ into parts each equal to $d\,v$, and through the points of division draw lines parallel to $a\,c$, and through their points of intersection with $a\,w$ draw lines parallel to $a\,g$; the spaces thus formed will be equal; and equal to $a\,c\,f\,b$, since they have the same base and altitude. Hence

$$(d\,\varphi)_\tau = a\,g\,h\,b = a\,c\,f\,b = \left(\frac{d\,p}{d\,\tau}\right)_v (d\,v)_\tau.$$

Dividing by $d\,v$,

$$\left(\frac{d\,\varphi}{d\,v}\right)_\tau = \left(\frac{d\,p}{d\,\tau}\right)_v . 1.$$

Let $a\,i$ represent unity of volume, or

$$a\,i = 1;$$

then

$$a\,c\,y\,m = a\,c \times a\,i = \left(\frac{d\,p}{d\,\tau}\right)_v . 1,$$

which is the right member of the preceding equation.

$\left(\frac{d\,\varphi}{d\,v}\right)_\tau$ is the number of $d\,\varphi$'s between τ and $\tau + 1$ for an increase of volume equal to unity, and is equivalent to the area $a\,g\,z\,m = a\,c\,y\,m$;

$$\therefore \left(\frac{d\,\varphi}{d\,v}\right)_\tau = 1 . \left(\frac{d\,p}{d\,\tau}\right)_v,$$

as deduced above, analytically.

It will be observed that the subscript of the parenthesis on one side of the equation is the same as the independent variable on the other side; that is, τ is a subscript on the left side, and the independent variable on the other. Writing the reciprocal of the preceding equation, and observing a similar order of subscripts, we have,

$$\left(\frac{d\,v}{d\,\varphi}\right)_p = \left(\frac{d\,\tau}{d\,p}\right)_\phi;$$

the geometrical interpretation of which we will proceed to show.

The equation

$$\varphi = \text{a constant}$$

is the equation of an adiabatic, and the expression $\left(\frac{d\,\tau}{d\,p}\right)_\phi$ implies the rate of change of temperature due to a change in the pressure along the adiabatic $a\ t$. Let the pressure at a be p, and at t, $p + 1$, and $a\ q$, parallel to the axis of volumes; then

$$q\ t = 1.$$

The temperature at a being τ, that at t will be $\tau + \left(\frac{d\,\tau}{d\,p}\right)_\phi$;

$$\therefore \text{change of temperature from } a \text{ to } t = \left(\frac{d\,\tau}{d\,p}\right)_\phi.$$

The area $a\ g\ h\ b = d\ \varphi$; and if this area were extended to t, its value would be

$$\left(\frac{d\,\tau}{d\,p}\right)_\phi d\ \varphi;$$

and this also equals

$$q\ t\ .\ a\ e = 1\ .\ d\ v_p;$$

$$\therefore 1\ .\ \left(\frac{d\,v}{d\,\varphi}\right)_p = \left(\frac{d\,\tau}{d\,p}\right)_\phi . 1.$$

Assume any arbitrary area as $a\ g\ l\ k$ for an increase of $\varphi = 1$. Since $\left(\frac{d\,\tau}{d\,p}\right)$ is the increase of temperature for an increase of unity of pressure, it is the temperature of the isothermal $t\ o$ above that of $a\ k$;

$$\therefore \text{area } a\ t\ n\ k = \left(\frac{d\,\tau}{d\,p}\right)_\phi\ (\varphi = 1).$$

The expression $\left(\frac{d\,v}{d\,\varphi}\right)_p$ is the abscissa of v for φ unity, and hence is the abscissa of k in reference to a; and this multiplied by $q\ t = 1$, gives

$$1 . \left(\frac{d\,v}{d\,\varphi}\right)_p = a\,t\,n\,k;$$

hence the equality of the expressions.

The second equation of thermodynamics is

$$d\,\varphi = \frac{d\,H}{\tau} = C_p\,\frac{d\,\tau}{\tau} - \left(\frac{d\,v}{d\,\tau}\right)_p d\,p,$$

which becomes for τ constant,

$$d\,\varphi = -\left(\frac{d\,v}{d\,\tau}\right) d\,p,$$

or

$$\left(\frac{d\,\varphi}{d\,p}\right)_\tau = -\left(\frac{d\,v}{d\,\tau}\right)_p.$$

If a and b be consecutive states on the isothermal, for an increase of volume $d\,v = a\,e$, the pressure decreases an amount $a\,d = -\,d\,p$.

$$\left(\frac{d\,v}{d\,\tau}\right)_p = a\,p,$$

where $a\,p$ is the increase of volume in passing from one isothermal, τ, to another a unit higher, $\tau + 1$, measured on a horizontal line; and this multiplied by an ordinate $q\,t = p = 1$, gives the area $a\,t\,u\,p = a\,t\,v\,w$. $\left(\frac{d\,\varphi}{d\,p}\right)_\tau$ is the number of φ's between two isothermals for an increase unity in p.

$$d\,\varphi_\tau = a\,b\,h\,g = a\,b\,j\,p,$$

which divided by $-\,d\,p$ gives

$$-\,a\,t\,u\,p = -\left(\frac{d\,\varphi}{d\,p}\right)_\tau = a\,t\,v\,w.$$

Taking the reciprocal, we have

$$\left(\frac{d\,p}{d\,\varphi}\right)_v = -\left(\frac{d\,\tau}{d\,v}\right)_\phi,$$

which may be geometrized in a similar manner.

Thus, the right-hand member implies the increase of temperature for a change of unity of volume on an adiabatic. Let $a\ q = -v = -1$, and from q erect a perpendicular to meet the adiabatic through a at t; then will the increase of temperature at t above that at a be

$$\left(\frac{d\ \tau}{d\ v}\right)_{\phi}.$$

If the increase unity of φ be represented by $a\ g\ l\ k$, then

$$a\ t\ n\ k = \left(\frac{d\ \tau}{d\ v}\right)_{\phi}. \ (\varphi = 1).$$

Also

$$\left(\frac{d\ p}{d\ \varphi}\right)_{v}$$

will be the ordinate of g above a, which multiplied by $v =$ unity will give the same result as the preceding.

113. Cut-off. In the analysis on page 182 no account is taken of the condensation of steam. In order to determine approximately the modification of the preceding analysis which would result from this cause, assume that sufficient steam is admitted to produce the same pressure up to the point of cut-off as if there were no condensation, and that the mean effective pressure is the same; then will all the quantities in expression (191), page 183, remain the same except r and h. The value of h will be increased, for larger boilers may be required, and more fuel will be required, and other incidentals may be increased. In the solution, if h be increased, $O\ C$, Fig. 50, page 184, will be increased, and $C\ D$ will be increased in like proportion, since

$$C\ D = \frac{F}{p} \times O\ C.$$

Hence, the point D will be found on the same line $N\ O\ D$, prolonged; but the more remote the point D is, the further to the right will be the tangent point P, thus requiring a later cut-off. In theory or practice, however,

the cut-off in case of condensation will not be very much larger than if there were no condensation. This case was well discussed in the paper by Wolff and Denton in their paper read before the *Society of Mechanical Engineers*, referred to on page 189. It appears from the examples there given, that the first cost has an important influence on the most economical point of cut-off, and that in stationary engines the ratio of expansion should not be very high for greatest economy—all elements involved being considered. But in marine engines another element enters—that of storage room for fuel. Here, all space saved by reducing the amount of coal necessary for making a voyage may become profitable by furnishing room for the storage of merchandise, or, otherwise, lessening the non-paying burden of the ship. Under these conditions very high expansions may be economical when they would not be on land service, where storage room is of no value.

Miscellaneous.

The following are some average results for the total heat of combustion per pound of some liquid fuels. (*Thesis*, by James Beatty, Jr., Stevens Institute, 1884, p. 72.)

Petroleum	20,300	B. thermal	units.
Coal gas	20,200	"	"
Creosote	16,400	"	"
Asphalt	15,900	"	"
Anthracite	14,500	"	"
Carbon	14,450	"	"
Bituminous	14,200	"	"
Lignite	11,800	"	"
Peat	9,600	"	"
Water gas	8,500	"	"
Wood	6,800	"	"
Generator gas	3,100	"	"

Air liquefied at high pressures—say with about 40 atmospheres and at — 142° C.—may be maintained in a liquid state at a pressure of one atmosphere, provided the temperature be lowered to about — 191° C. (*Phil. Mag.*, 1885, (2), 463.)

This is similar to the action of steam. If steam be at a pressure of

200 pounds to the square inch and temperature 390° F., it cannot be liquefied by mere compression ; but if at that pressure the temperature be diminished to 380°, it will liquefy, and the pressure may be gradually removed down to one atmosphere, provided the temperature be reduced to 212° F. or less.

Curve of saturation of oxygen. The experiments of Wroblewski for the boiling points of oxygen give the following results (*Phil. Mag.*, 1884, (1), 491) :

Pressure.	Temperature.	
Atm.	Degrees C.	Degrees F.
50	— 113	— 171.4
27.02	— 129.6	— 201.3
25.85	— 131.6	— 204.9
24.40	— 133.4	— 208.1
23.18	— 134.8	— 210.6
22.2	— 135.8	— 212.4
1	— 184.0	— 299.2

These plotted on the plane of p and τ would be the curve of saturation on that plane.

Evaporation and dissociation. (*Phil. Mag.*, 1887, (2), 196. This is Part IV. of a series of articles upon liquids and vapors.)

Boiling points of gases at atmospheric pressure, or the points of liquefaction of gases. M. S. Wroblewski has determined the following values, the temperature being measured by means of a thermo-electrical apparatus which was considered correct to within $\frac{1}{5}$ of a degree down to — 200° C.

Substance.	Boiling Point at Atmospheric Pressure. Degrees.	
	C.	F.
Air	— 194.2	— 317.5
Oxygen, O	— 184.0	— 299.2
Nitrogen, N	— 193.1	— 315.6
Carbonic oxide, CO	— 193.0	— 315.4
Ethyline	— 193.0	— 315.4

(*Phil. Mag.*, 1884, (1), 490.)

The conductivity of heat in several liquids is given in the *Phil. Mag.*, 1887, (2), 1-27. The conductivity of water is the highest of all those reported. If that of water be 100, that of benzine is about 26.8, p. 18. If that of air be 0 00049 C. G. S., that of water will be 0.0645, p. 22 ; and that of petroleum about 0.022 ; ether, 0.023 ; carbon disulphide, 0.016, p. 27. Accordingly, it appears that water is one of the best of the liquids for absorbing heat in a boiler.

Watt's law. *The sum of the sensible and latent heats of steam is constant.*

This is one of the earlier laws deduced from crude experiments, but Regnault's experiments showed it to be erroneous. (Eq. (78), p. 95. Also Browne's translation of Clausius *On Heat*, pp. 131–134.)

The mechanical equivalent of light, as determined by Julius Thomsen, is about 12.28 foot-pounds per minute for one candle. (*Phil. Mag*, 1865, Vol. XXX., p. 246.)

According to Moses G. Farmer, it is about 10.1 foot-pounds per candle per minute. (*Am. Jour. Sc. and Arts*, 1886, Vol. XLI., p. 214.)

Density of liquefied gases. The density of liquid hydrogen at 0° C. is 0.025; at − 23° C., 0.032; of liquid nitrogen at 0° C., 0.37; of liquid oxygen, 0.65 at 0° C. (Caillett and Hautefeuelle in *Comptes Rendus*, Vol. XCII., p. 1086.)

According to Wroblewski, the density of liquefied oxygen is 0.899 at − 100° C., and 1.24 at − 200° C. (*Comptes Rendus*, 102, 1010.)

The specific gravity of ice at 0° C. is 0.91886. (*Smithsonian Contributions*, 1888, Vol. XXXIII., p. 40.)

Sulphurous acid. Latent heat of evaporation of sulphurous acid is

$$h_e = 91.2 - 0.37\,t \text{ calories},$$

very nearly from $t = -10°$ C. to $t = 70°$ C.

Density of the saturated gas is 0.00624 at 7.3° C.; 0.4017 at 154.9° C.

Density of the liquid, 1.4338 at 0° C.; 0.6370 at 155.05° C. (*Comptes Rendus*, Vol. CIV., p. 1564.)

For more recent and more reliable values, see page 357, and paper by the author in the *Trans.* of the Society of Mechanical Engineers, Vol. XI., 1890.

Short trips across the Atlantic. In May, 1889, the *City of Paris* crossed westward in 5 days, 23 hours and 7 minutes. In August, eastward, in 5 days, 23 hours and 44 minutes; westward, in 5 days, 19 hours and 18 minutes. In August, 1890, the *Teutonic* made a westward trip in 5 days, 19 hours and 5 minutes, and claimed to have sailed 40 miles farther than the *City of Paris* when the latter made her quickest trip. The greatest reported speed of the latter exceeded that of the former by 5 miles for 24 hours. In August, 1891, the *Teutonic* made a trip in 5 days, 16 hours and 31 minutes. July, 1892, *City of Paris*, 5 days, 15 hours, 58 minutes.

Vertical distribution of temperature in the atmosphere is very variable. (*Engineering* [London], 1889, November, pp. 522, 566.)

Temperature of space has no sensible existence. (S. P. Langley, *Phil. Mag.*, January, 1890, p. 33.) The mean temperature of sunlit lunar

soil is much lower than has been supposed, and is most probably not greatly above zero centigrade. (*Ibid*, p. 53.)

Heat transmitted through cast-iron plates pickled in nitric acid is about 70 per cent. of clean untreated plates. (Carpenter, in *Trans. Soc. Mech. Eng'rs*, 1890.)

History of the gas-engine. (*Scientific American Supplement*, 1889, p. 11,416.)

Steam-engine practice in 1884 by J. C. Hoadley, C. E. (*Trans. Am. Asso. for Ad. of Science*, 1884, pp. 289–359.)

While breathing, a man consumes about 215 cubic feet of air per hour, or about 17 pounds.

Page 74.—To show that the elasticity of the air equals its tension, let p_1 be the pressure on unity of area, producing a compression λ and l the length of the column; then, from *Resistance of Materials*, we have

$$p_1 = E\frac{\lambda}{l}. \qquad (a)$$

For isothermal change we have

$$p\,v = R\,\tau;$$

$$\therefore p\,d\,v = -\,v\,d\,p;$$

or, making $v = l$, $d\,v = -\,\lambda$ and $d\,p = p_1$, then

$$p\,\lambda = l\,p_1;$$

$$\therefore p_1 = \frac{p\,\lambda}{l},$$

which compared with equation (*a*) gives

$$p = E.$$

If the change be adiabatic, then

$$p\,v^{\gamma} = a,$$

and by a similar process it will be found that

$$E = \gamma\,p.$$

Page 83.—To obtain equation (65), let τ_2 be variable, and dropping the subscript $_2$, the variable factor in (64) will be

$$\tau^{\frac{1}{\gamma-1}}\sqrt{\tau_1 - \tau}.$$

Squaring, differentiating, placing equal to zero, and equation (65) is readily found.

Page 103. EXERCISE 9.—Let $O\ v_1$ be the volume of a pound of water; when it is all evaporated, let it be $O\ v_2 = 26.5$ cubic feet. Let $v_1\ A$ represent the pressure of one atmosphere $= 2116.3$ pounds per square foot; then the external work will be $O\ v_2 \times v_1\ A$. Let $v_1\ a$ be the *virtual* pressure, which would, if worked to the volume v_2, represent both the *external* and *internal* work; then, in this exercise, $A\ a$ will be 12.37 times $v_1\ A$. The heat absorbed in evaporating the pound of water will be represented by the area $\varphi_1\ A\ B\ \varphi_2$, which in this exercise will be 751672 foot-pounds.

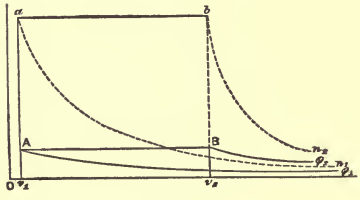


FIG. *f*.

The line $A\ B$ is an isothermal of the vapor whose equation is of the form

$$p = o \cdot v + b.$$

The value of p (or b) is given by equation (80), page 97; or by the equations on pages 413 and 414. The equation of $a\ b$ is of the same form, but for steam the last term will be about 11 to 12 times $v_1\ A$. The equations of the adiabatics $A\ \varphi_1$ and $B\varphi_2$ will be given by equation (149), page 153, or equation (a), page 184.

Page 109. EXERCISE 1.—Let t be the required temperature; then will the heat lost by the water equal that gained by the other substances; hence

$$3\,(90 - t) = 10\,(t - 30) \times 0.655 + 20\,(t - 60) \times \tfrac{1}{33}\,;\ \therefore\ t = 49.56^\circ \text{ F.}$$

Another way of solving these problems, when there is no change of state, as from solid to liquid, is to find the heat in all the substances above some assumed initial temperature, and divide the result by the heat necessary to raise the several substances one degree, as follows:

Since the substances are all liquid in this Exercise, we will treat them as if they were liquid from and above 0° F.; then will the heat units in

3 lbs. of water at 90° F. above 0° F. be $3 \times 90 \times 1 = 270$, since the specific heat of water is unity. In this way find

Sp. h.	Lbs.		B. T. U. per deg.		Deg.		B. T. U. Total.
1	× 3	=	3.000	×	90	=	270
0.655	× 10	=	6.550	×	30	=	196.5
$\frac{1}{33}$	× 20	=	0.606	×	60	=	36.36
	Total.		10.156				502.86

$$\frac{502.86}{10.156} = 49.56° \text{ F.}$$

EXERCISE 2.—In this exercise there will be a change of state from ice to water. The heat units above 0° F. will be

heat in 3 lbs. ice from 0° to 10°	$0.5 \times 3 \times 10 =$	15 heat units
" " 12 " solid water at 32°	$0.5 \times 12 \times 32 =$	192 " "
heat of fusion 12 lbs.	$12 \times 144 =$	1728 " "
heat in 12 lbs. H_2O from 32 to 60°	$12 \times 28 =$	336 " "
Total heat in the mixture above 0° F.		= 2271 " "

The heat in the whole mass (15 lbs.) above zero to water at 32° will be

$$0.5 \times 15 \times 32 + 15 \times 144 = 2400 \text{ heat units}$$

This shows that it would require $2400 - 2271 = 139$ more heat units to melt all the ice at 32°. If all be melted, then the temperature of melting would be $129 \div 15 = 8.6°$ F. below 32°, or 23.4° F. The wording of the problem should be changed in order to obtain a consistent result. Thus, let it be required to find how much ice would be melted; then, if y be the pounds melted, we have

$$3\,(32 - 10) \times 0.5 + 144\,y = 12\,(60 - 32);$$
$$\therefore y = 2.1.$$

EXERCISE 3.—In this there will be two changes of state from 0° to 225° F. We have for heat in the two substances

		B. T. U.
6 lbs. ice above 0° F.	$0.504 \times 6 \times 20 =$	60.48
1 lb. H_2O from 0° to 32°	$0.504 \times 1 \times 32 =$	16.128
plus heat of fusion	$1 \times 144 =$	144.
" from 32° to 212°	$1 \times 180 =$	180.
" heat of vaporization	$1 \times 966 =$	966.
" steam from 212 to 225	$0.48 \times 1 \times 13 =$	5.24
		1371.368

Assuming that the resultant mixture is liquid, we next find how much heat would be necessary to raise the temperature of 7 lbs. from 0° to water at 32°, we have

$$0.504 \times 7 \times 32 = 112.896 \text{ heat units.}$$
$$7 \times 144 = 1008. \quad " \quad "$$
$$\text{Heat required } 1120.896 \quad " \quad "$$

There will remain $1371.368 - 1120.896 = 250.472$ thermal units, and this would raise the temperature of the 7 lbs. of water $250.472 \div 7 = 35.75$ degrees; hence the temperature will be $35.75 + 32 = 67.75$ degrees F., and as this is less than 212 degrees it is the result sought.

EXERCISE 5.—Let x be the required number of pounds, then

$$(212 - 200)\, x = 966.$$

Page 120.—In order to reduce equations (107) and (108), it will be necessary to find v in terms of p and τ; but the equations for imperfect fluids usually being $p = f(\tau, v)$, the resolution in terms v will be more or less complex. Thus, one of the simplest forms is that frequently used in this work, $p\,v = a\,\tau - \frac{b}{\tau\,v}$, which resolved in terms of v is a radical of the second degree, and the $\frac{d^2v}{d\tau^2}$ is so complex that when multiplied by $d\,p$ the integral cannot be performed in finite terms. If, however, the equation be that given by Zeuner, equation (21), page 414, v will be found as a simple function of τ and p, and equations (107) and (108) will be more easily reduced than (105) and (106).

Page 123.—To show the geometrical signification of the quantities in the equation

$$d\,v = \left(\frac{d\,v}{d\,p}\right)_\tau d\,p + \left(\frac{d\,v}{d\,\tau}\right) d\,\tau,$$

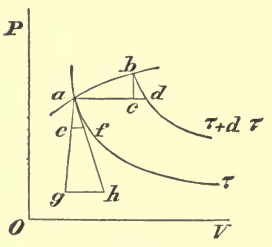


FIG. g.

let $a\,b$ be the path of the fluid. Intersect it by two consecutive isothermals, τ and $\tau + d\,\tau$, and draw $a\,d$ parallel to the v axis, $b\,c$ and $a\,g$ parallel to the p axis. Then the total differential of v, or $d\,v$ (which is the left member of the equation), will be the abscissa of b in refer-

ence to a, and the total $d\,p$ will be the ordinate of b in reference to a; hence

$$d\,v = a\,c,\ d\,p = b\,c.$$

At a draw the tangent $a\,h$, and let $a\,g = p = 1$, and make $a\,e = b\,c$ and draw $e\,f$ and $g\,h$ parallel to $a\,d$, then

$$g\,h = \left(\frac{d\,v}{d\,p}\right)_\tau;$$

that is, if the isothermal were the straight line $a\,h$, then by passing down it until $p = 1 = a\,g$, the abscissa $g\,h$ in reference to a will be as written above. It not being generally a straight line, take $a\,e = b\,c = d\,p$, then, by the similarity of triangles,

$$e\,f = \frac{g\,h}{a\,g}\,a\,e = \left(\frac{d\,v}{d\,p}\right)_\tau d\,p.$$

Since $a\,e$ is negative, being measured downward, we really have

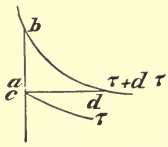
$$e\,f = \left(\frac{d\,v}{d\,p}\right)_\tau \left(-\,d\,p\right) = -\left(\frac{d\,v}{d\,p}\right)_\tau d\,p = d\,c$$

by construction. This is the first term of the second member. In the third term $\left(\frac{d\,v}{d\,\tau}\right)_p$ would be represented by $a\,d$, since p is constant, *provided* the isothermals were unity apart; but as they are only $d\,\tau$ apart we have, on the principle just given,

$$a\,d = \left(\frac{d\,v}{d\,\tau}\right)_p d\,\tau;$$

hence,

$$d\,v = a\,c = a\,d - c\,d = \left(\frac{d\,v}{d\,\tau}\right)_p d\,\tau + \left(\frac{d\,v}{d\,p}\right)_\tau \left(-\,d\,p\right).$$



If v be constant, the path of the fluid will be perpendicular to the axis of v. Let a of the preceding figure be moved to the right until it falls on c, as shown in the annexed figure, when total $d\,v$ becomes zero, and we have

$$d\,v = o, \left(\frac{d\,v}{d\,\tau}\right)_{\mathrm{p}} d\,\tau = c\,d, \left(\frac{d\,v}{d\,p}\right)_{\tau} d\,p = -\,c\,d.$$

Page 132.—The equation

$$\left(\frac{d\,H}{d\,v}\right)_{\tau} = \tau \left(\frac{d\,p}{d\,\tau}\right)_{\mathrm{v}}$$

was deduced from equation (A), p. 48, by dividing both members by $d\,v$; hence, in the right member the factor 1 should be retained in order to make the terms homogeneous, giving

$$\left(\frac{d\,H}{d\,v}\right)_{\tau} = \tau \left(\frac{d\,p}{d\,\tau}\right)_{\mathrm{v}} \cdot 1,$$

in which 1 represents unity of volume.

In representing these quantities geometrically, we will for convenience use straight lines, a method strictly correct *at* a state.

Let A be the initial state, $O\,V_2 - O\,V_1 = 1$, be the increase of volume, $A\,B$ the isothermal through A, or if the isothermal be a curved line, then $A\,B$ will be tangent to the isothermal, $A\,\varphi_1$, $B\,\varphi_2$, adiabatics indefinitely extended; then

$$\varphi_1\,A\,B\,\varphi_2 = \left(\frac{d\,H}{d\,v}\right)_{\tau}.$$

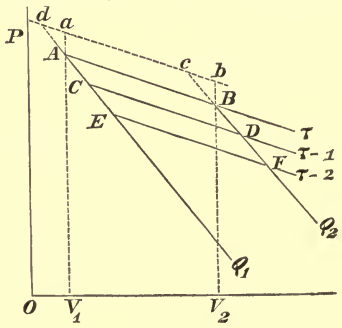


FIG. *h*.

If $a\,b$ be an isothermal one degree higher than $A\,B$, then

$$A\,a = \left(\frac{d\,p}{d\,\tau}\right)_{\mathrm{v}},$$

and

$$A\,a\,b\,B = A\,a \cdot v_1\,v_2 = \left(\frac{p\,d}{d\,\tau}\right)_{\mathrm{v}} \cdot 1.$$

If the area $\varphi_1 A B \varphi_2$ be cut by isothermals $C D$, $E F$, etc., differing by unity, then

$$A a b B = A B D C = C D F E, =, \text{ etc.},$$

and

$$\varphi_1 A B \varphi_2 = \left(\frac{d H}{d v}\right)_\tau = \tau \left(\frac{d p}{d \tau}\right)_v \cdot 1.$$

The next equation on page 132, becomes

$$\frac{d}{d \tau}\left(\frac{d H}{d v}\right)_\tau = \tau \left(\frac{d^2 p}{d \tau^2}\right)_v \cdot 1 + \left(\frac{d p}{d \tau}\right)_v \cdot 1.$$

In this case, the temperature of the working fluid is increased. Let $A a$ be the increase of pressure for an increase of temperature of one degree and $v_1 v_2 = 1$. Then $\left(\frac{d H}{d v}\right)_\tau = \varphi_1 A B \varphi_2$; and the left member is the heat necessary to be added to that of $\varphi_1 A B \varphi_2$ in order to increase the temperature one degree, and maintain it at that temperature while it works under the pressure $v_1 a$, through the space $v_1 v_2$, provided the pressure is increased uniformly with increase of temperature. The right member represents the difference of the external works during the expansions at temperature τ and $\tau + 1$ plus the difference of the internal works during the same expansions. We have

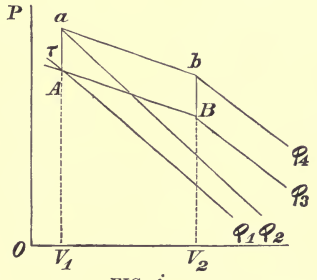


FIG. i.

$$\left(\frac{d p}{d \tau}\right)_v \cdot 1 = A a b B.$$

The difference of the internal works in expanding along the two isothermals, $A B$ and $a b$, respectively, equals the difference of the internal works along the paths $A a$ and $B b$, respectively, and the latter is as given on page 119,

$$K_A - K_B = \tau \frac{d^2 p}{d \tau^2} (v_2 - v_1 = 1) = \tau \frac{d^2 p}{d \tau^2} \cdot 1.$$

The next equation on page 132 is

$$\left(\frac{d H}{d \tau}\right)_v = C + \tau \int_\infty^v \left(\frac{d^2 p}{d \tau^2}\right)_v d v.$$

This represents the heat absorbed at constant volume for an increase of one degree of temperature. It is, as shown by the right member, the heat which makes the substance hotter, represented by C, plus that which does internal work. It may be represented on a diagram of energy by the area $\varphi_1\ A\ B\ \varphi_2$, in which $A\ B$ is the increase of pressure due to an increase unity of temperature and

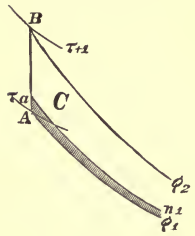


FIG. *j*.

$$C = n_1\ a\ B\ \varphi_2.$$

The sum of these two quantities is the specific heat *at* state A. The heat absorbed in raising the temperature one degree will be, making $\tau_2 = \tau + 1$ in equation (106), or

$$C + \varphi_1\ A\ a\ n_1 = C + \int_{\tau}^{\tau+1} \int_{\infty}^{v} \tau \left(\frac{d^2 p}{d \tau^2}\right)_v\ d\tau\ dv.$$

The last term, generally, cannot be integrated with τ and v, mutually dependent variables as they will be, except for isothermal expansion. The *internal* work will be the same whether the expansion be isothermal or adiabatic, if indefinitely extended; for the two paths are asymptotic to each other, and the initial states are the same, so that the two paths will form a closed cycle.

Let $A\ \tau_1$ and $B\ \tau_2$ be two isothermals differing by unity; then may the difference of the internal works in expanding along these isothermals be represented by the shaded area $A\ a m_1 \tau$, and this area will equal the shaded one in figure *j*, or

$$\tau_1\ A\ a\ m_1 = \varphi_1,\ A\ a\ n_1.$$

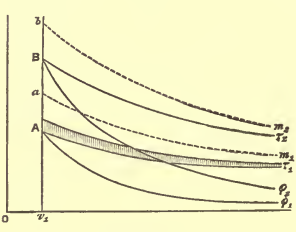


FIG. *k*.

Conceiving the expansion to be isothermal, τ will be constant during the v-integration, and v constant for the τ-integration. If the equation of the gas be $p\ v = R\ \tau - \frac{a}{\tau\ v}$, then for an increase of one degree of temperature

$$\tau_1\ A\ a\ m_1 = \frac{2\ a}{v\ \tau\ (\tau + 1)}.$$

Page 132.—In the equation

$$\frac{d}{d\ v}\left(\frac{d\ H}{d\ \tau}\right)_v = \tau \left(\frac{d^2 p}{d\ \tau}\right) \cdot 1,$$

the value $\left(\frac{d\,H}{d\,\tau}\right)_v$ is the heat absorbed for an increase of temperature of one degree (or strictly it is the rate at which heat is absorbed per degree of temperature) at constant volume. $d\left(\frac{d\,H}{d\,\tau}\right)_v$ is an elementary increase of this heat, and $\frac{d}{d\,v}\left(\frac{d\,H}{d\,\tau}\right)_v$ is the elementary amount for an increase of volume equal to unity. The value is more readily seen from the right member. Referring to equation (105), page 119, it will be seen that the right member is the heat absorbed in doing internal work for an expansion unity of volume, provided it is uniform throughout that volume. Therefore $\tau\left(\frac{d^2\,p}{d\,\tau^2}\right)_v$ may be considered as the ordinate at any point whose abscissa is v of the shaded part $\varphi_1,\ A\,a\,n_1$, or of $\tau_1\ A\,a\,m_1$, the latter of which will be used in the solution of problems since τ will then be constant.

In the equation (page 132)

$$\left(\frac{d\,H}{d\,p}\right)_\tau = -\,\tau\left(\frac{d\,v}{d\,\tau}\right)_p \cdot 1;$$

the left member is the heat absorbed during isothermal expansion for a fall of pressure equal to unity. Let $A\,B$ be an isothermal, $B\,b = 1$, then will

$$\left(\frac{d\,H}{d\,p}\right)_\tau = \varphi_1\ A\,B\,\varphi_2.$$

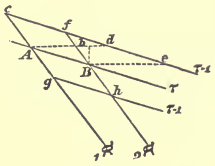


FIG. *l*.

Let $c\,e$ be an isothermal one degree higher than $A\,B$; prolong $A\,b$ to d, and draw $B\,e$ parallel to $A\,d$; then

$$A\,d = \left(\frac{d\,v}{d\,\tau}\right)_p$$

Since $b\,B$ is negative, we have

$$A\,d\,e\,B = A\,d \cdot b\,B = \left(\frac{d\,v}{d\,\tau}\right)_p \cdot (-1) = A\,c\,f\,B = A\,B\,h\,g;$$

$$\therefore\ \varphi_1\ A\,B\,\varphi_2 = \tau\left(\frac{d\,v}{d\,\tau}\right)_p (-1).$$

Page 134.—The differential of a function of any number of variables may be found by well-known rules, the result being an *exact differential*; but it is often difficult to find the primitive of a differential. A transformation is often necessary in order to render an equation or an

expression an exact differential. Thus, the first member of the equation

$$x\,y\,d\,y + (1 + y^2)\,d\,x = o,$$

is not an exact differential as it stands, for it does not satisfy the condition $\frac{d\,(x\,y)}{d\,x} = d\,\frac{(1+y^2)}{d\,y}$; but multiplying the equation by 2 x it becomes

$$2\,x^2\,y\,d\,y + 2\,(1 + y^2)\,x\,d\,x = o,$$

and then

$$\frac{d\,(2\,x^2\,y)}{d\,x} = \frac{d\,[2\,x\,(1+y^2)]}{d\,y},$$

for it reduces to

$$4\,x\,y = 4\,x\,y\,;$$

and the integral will be

$$x^2\,(1 + y^2) = c.$$

If the expression does not equal zero we may write it

$$d\,\varphi - M\,d\,x - N\,d\,y = o,$$

which is a differential equation of three variables, of which two may be independent. According to the theory of Differential Equations, the last equation can be integrated if

$$M\,d\,x + N\,d\,y = o$$

can be integrated (Boole's *Dif. Eqs.*, 4th ed., page 276). An integrating factor always exists for the latter equation, although it cannot always be found. The equation

$$d\,H = K_v\,d\,\tau + \tau\left(\frac{d\,p}{d\,\tau}\right)_v d\,v$$

on page 48 is an equation of three variables, of which τ and v are independent of each other. If

$$K_v\,d\,\tau + \tau\left(\frac{d\,p}{d\,\tau}\right)_v d\,v = o$$

is integrable, then is the former equation also integrable. It has been found by some writers that the integrating factor is $\frac{1}{\tau}$; hence,

$$d\,\varphi = \frac{d\,H}{\tau} = K_v\,\frac{d\,\tau}{\tau} + \left(\frac{d\,p}{d\,\tau}\right)_v d\,v$$

is integrable. For instance, substituting the value of K_v from page 119, it may be found that

$$\varphi_2 - \varphi_1 = C \log \tau + f(v).$$

The function of v may be found in some cases when the equation of the gas is known. (Boole's *Dif. Eqs.*, page 49.) The integral is of little value except when the temperature is constant.

If a substance be worked in a cycle the resultant internal work will be zero, and the resultant internal energy will be constant; hence the expressions

$$\frac{d\,H}{\tau}, \qquad d\,E, \qquad d\,H - p\,d\,v,$$

page 135, etc., are exact differentials.

Page 142.—The thermodynamic function of a perfect gas between finite limits becomes

$$\varphi_2 - \varphi_1 = C \log \frac{\tau_2}{\tau_1} + R \log \frac{v_2}{v_1}.$$

Let the expansion from v_1 to v_2 be isothermal along $A\ C$. Let $E\ D$ be an isothermal one degree lower than $A\ C$, $A\ \varphi_1\ C\ \varphi_3$ adiabatics; then

$$A\ C\ D\ E = R \log \frac{v_2}{v_1}.$$

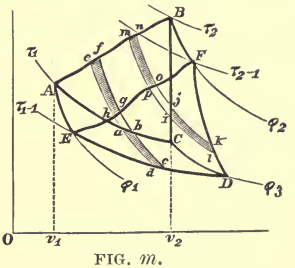


FIG. *m*.

Equation (36), page 55, makes this evident. Take F on the adiabatic through B, one degree lower than the temperature of B, and similarly for all points between F and D and draw $F\ D$; then

$$C \log \frac{\tau_2}{\tau_1} = C\ B\ F\ D.$$

Let $A\ B$ be any arbitrary path of the fluid; between A and B draw consecutive adiabatics, $e\ d$ and $f\ c$, etc., and make $e\ f\ g\ h = a\ b\ c\ d$, etc., thus determining a point g, and in a similar manner find other points as $p\ o$, etc., and draw $E\ g\ o\ F$; then

$$\varphi_2 - \varphi_1 = A\ B\ F\ E = A\ C\ D\ E + C\ B\ F\ D.$$

Page 179.—The theoretical efficiency for each of the three cases considered is about 20 per cent. But no allowances were made for clearance, condensation, nor for the imperfection of the indicator diagram. In order to reach more nearly to actual conditions, assume that clearance

reduces the efficiency 3 per cent. ; condensation, say 25 per cent. ; imperfection of the diagram, say, 4 per cent. Then will the practical efficiency be

$$0.97 \times 0.75 \times 0.96 = 0.70 \text{ nearly}$$

of the theoretical value ; hence the actual would be

$$20 \times 0.70 = 14 \text{ per cent,}$$

which has been exceeded in practice, as shown on page 221.

Page 234.—EXERCISE 2. If $\frac{7}{10}$ of the heat is abstracted by the refrigerator, then $\frac{3}{10}$ does work, and the efficiency, neglecting friction and leakage, will be 30 per cent.

If 1670000 be $\frac{3}{10}$, then will the entire heat absorbed be $1670000 \div 0.3 = 5566666$ ft. lbs. The heat removed by the refrigerator will be $250 \times 18 \times 778 = 3501000$ ft. lbs. ; hence, if the data is correct, the total heat would be $1670000 + 3501000 = 5{,}171{,}000$; and the heat unaccounted for would be $5566666 - 5171000 = 395666$ foot-pounds, which is about 7 per cent. of the whole. The data are inconsistent unless it be assumed that about 7 per cent is consumed by the friction of the engine.

During a test of a De La Vergne ice-making machine at Memphis, Tenn., from July 21st to August 10th, 1888, 1,221,172 pounds of commercial ice were produced by a consumption of 180.597 pounds of bituminous coal ; giving an average for the 20 days of 6.76 pounds of ice per pound of coal. If the coal cost $4 per 2000 pounds, then would the coal bill be $\frac{1}{3}$ of a cent per pound of ice.

Latent heat of vaporization of Ammonia, NH_3.—Dr. Von Strombeck determined this value at 32.45° F. and Regnault found the same at 53.01° F. (For the latter see a paper by Professor Jacobus in the *Trans.* of the *Am. Soc. Mech. Eng.*, vol. xii.) These results furnish a check to the theoretical formula (360), page 332. The following are the results :

At 32.45° F. the author finds by Table VI.	h_e	= 535.18
At 32.45° F. Dr. Von Strombeck finds	h_e	= 534.2
Difference		0.98

or about $\frac{1}{6}$ of one per cent.

At 53° F. the author finds	h_e	= 522.39
At 53° F. Regnault exp. gives	h_e	= 521.64
Difference		0.75

or about $\frac{1}{7}$ of one per cent.

These results indicate that Table VI. is practically exact.

Latent heat of the vaporization of Sulphur Dioxide, SO_2.

Deg. F.	Experimental results.	Formula (*c*) page 357.	Difference.
−10	170.21	173.68	3.47
0	165.06	171.26	6.20
5.74	160.88	169.92	8.04
10.50	157.18	168.56	11.38
20.	152.06	165.59	13.53

From these results it will be seen that the experimental and theoretical results depart more and more with increase of temperature. The following formula represents the results of these experiments, for the 30° here given, with sufficient accuracy

$$h_e = 177.65 - 0.385\ T,$$

in which T is degrees F. At 140° F. this gives $h_e = 123.75$, while the theoretical table gives 108.37, a difference of 15.38 heat units, or 14 per cent. It is probable, therefore, that the above formula gives more accurate results than formula (*c*), page 357; and this would diminish the computed volume of the pound of vapor in Table VII. in the same ratio, being an average of about 5 per cent. within the limits for which it is ordinarily used in practice.

The equation for the flow of steam from a straight uniform tube of large diameter into a straight uniform tube of small diameter is:

$$\frac{1}{J}\left(\frac{r^2_b}{2g} - \frac{v^2_a}{2g}\right) = Q + x_a h_a - x_b h_b + q_a - q_b + \frac{1}{J}\left(\frac{p_a - p_b}{\delta}\right) \quad (1)$$

in which J is the mechanical equivalent of heat, and g is the acceleration due to gravity; Q is the heat given to the steam at the orifice where the small tube joins the large one; r_a is the velocity in the large tube at a distance from the orifice, and r_b the velocity in the small tube also at a distance from the orifice; p_a and p_b are the pressures in the tubes A and B, p_a being the larger, and h_a and h_b are the latent heats of vaporization, and q_a and q_b the heats of the liquid corresponding; x_a is the part of one unit of weight of the fluid in the tube A which is dry steam, and $1 - x_a$ is the part which is water mingled with the steam; x_b is the corresponding quantity for the tube B; δ the weight of unity of volume of the liquid.

It is assumed that neither tube gives heat to the steam or receives heat from them, and that the friction of the fluid on the sides of the wall can be neglected. The heat Q is supposed to be given at the orifice; it is commonly assumed to be zero, in which case the flow is said to be adiabatic.

The value of x_a must be determined by experiment. x_b can then be determined by the equation:

$$\frac{x_a r_a}{r_a} + \int_{r_0}^{r_a} \frac{c\,d\,r}{r} = \frac{x\,r_b}{r_b} + \int_{r_0}^{r_b} \frac{c\,d\,r}{r}, \quad (2)$$

TEST OF A REFRIGERATING MACHINE OF THE AMMONIA COMPRESSOR TYPE, BY PROFESSOR J. E. DENTON. (*Trans. Am. Soc. Mech. Engineers*, Vol. XII, 1890.)

	OBJECT OF TEST.	Maximum Capacity and Economy at 28 lbs. Back Pressure. Exact.	Maximum Capacity and Economy at Zero, Brine, and 8 lbs. Back Pressure.	Influence of Reducing Quantity of Brine Circulated. Not Exact for Condensing Water.	Maximum Capacity and Economy for Zero, Brine, 13 lbs. Back Pressure. Exact for all Quantities.	Maximum Capacity and Economy at 27.5 lbs. Back Pressure. Exact for all Quantities.
1	DURATION OF TEST, HOURS.	24	24	13½	18	18
2	Forward Ammonia pressure above Atmosphere.. Beginning	141 lbs.	154 lbs.	119 lbs.	133 lbs.	....
3	Forward Ammonia pressure above Atmosphere.. Ending	148 "	150 "	108 "	151 "	.....
4	Forward Ammonia pressure above Atmosphere.. Average	151 "	152 "	105 "	147 "	161 lbs.
5	Back Ammonia pressure above Atmosphere..... Beginning	25 "	9 "	10 "	12 "	.....
6	Back Ammonia pressure above Atmosphere..... Ending	21 "	9 "	9 "	14 "	.....
7	Back Ammonia pressure above Atmosphere..... Average	28 "	8.2 "	7.6 "	13 "	27.5 lbs.
8	Temperature Brine Inlet. Average	36.76°	6.27°	6.4°	14.29°	.....
9	Temperature Brine Outlet ..... Beginning	27.0°	2.6°	0.1°	2.6°	36.36
10	Temperature Brine Outlet ..... Ending	29.8°	1.6°	—2.9°	1.6°	....
11	Temperature Brine Outlet ..... Average	28.86°	2.03°	—2.22°	2.29°	28.45°
12	Average range of Temperature	7.9°	4.24°	8.62°	12.00°	7.91°
13	Pounds of Brine circulated per minute.....	2281.0	2173.0	1030.0	942.8	2374.0
14	Temperature of condensing water at Inlet ..... Beginning	46.6°	56.25°	46.0°	46.0°	....
15	Temperature of condensing water at Inlet ..... Ending	48.0°	56.0°	46.0°	47.0°	....
16	Temperature of condensing water at Inlet ..... Average	44.65°	56.65°	46.0°	46.9°	54.00°
17	Temperature of condensing water at Outlet..... Beginning	80.2°	88.0°	62.0°	74.0°	.....
18	Temperature of condensing water at Outlet..... Ending	81.8°	84.0°	62.0°	87.0°	. . ..
19	Temperature of condensing water at Outlet..... Average	83.66°	85.4°	60.22°	85 46°	82.86°
20	Average range of Temperature.	39.01°	28.75°	14.22°	38.56°	28.80°
21	Pounds of water circulated per minute through Condenser.	442.0	315.0	703.0	257.0	601.5
22	Pounds of water circulated per minute through Jackets	25.0	44.0	37.0	40.0	14 0
23	Average temperature of water entering Jackets	44.65°	56.7°	46.0°	46.9°	54.3°
24	Average temperature of water leaving Jackets.....	69.0°	72 9°	60.3°	63.3°	83.45°
25	Range of temperature in Jackets	24.0°	16.2°	14.3°	16.4°	29.1°
26	Cubic feet of Liquid Ammonia circulated per hour.....	.....	24.96	25.16	28.26	48.01
27	Weight of Ammonia circulated per minute pounds	*28.17	14.68	14.84	16.67	28.32
28	Probable temperature of Liquid Ammonia, entrance to Brine tank.	*71.3°	*68.0°	*52.5°	*63.7°	76.7°

29	Pounds of Amm. per minute due to displacement of Comp. Pistons.	.....	.....	19.02	22.10	34.51
30	Difference unaccounted for by Compressor displacement.....	.....	.....	4.18	5.43	5.19
31	Degrees Fahr. of superheating by walls of Cylinder.....	.....	.....	108.°	160.°	66.°
32	Temperature of Ammonia corresponding to average back pressure.	+14.0	−8.0	−11.0	−5.0	14.0
33	Average temperature of Gas after leaving Brine tanks.....	34.2	14.7	4.7	3.0	29.2
34	Temperature of Gas entering Compressor.....	*39.°	25.°	15.°	10.13°	34.°
35	Average temperature of Gas leaving Compressor.....	213.0°	263.0°	222.0°	239.0°	221.°
36	Theoretical temperature $t = t_0\left(\frac{p_1}{p_0}\right)^{0.24}$	*229.°	304.°	258.°	260.°	237.°
37	Average temperature of gas entering Condenser	200.0°	218.0°	192.0°	209.0°	168.0°
38	Temperature due to Condensing pressure.....	84.5°	84.0°	66.0°	82.5°	88.0°
39	Temperature of Engine room.....	68.°	64.°	63.°	67.°	68.°
40	Specific Gravity of Brine.....	1.163	1.174	1.174	1.174	1.174
41	Specific heat of Brine.....	0.82	0.78	0.78	0.78	0.783
42	Barometer, inches.....	30.07	29.59	30.28	29.87	30.01
43	Strokes of Brine pump per minute.....	.....	21.66	11.06	11.04	23.55
44	Heat given to Ammonia by Brine, B. T. U. per minute.....	14776	7186	6925	8824	14647
45	Heat given to Ammonia by Compressor B. T. U. per minute.....	2786	2320	2074	2518	3020
45*a*	" " " " Atmosphere, B. T. U. " "	140	147	148	167	141
46	Total heat received by Ammonia, B. T. U.	17702	9653	9147	11409	17708
47	Heat taken from Ammonia by Condenser, B. T. U. " "	17242	9056	9097	9910	17359
48	Heat taken from Ammonia by Jackets, B. T. U. " "	608	712	529	656	406
48*a*	" " " " Atmosphere, B.T.U. " "	182	338	222	250	252
49	Total heat rejected by Ammonia, B. T. U. " "	18032	10106	10748	10816	18017
50	Difference of heat received and rejected, B. T. U. " "	330	453	1601	407	309
51	Difference in per cent. of heat received	2%	5%	17%	3.5%	2%
52	Per cent. of work of Compression removed by Jackets.....	22%	31%	25%	20%	13%
53	Boiler pressure above atmosphere, pounds.....	87.0	83.0	86.	84.	93.
54	Average revolutions per minute.....	58.09	57.7	59.2	57.88	58.89
55	Mean effective Pressure Steam Cylinder pounds per square inch..	32.5	27.17	22 866	27.83	32.97
56	Mean effective Pressure Ammonia Cylinder, pounds per square in.	65.9	53.3	47.51	59.86	70.54
57	Average Horse-power Steam Cylinder.....	85.00	71.7	61.79	73.6	88.63
58	Average Horse-power Ammonia Cylinder	65.7	54.7	48.19	59.37	71.20
59	Average Horse-power in Friction.....	19.3	17.0	13.6	14.23	17.43
60	Friction in per centum of steam horse-power	23 0	24.0	22.0	20.0	19.67
61	Total cooling water gallons per minute per ton per 24 hours.....	0 75	1.185	2.527	0 797	0.990
62	Latent Heat per lb. of ammonia for condensing pressure, B. T. U..	537.0	569.0	529.0	596.0	574.0
63	Latent heat per pound of ammonia for back pressure, B. T. U.....	577.0	535.0	592.0	508.0	538.0
64	Jacket water gallons per minute per ton in 24 hours.....	0 04	0.145	0 127	0.107	0.022
65	Condensing water gallons per minute per ton	0.71	1.04	2.40	0 69	0.968
66	Tons Ice melting capacity per 24 hours.....	74.8	36.43	35.11	44.64	74.56
67	Lbs of Ice refrig. effect per lb. of Coal at 3 lbs. per horse power...	24.1	14.1	16.03	17.27	23.37
68	Cost of Coal per 1 ton of Ice refrigerating effect at $4 00 per ton..	$0.166	$0.283	$0.245	$0.231	$0.170
69	Cost of Water per 1 ton of Ice refrig. effect at $1 per $1.000 cu. ft..	$0.128	$0.200	$0.360	$0.136	$0.169
70	Total cost of one ton of Ice, refrigerating effect.....	$0.294	$0.483	$0.605	$0.467	$0.339

* Computed.

TEST OF A PULSOMETER BY C. G. ATWATER AND CHARLES B. HODGES, OF THE CLASS OF '91, STEVENS INSTITUTE, under the supervision of the Department of Tests. The pump was taken from the ordinary stock of machines on hand, and was known as No. 6. Tests were for three hours each.

DATA AND RESULTS.	NUMBER OF THE TEST.			
	1	2	3	4
Strokes per minute	71.	60.	57.	64.
Steam pressure in pipe before throttling, lbs.	114.	110.	127.	104.3
" " " " after " "	19.	30.	43.8	26.1
" temperature after throttling, Deg. F.	270.4	277.4	309.	270.1
" amount of superheating, Deg. F.	3.1	3.	17.4	1.4
" total heat of, above highest temperature of water, B. T. U.	1118.67	1112.44	1127.	1121.2
" used as determined from temp., lbs	1617.	921.	1518.	1019.9
Water pumped, lbs.	404786.	186362.	228485.	248053.
" temp., before entering pump, Deg. F.	75.15	80.6	76.3	70.25
" temperature after leaving pump, Deg. F.	79.62	86.1	83.79	74.8
" " rise of, Deg. F.	4 47	5.5	7.49	4.55
" heat absorbed, B. T. U.	1815127.	1024993.	1704498.	1143527.
" head by gauge on lift, ft.	29.90	54.05	54.05	29.9
" " " " " suction, f.	12.26	12.26	19.67	19 67
" " " " total (H)	42.16	66.31	73.72	49 57
" " measure on lift.	25 3	50.3	50.3	25.30
" " " " " suction	7.5	7.5	16.3	16.30
" " " " total (h)	32.8	57.8	66.6	41.60
Efficiency of pump compared with total work, (h) ÷ (H)	0.777	0.877	0.911	0.839
Total work as per gauges, ft. lbs	16966886.	12335940.	16795192.	12275923.
Efficiency of the pulsometer	0.012	0.0155	0.0126	0.0138
" " plant exclusive of boiler	0.0093	0.0136	0.0115	0.0116
" " " if that of boiler and furnace be 0.7.	0.0065	0.0095	0.0080	0.0081
Duty of pump per 100 lbs. coal if 1 lb. evaporates 8 lbs. water.	8409120.	10712800.	8847200.	9629040.
Duty if 1 lb. coal evaporates 10 lbs. water	10511400.	13391000	11068500.	12036200.

Of the two tests having the highest lift (54.05 ft.), that was more efficient which had the smaller suction (12.26 ft.), and this also was the most efficient of all the tests. But in the other two tests having the same lift (29.9), that was most efficient which had the greater suction (19.67). The pressures used, 19, 30, 43.8, 26.1, were made to follow the order of total heads, but are not proportional thereto. They doubtless have much to do with the efficiency, but no attempt was made to determine the pressure which would give the highest efficiency. The first test compared with the other three is somewhat paradoxical. Thus, it is peculiar that a pressure of 19 pounds of steam should produce a greater number of strokes and pump over 50 per cent. more water than 26.1 pounds of steam, the lift being the same.

REESE LIBRARY OF THE UNIVERSITY OF CALIFORNIA

TEST OF THE NEW YORK HYÉIA ICE-MAKING PLANT, BY A. G. HUPFEL, H. E. GRISWOLD, AND WILLIAM P. MACKENZIE, FOR GRADUATING THESIS, 1893, under the supervision of the Department of Tests of Stevens Institute :

Net ice made per pound of coal in pounds	7.12
Pounds of net ice per hour per horse-power	37.8
Net ice manufactured per day (12 hours) in tons	97
Average pressure of ammonia gas at condenser in pounds per square inch above the atmosphere	135.2
Average back pressure of ammonia gas in pounds per square inch above the atmosphere	15.8
Average temperature of brine in freezing tanks in degrees Fahr.	19.7
Total number of cans filled per week	4,389
Ratio of cooling surface of coils in brine tank to can surface	7 to 10
Ratio of brine in tanks to water in cans	1 to 1.2
Ratio of circulating water at condensers to distilled water	26 to 1
Pounds of water evaporated at boilers per pound of coal	8.085
Total horse-power developed by compressor engines	444
Percentage of ice lost in removing from cans	2.2

APPROXIMATE DIVISION OF STEAM IN PER CENTS. OF TOTAL AMOUNT.

Compressor engines	60.1
Live steam admitted directly to condensers	19.7
Steam for pumps, agitator and elevator engines	7.6
Live steam for reboiling distilled water	6.5
Steam for blowers furnishing draught at boilers	5.6
Sprinklers for removing ice from cans	0.5

EXERCISES.

1. Required the specific heat of air at 500° F. absolute, the path being $p = 10\ v$ (p, pounds per square foot ; v, volume of a pound in cubic feet). The specific heat is the heat absorbed in raising one pound one degree.

2. If the equation of a superheated vapor be $p\ v = R\ \tau - C\ p^n$ (see p. 414) ; required the heat absorbed at the constant pressure p_1 in expanding from v_1 to v_2.

3. If the equation of the gas be $p\ v = R\ \tau - \frac{a}{\tau\ v}$ and of the path of the fluid, $p = m\ v + n$; required the heat absorbed in expanding from state $v_1 = 12$, $p_1 = 4000$ to state $v_2 = 24$, $p_2 = 5000$.

4. Find the internal and the external work of expanding the gas

$p\,v = R\,\tau - \frac{a}{\tau\,v^2}$ at the constant pressure p_1 from v_1 to v_2, and leave the final result without τ.

5. Find the thermodynamic function for the gas whose equation is $p\,v = R\,\tau - \frac{a}{\tau\,v^2}$ for isothermal expansion.

6. Find the velocity of discharge of a perfect gas from an orifice, the temperature remaining constant. Also the weight per second.

7. Write a formula for the pressure of a saturated vapor in terms of the volume of a pound.

8. Required the difference in the elevation of two stations at which water boils at atmospheric pressure respectively at 212° F. and 180° F.

9. A vessel containing two cubic feet of fluid, one-fourth of which by weight is steam and the remainder water; required the work necessary to compress the vapor to water—in one case adiabiatically, and in another isothermally.

10. A frictionless piston, in an upright cylinder, rests on a pound of water. Heat is absorbed under a pressure of 6 atmospheres absolute until the piston has swept over two cubic feet, then expansion is adiabatic until the pressure is reduced to two atmospheres absolute, then compressed isothermally until by adiabatic compression the vapor will be reduced to water at the initial pressure. Required (1) the heat absorbed; (2) the clearance; (3) the entire stroke of the piston; (4) the heat emitted during compression; (5) the work done in the cycle; (6) the efficiency of the cycle.

11. Find difference of external works in expanding a gas adiabatically and isothermally from v_1 to $2\,v_1$.

12. Find heat absorbed by a liquid in raising the temperature from 40° to 200°, if specific heat $c = 1 + a\,T^2$. If $c = 1 - a\,(T - T_0)$. If $c = 1 - a\,(T - T_0)^2$.

13. What is the specific heat of a gas at constant temperature?

14. Find the thermodynamic function for air from state τ_1, p_1 to τ_2, p_2, independently of v.

15. A single-acting engine (vertical) has one pound of fluid (water) at the lower end, on which rests a frictionless piston. By the absorption of heat the piston is raised against an absolute pressure of 9 atmospheres until the volume swept over by the piston is twice the volume of the dry saturated vapor at that pressure, then it expands adiabatically as a perfect gas until the pressure is reduced to 3 atmospheres, then refrigerated at constant volume until the pressure is reduced to 2 atmospheres, then refrigerated at constant pressure until the volume is such that when compressed adiabatically it will be reduced to a liquid at 7 atmospheres, and its temperature then raised under a pressure of 9 atmospheres to the initial state.

Find :

1. Indicator diagram.
2. Volume at the beginning of adiabatic expansion.
3. Degrees of superheating.
4. Volume swept over by the piston.
5. Ratio of expansion.
6. Temperature at end of expansion.
7. Temperature at beginning of back stroke.
8. Degrees of superheating on back stroke.
9. Volume at end of refrigeration, so that when compressed adiabatically it will all be reduced to liquid at 7 atmospheres.
10. Temperature at end of back stroke when fluid is reduced to water.
11. Clearance.
12. Work done in the cycle.
13. Mean effective pressure.
14. Mean forward pressure.
15. Heat absorbed in the cycle.
16. Heat emitted.
17. Pounds of coal necessary to supply the heat for 100 cycles, if each pound contains 14,500 heat units.
18. Pounds of water necessary to produce the refrigeration for 100 cycles—temperature ranging from 60° to 80°.
19. If length of stroke equals diameter of cylinder, find diameter of cylinder
20. The pounds of water necessary to develop a horse power if piston speed be 200 feet per minute.
21. The pounds of coal necessary to produce the horse power if a pound of coal has 1200 heat units and the efficiency of the furnace be 0.70.

16. A variety of exercises may be made similar to the preceding. We suggest the following : Let the volume of the vapor be the fraction of a pound. Let the expansion be entirely with saturated vapor and adiabatic. Let it be the curve of saturation, or a straight line entirely within the curve of saturation, or entirely without (superheated) ; or a line crossing the curve of saturation. Let the indicator card be entirely within the curve of saturation and bounded by any assumed lines.

17. A pound of water at 60° F. is within a closed vessel containing two cubic feet ; required the heat absorbed by the fluid in raising its temperature 335° F.

From pp. 146 and 147 :

$$d\,H = C\,d\,\tau + x\,d\,H_e - \frac{H_e}{\tau}\,d\,\tau + H_e\,d\,x.$$

Let

v = volume of mixture of liquid and vapor = 2 cubic feet.
v_2 = specific volume of 1 pound vapor (see table).
v_1 = specific volume of liquid = 0.017 nearly for water.

Then,

$$x = \frac{v - v_1}{v_2 - v_1} = (v - v_1)\frac{\tau \frac{d\,p}{d\,\tau}}{a - b\,\tau}$$

$$v_2 - v_1 = \frac{H_e}{\tau\left(\frac{d\,p}{d\,\tau}\right)} \text{ (Eq. (84))}; \; H_e = a - b\,\tau,$$

$$d\,x = \frac{v - v_1}{(v_2 - v_1)^2}\,d\,v_2\,; \; d\,v_2 = \frac{d\,H_e}{\tau\left(\frac{d\,p}{d\,\tau}\right)} - \frac{H_e\,d\left(\tau\left(\frac{d\,p}{d\,\tau}\right)\right)}{\tau^2\left(\frac{d\,p}{d\,\tau}\right)^2}.$$

Reduce and find,

$$H = C(\tau_2 - \tau_1) + (v - v_1)\left[-\frac{H_{e1}\,\tau_2}{H_{e2}\,\tau_1}\,log\frac{\frac{d\,p_2}{d\,\tau_2}}{\frac{d\,p_1}{d\,\tau_1}}\right] - a\,(v - v_1)\int_{\tau_1}^{\tau_2}\frac{\frac{d\,p}{d\,\tau}\,d\,\tau}{a - b}.$$

18. If gas flows from one vessel into another through a short pipe, what must be the cross-section of the pipe that q pounds will flow in t seconds at constant temperature?

Let Q and Q' be the volumes of the vessels, k the section, P' and p' the initial pressure, $P' > p'$, P and p the pressures at time t, μ the coefficient of velocity, V the velocity at the time t, and w the weight per cubic foot.

Then,

$$V = \sqrt{\frac{2\,p_0}{\delta_0}\,log\,\frac{P}{p}}.$$

$$d\,q = \mu\,k\,w\,V\,d\,t.$$

$$P\,Q + p\,Q' = P'\,Q + p'\,Q'.$$

$$w = \frac{w_0}{p_0}P.$$

$$\frac{w_0}{p_0}P'\,Q - q = \frac{w_0}{p_0}P\,Q.$$

$$\therefore P = P' - \frac{p_0}{w_0\,Q}q.$$

These give

$$\mu k g \sqrt{\frac{2\delta_0}{p_0}}\, dt = \frac{dq}{\left[P' - \frac{p_0}{Q w_0} q\right] \cdot \left[\log Q' \frac{P' - \frac{p_0}{Q w_0} q}{p' Q' - \frac{p_0}{w_0} q}\right]^{\frac{1}{2}}}$$

which integrated between o and t and o and q will give an equation from which k may be found.

19. Find the path of a fluid considered as a perfect gas when the heat absorbed is n times the work done.

Make $n p\, dv = dH$ in equations in p. 50, and find

$$p v^{\frac{\gamma - n(\gamma - 1)}{}} = B \text{ (a constant).}$$

Discuss making $n = 1, 2, 5{\cdot}9, \infty$. What value of n makes the path a right line?

Specific Heat of Aqua Ammonia.—The mean of six determinations by Ludeking and Starr gives 0.886 (Am. Jour. Arts and Sc.). The value found by Hans von Strombuck was 1.22876 (page 337), which is nearly 50 per cent. larger than the above value. The value found by theory is nearly the mean of the two. The above experimenters inform the author that they are not aware of any error in their own work, neither do either know of an error in the work of the other. This leaves the correct value in doubt, and one may consider it as unity until determined by further experiments.

N	0	1	2	3	4	5	6	7	8	9	D
10	0000	043	086	128	170	212	253	294	334	374	42
11	414	453	492	531	569	607	645	682	719	755	38
12	792	828	864	899	934	969	1004	1038	1072	1106	35
13	1139	173	206	239	271	303	335	367	399	430	32
14	461	492	523	553	584	614	644	673	703	732	30
15	1761	790	818	847	875	903	931	959	987	2014	28
16	2041	068	095	122	148	175	201	227	253	279	26
17	304	330	355	380	405	430	455	480	504	529	25
18	553	577	601	625	648	672	695	718	742	765	24
19	788	810	833	856	878	900	923	945	967	989	22
20	3010	032	054	075	096	118	139	160	181	201	21
21	222	243	263	284	304	324	345	365	385	404	20
22	424	444	464	483	502	522	541	560	579	598	19
23	617	636	655	674	692	711	729	747	766	784	19
24	802	820	838	856	874	892	909	927	945	962	18
25	3979	997	4014	4031	4048	4065	4082	4099	4116	4133	17
26	4150	166	183	200	216	232	249	265	281	298	16
27	314	330	346	362	378	393	409	425	440	456	16
28	472	487	502	518	533	548	564	579	594	609	15
29	624	639	654	669	683	698	713	728	742	757	15
30	4771	786	800	814	829	843	857	871	886	900	14
31	914	928	942	955	969	983	997	5011	5024	5038	14
32	5051	065	079	092	105	119	132	145	159	172	13
33	185	198	211	224	237	250	263	276	289	302	13
34	315	328	340	353	366	378	391	403	416	428	13
35	5441	453	465	478	490	502	514	527	539	551	12
36	563	575	587	599	611	623	635	647	658	670	12
37	682	694	705	717	729	740	752	763	775	786	12
38	798	809	821	832	843	855	866	877	888	899	11
39	911	922	933	944	955	966	977	988	999	6010	11
40	6021	031	042	053	064	075	085	096	107	117	11
41	128	138	149	160	170	180	191	201	212	222	10
42	232	243	253	263	274	284	294	304	314	325	10
43	335	345	355	365	375	385	395	405	415	425	10
44	435	444	454	464	474	484	493	503	513	522	10
45	6532	542	551	561	571	580	590	599	609	618	10
46	628	637	646	656	665	675	684	693	702	712	9
47	721	730	739	749	758	767	776	785	794	803	9
48	812	821	830	839	848	857	866	875	884	893	9
49	902	911	920	928	937	946	955	964	972	981	9
50	6990	998	7007	7016	7024	7033	7042	7050	7059	7067	9
51	7076	084	093	101	110	118	126	135	143	152	8
52	160	168	177	185	193	202	210	218	226	235	8
53	243	251	259	267	275	284	292	300	308	316	8
54	324	332	340	348	356	364	372	380	388	396	8

N	0	1	2	3	4	5	6	7	8	9	D
55	7404	412	419	427	435	443	451	459	466	474	8
56	482	490	497	505	513	520	528	536	543	551	8
57	559	566	574	582	589	597	604	612	619	627	8
58	634	642	649	657	664	672	679	686	694	701	7
59	709	716	723	731	738	745	752	760	767	774	7
60	7782	789	796	803	810	818	825	832	839	846	7
61	853	860	868	875	882	889	896	903	910	917	7
62	924	931	938	945	952	959	966	973	980	987	7
63	993	8000	8007	8014	8021	8028	8035	8041	8048	8055	7
64	8062	069	075	082	089	096	102	109	116	122	7
65	8129	136	142	149	156	162	169	176	182	189	7
66	195	202	209	215	222	228	235	241	248	254	7
67	261	267	274	280	287	293	299	306	312	319	6
68	325	331	338	344	351	357	363	370	376	382	6
69	388	395	401	407	414	420	426	432	439	445	6
70	8451	457	463	470	476	482	488	494	500	506	6
71	513	519	525	531	537	543	549	555	561	567	6
72	573	579	585	591	597	603	609	615	621	627	6
73	633	639	645	651	657	663	669	675	681	686	6
74	692	698	704	710	716	722	727	733	739	745	6
75	8751	756	762	768	774	779	785	791	797	802	6
76	808	814	820	825	831	837	842	848	854	859	6
77	865	871	876	882	887	893	899	904	910	915	6
78	921	927	932	938	943	949	954	960	965	971	6
79	976	982	987	993	998	9004	9009	9015	9020	9025	5
80	9031	036	042	047	053	058	063	069	074	079	5
81	085	090	096	101	106	112	117	122	128	133	5
82	138	143	149	154	159	165	170	175	180	186	5
83	191	196	201	206	212	217	222	227	232	238	5
84	243	248	253	258	263	269	274	279	284	289	5
85	9294	299	304	309	315	320	325	330	335	340	5
86	345	350	355	360	365	370	375	380	385	390	5
87	395	400	405	410	415	420	425	430	435	440	5
88	445	450	455	460	465	469	474	479	484	489	5
89	494	499	504	509	513	518	523	528	533	538	5
90	9542	547	552	557	562	566	571	576	581	586	5
91	590	595	600	605	609	614	619	624	628	633	5
92	638	643	647	652	657	661	666	671	675	680	5
93	685	689	694	699	703	708	713	717	722	727	5
94	731	736	741	745	750	754	759	763	768	773	5
95	9777	782	786	791	795	800	805	809	814	818	5
96	823	827	832	836	841	845	850	854	859	863	5
97	868	872	877	881	886	890	894	899	903	908	4
98	912	917	921	926	930	934	939	943	948	952	4
99	956	961	965	969	974	978	983	987	991	996	4

Proportional Parts.

D	1	2	3	4	5	6	7	8	9
42	4.2	8.4	12.6	16.8	21.0	25.2	29.4	33.6	37.8
38	3.8	7.6	11.4	15.2	19.0	22.8	26.6	30.4	34.2
35	3 5	7.0	10.5	14.0	17.5	21.0	24.5	28.0	31.5
32	3.2	6.4	9.6	12.8	16.0	19.2	22.4	25.6	28.8
30	3.0	6.0	9.0	12.0	15.0	18.0	21.0	24.0	27.0
28	2.8	5.6	8.4	11.2	14.0	16.8	19.6	22.4	25.2
26	2.6	5.2	7.8	10.4	13.0	15.6	18.2	20.8	23.4
25	2.5	5.0	7.5	10.0	12.5	15.0	17.5	20.0	22.5
24	2.4	4.8	7.2	9.6	12.0	14.4	16.8	19.2	21.6
22	2.2	4.4	6.6	8.8	11.0	13.2	15.4	17.6	19.8
21	2.1	4.2	6.3	8.4	10.5	12.6	14.7	16.8	18.9
20	2.0	4.0	6.0	8.0	10.0	12.0	14.0	16.0	18.0
19	1.9	3.8	5.7	7.6	9.5	11.4	13.3	15.2	17.1
18	1.8	3.6	5.4	7.2	9.0	10.8	12.6	14.4	16.2
17	1.7	3.4	5.1	6.8	8.5	10.2	11.9	13.6	15.3
16	1.6	3.2	4.8	6.4	8.0	9.6	11.2	12.8	14.4
15	1.5	3.0	4.5	6.0	7.5	9.0	10.5	12.0	13.5
14	1.4	2.8	4.2	5.6	7.0	8.4	9.8	11.2	12.6
13	1.3	2.6	3.9	5.2	6.5	7.8	9.1	10.4	11.7
12	1.2	2.4	3.6	4.8	6.0	7.2	8.4	9.6	10.8
11	1.1	2.2	3.3	4.4	5.5	6.6	7.7	8.8	9.9
10	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0
9	.9	1.8	3.7	3.6	4.5	5.4	6.3	7.2	8.1
8	.8	1.6	2.4	3.2	4.0	4.8	5.6	6.4	7.2
7	.7	1.4	2.1	2.8	3.5	4.2	4.9	5.6	6.3
6	.6	1.2	1.8	2.4	3.0	3.6	4.2	4.8	5.4
5	.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
4	.4	.8	1.2	1.6	2.0	2.4	2.8	3.2	3.6
3	.3	.6	.9	1.2	1.5	1.8	2.1	2.4	2.7
2	.2	.4	.6	.8	1.0	1.2	1.4	1.6	1.8

Modulus of the common system = 0.4342945.

$$log_e 10 = 2.302585 = \frac{1}{0.4342945}.$$

$$e = 2.71828128$$

$$log_{10} e = 0.4342945.$$

$$log_e N = 2.302585\ log_{10} N.$$

TABLE II.

REDUCTION TABLES.

Conversion of English and Metric Units.

1 Foot = 0.3048 metre.
1 Litre (vol. of 1 kilog. water) = 0.2202 gal.
1 Gallon (vol. of 10 lbs. water) = 4.541 litres.
1 Kilogramme per sq. mm. = 1422.28 lbs. per sq. in.
1 Lb. per sq. inch = 703.0958 kilog. per sq. metre.
1 Grain = 0.0648 gram.
1 Foot-pound = 0.1383 metre-kilog.
1 Atmosphere = 14.7 lbs. per sq. in. = 10334 kilog. per sq. metre = 29.922 inches, or 760 mm. of mercury = 33.9 ft., or $10\frac{1}{3}$ metres of water.
1 Pound av. = 0.4536 kilog.
1 Calorie (kilog. water raised 1° C.) = 424 metre-kilog. = 3.9683 B. T. U.
1 Eng. heat unit (lb. water raised 1° F.) = 778 ft.-lbs. = 0.254 calorie.

French Measures in Equivalent English Measures.

MEASURES OF LENGTH.

1 Millimetre = 0.03937079 inch, or about $\frac{1}{25}$ inch.
1 Centimetre = 0.3937079 inch, or about 0.4 inch.
1 Decimetre = 3.937079 inches.
1 Metre = 39.37079 inches = 3.28 feet nearly.
1 Kilometre = 39370.79 inches.

MEASURES OF AREA.

1 sq. decimetre = 15.5006 sq. inches.
1 sq. metre = 1550.06 sq. inches, or 10.764 sq. ft.

MEASURES OF WEIGHT.

1 Gramme = 15.432349 grains.
1 Kilogramme = 15432.349 grains, 2.2046 lbs., or 2.2 lbs. nearly.

TWO UNITS INVOLVED.

1 Gramme per sq. centimetre = 2.048098 lbs. per sq. foot.
1 Kilogramme per sq. metre = 0.2048098 " "
1 Kilogramme per sq. millimetre = 2.048098 " "
1 Kilogramme metre = 7.23314 ft.-lbs.
= $7\frac{1}{4}$ ft. lbs. nearly.

1 force de cheval = 75 kilogrammetres per second, or $542\frac{1}{2}$ foot-pounds per second nearly. 1 horse-power = 550 foot pounds per second.

TABLE III.

Liquids and Solids.

	Weight of a cubic foot of the substance.	Specific Gravity.	Expansion. —Liquids per unit of volume, Solids per unit of length, from 32° to 212° F.	Specific Heat.
	W.	S. G.	E.	C.
Water, pure (at 39°.1 F.).	62.425	1.000	0.04775	1.000
" sea, ordinary.....	64.05	1.026	0.05	....
Alcohol, pure.....	49.38	0.791	0.1112	0.622
" proof spirit.....	57.18	0.916	...	...
Ether.....	44.70	0.716	....	0.517
Mercury.....	848.75	13.596	0.018153	0.033
Naphtha.....	52.94	0.848	....	0.434
Oil, linseed.....	58.68	0.940	0.08	...
" olive.....	57.12	0.915	0.08	....
" whale.....	57.62	0.923	....	....
" of turpentine.....	54.31	0.870	0.07	...
Petroleum.....	54.81	0.878	....	.434
Ice.....	57.5	0.92	....	.504
Brass...	487 to 533	7.8 to 8.5	.00216	....
Bronze	524	8.4	.00181	....
Copper	537 to 556	8.6 to 8.9	.00184	.0951
Gold	1186 to 1224	19 to 19.6	.0015	.0298
Iron, cast.....	444	7.11	.0011	....
Iron, wrought	480	7.69	.0012	.1138
Lead	712	11.4	.0029	.0293
Platinum	1311 to 1373	21 to 22	.0009	.0314
Silver.....	655	10.5	.002	.0557
Steel.....	490	7.85	.0012	....
Tin.....	462	7.4	.0022	.0514
Zinc.....	436	7.2	.00294	.0927

TABLE IV.

Properties of Gases Considered Perfect.

I. Name of the Gas.	II. Chemical Composition.	III. Density.	IV. Specific Heat at Constant Pressure		V. Specific Heat at Constant Volume	
			compared Weight for Weight with Water.	compared Volume for Volume with Air.	compared Weight for Weight with Water.	compared Volume for Volume with Air
Atmospheric Air		1	0.2375	1	0.1689	1
Oxygen	O_2	1.1056	0.21751	1.013	0.1551	1.018
Nitrogen	N_2	0.9713	0.24380	0.997	0.1727	0.996
Hydrogen	H_2	0.0692	3.40900	0.993	2.411	0.990
Chlorine	Cl_2	2.4502	0.12099	1.248	0.0928	1.350
Bromine	Br_2	5.4772	0.05552	1.280	0.0429	1.395
Nitric Oxide	N O	1.0384	0.2317	1.013	0.1652	1 018
Carbonic Oxide	C O	0.9673	0.2450	0.998	0.1736	0.997
Hydrochloric Acid	H Cl	1.2596	0.1852	0.982	0.1304	0.975
Carbonic Acid	$C O_2$	1.5201	0.2169	1.39	0.172	1.55
Nitric Acid	$N_2 O$	1.5241	0.2262	1.45	0.181	1.64
Steam	$H_2 O$	0.6219	0.4805	1.26	0.370	1.36
Sulphuric Acid	$S O_2$	2.2113	0.1544	1.44	0.123	1.62
Hydro-sulphuric Acid	$H_2 S$	1.1747	0.2432	1.20	0.184	1.29
Carbonic Di-sulphide	$C S_2$	2.6258	0.1569	1.74	0.131	2.04
Carburetted Hydrogen	$C H_4$	0.5527	0.5929	1.38	0.468	1.54
Chloroform	$C H Cl_3$	4.1244	0.1567	2.72	0.140	3.43
Olefiant Gas	$C_2 H_4$	0.9672	0.4040	1.75	0.359	2.06
Ammonia Gas	$N H_3$	0.5894	0.5084	1.26	0.393	1.37
Benzine	$C_6 H_6$	2.6942	0.3754	4.26	0.350	5 60
Oil of Turpentine	$C_{10} H_{16}$	4.6978	0.5061	10.01	0.491	13.71
Wood Spirit	$C H_4 O$	1.1055	0.4580	2.13	0.395	2.60
Alcohol	$C_2 H_6 O$	1.5890	0.4534	3.03	0.410	3.87
Ether	$C_4 H_{10} O$	2.5573	0.4797	5.16	0.453	6.87
Ethyl Sulphide	$C_4 H_{10} S$	3.1101	0.4008	5.25	0.379	6.99
Ethyl Chloride	$C_2 H_5 Cl$	2.2269	0.2738	2.57	0.243	3 21
Ethyl Bromide	$C_2 H_5 Br$	3.7058	0.1896	2.96	0.171	3.76
Dutch Liquid	$C_2 H_4 Cl_2$	3.4174	0.2293	3.30	0.209	4.24
Aceton	$C_3 H_6 O$	2.0036	0.4125	3.48	0.378	4.50
Butyric Acid	$C_4 H_8 O_2$	3.0400	0.4008	5.13	0.378	6.82
Tri chloride of Silicon	$Si Cl_3$	5.8833	0.1322	3.27	0.120	4.21
Tri-chloride of Phosphorus	$P Cl_3$	4.7464	0.1347	2.69	0.120	3.39
Tri-chloride of Arsenic	$As Cl_3$	6.2667	0.1122	2.96	0.101	3.77
Tetra-chloride of Titanium	$Ti Cl_4$	6.6402	0.1290	3.61	0.119	4.67
Tetra-chloride of Tin	$Sn Cl_4$	8.9654	0.0939	3.54	0.086	4.59

TABLE V.

Saturated Steam.

p, Pressure per square inch.

T, Temperature degrees F., Eq. (81), page 97.

v, Volume of a pound of saturated steam in cubic feet, equation (84), in which p is pounds per *square foot.*

w, Weight of a cubic foot of steam $= \frac{1}{v}$.

h, The heat in one pound of liquid above 32° F. in thermal units, equation (95), page 111. The values, however, are the direct results of Regnault's experiments.

h_e, The latent heat of evaporation in thermal units, equation (10), page 409.

h, The total heat of steam above 32° F. being equal to $h + h_e$.

SATURATED STEAM.

p.	T.	v.	w.	h.	he.	**h.**
0.089	32.0	3387.	0 00029	0.0	1091.7	1091.7
0.2	54.0	1482.	0.00066	22.1	1076.3	1098.4
1	102.0	335.	0.00299	70.0	1043.0	1113.1
5	162.3	73.	0.01366	130.7	1000.8	1131.5
10	193.2	38.	0.02621	161.9	979 0	1140.9
15	213.0	26.2	0.03826	181.8	965.1	1146.9
20	228.0	19.9	0.05023	196.9	954.6	1151.5
25	240.0	16.1	0.06199	209.1	946.0	1155.1
30	250.3	13.6	0.07360	219.4	938.9	1158.3
35	259.2	11.8	0.08508	228.4	932.6	1161.0
40	267.1	10.4	0.09644	236.4	927.0	1163.4
45	274.3	9.3	0.1077	243.6	922.0	1165.6
50	280.9	8.4	0.1188	250.2	917.4	1167.6
55	286.9	7.7	0.1299	256.3	913.1	1169.4
60	292.5	7.1	0.1409	261.9	909.3	1171.2
65	297.8	6.6	0.1519	267.2	905.5	1172.7
70	302.7	6.15	0.1628	272.2	902.1	1174.3
75	307.4	5.70	0.1736	276.9	898.8	1175.7
80	310.9	5.49	0.1843	281.4	895.6	1177.0
85	315.2	5.18	0.1951	285.8	892.5	1178.3
90	320.0	4.858	0.2058	290.0	889.6	1179.6
95	323.9	4.619	0.2165	294.0	886.7	1180.7
100	327.6	4.403	0.2271	297.9	884.0	1181.9
105	331.1	4.206	0.2378	301.6	881.3	1182.9
110	334.56	4.026	0.2488	305.2	878.8	1184.0
115	337.86	3.862	0.2589	308.7	876.3	1185.0
120	341.05	3.711	0.2695	312.0	874.0	1186 0

SATURATED STEAM—*Continued.*

p.	*T.*	*v.*	*w.*	*h.*	*he.*	**h.**
125	344.13	3.572	0.2800	315.2	871.7	1186.9
130	347.12	3.444	0.2904	318.4	869.4	1187.8
135	350.03	3.323	0.3009	321.4	867.3	1188.7
140	352.85	3.212	0.3113	324.4	865.1	1189.5
145	355.59	3.107	0.3218	327.2	863.2	1190.4
150	358.26	3.011	0.3321	330.0	861.0	1191.2
155	360.86	2.919	0.3426	332.7	859 3	1192.0
160	363.40	2.833	0.3530	335.4	857.4	1192.8
165	365.88	2.751	0.3635	338.0	855.6	1193.6
170	368.29	2 676	0.3737	340.5	853.8	1194.3
175	370.65	2.603	0.3841	343.0	852.0	1195 0
180	372.97	2.535	0.3945	345.4	850.3	1195.7
185	375.23	2.470	0.4049	347.8	848.6	1196.4
190	377.44	2.408	0.4153	350.1	847.0	1197.1
195	379.61	2.349	0.4257	352.4	845.3	1197.7
200	381.73	2.294	0.4359	354.6	843.8	1198.4
205	383.82	2.241	0.4461	356.8	842.2	1199.0
210	385.87	2.190	0.4565	358.9	840.7	1199.6
215	387.88	2.142	0.4669	361.0	839.2	1200.2
220	389.84	2.096	0.4772	363.0	837.8	1200.8
225	391.79	2.051	0.4876	365.1	836.3	1201.4
230	393 69	2.009	0.4979	367.1	834.9	1202.0
235	395.56	1.968	0.5082	369.0	833.6	1202.6
240	397.41	1.928	0.5186	371.0	832.2	1203.2
245	399.21	1.891	0.5289	372.8	830.9	1203.7
250	400.99	1.854	0.5393	374.4	829.5	1204.2
255	402.74	1.819	0.5496	376.5	828.3	1204.8
260	404.47	1.785	0.5601	378.4	826.9	1205.3
265	406.17	1.753	0.5705	380.2	825.6	1205.8
270	407 85	1.722	0.5809	381.9	824.4	1206.3
275	409.50	1.691	0.5913	383.6	823.2	1206.8
280	411.12	1.662	0.6020	385.3	821.8	1207.1
285	412.72	1.634	0.612	387.0	820.6	1207 6
290	414.32	1.607	0.622	388.6	819.5	1208.1
295	415.87	1.580	0.633	390.3	818.3	1208.6
300	417.42	1.554	0.644	391.9	817.2	1209.1
305	418.92	1.529	0.654	393.5	816.1	1209.5
310	420.42	1.505	0.664	395.0	815.1	1210.1
315	421.92	1.481	0.675	396.6	814.0	1210.6
320	423 37	1.459	0.685	398.1	813.0	1211.1
325	424.82	1.437	0.696	399.6	811.7	1211.3
330	426.24	1.415	0.707	401.1	810.6	1211.7
335	427.64	1.395	0.717	402.6	809.5	1212.1

TABLE VI.

Properties of the Saturated Vapor of Ammonia.

Temperature.		Pressure, Absolute, p.		Heat of Vaporization, Thermal Units. Eq. (360). h_e.	External Heat, Thermal Units. $\frac{p\,(v - v_1)}{J}$.	Internal Heat, Thermal Units. $p = h_e - \frac{p\,v}{J}$.	Volume of Vapor per lb., cu. ft. Eq (362). v.	Volume of Liquid per lb., cu. ft. v_1.	Weight of a cu. ft. of Vapor, Pounds. $\frac{1}{v}$.
Degree F. T.	Absolute. τ.	Lbs. per sq. ft. Eq. (350).	Lbs. per sq. in.						
— 40	420.66	1540.7	10.69	579.67	48.23	531.44	24.372	.0234	.0410
— 35	425.66	1773.6	12.31	576.69	48.48	528.21	21.319	.0236	.0468
— 30	430.66	2035.8	14.13	573.69	48.77	524.92	18.697	.0237	.0535
— 25	435.66	2329.5	16.17	570.68	49.06	521.62	16.445	.0238	.0608
— 20	440.66	2657.5	18.45	567.67	49.38	518.29	14.507	.0240	.0689
— 15	445.66	3022.5	20.99	564.64	49.67	514.97	12.834	.0242	.0779
— 10	450.66	3428.0	23.80	561.61	49.99	511.62	11.384	.0243	.0878
— 5	455.66	3877.2	26.93	558.56	50.31	508.25	10.125	.0244	.0988
0	460.66	4373.5	30.37	555.50	50.68	504.82	9.027	.0246	.1108
+ 5	465.66	4920.5	34.17	552.43	50.84	501.59	8.069	.0247	.1239
+ 10	470.66	5522.2	38.34	549.35	51.13	498.22	7.229	.0249	.1383
+ 15	475.66	6182.4	42.93	546.26	51.33	494.93	6.492	.0250	.1544
+ 20	480.66	6905.3	47.95	543.15	51.61	491.54	5.842	.0252	.1712
+ 25	485.66	7695.2	53.43	540.03	51.80	488.23	5.269	.0253	.1898
+ 30	490.66	8556.6	59.41	536.92	52.01	484.91	4.763	.0254	.2100

+ 35	495.66	9493.9	65.93	533.78	52.22	481.56	4.313	.0256	.2319
+ 40	500.66	10512	73.00	530.63	52.42	478.21	3.914	.0257	.2555
+ 45	505.66	11616	80.66	527.47	52.62	474.85	3.559	.0259	.2809
+ 50	510.66	12811	88.96	524.30	52.82	471.48	3.242	.0261	.3085
+ 55	515.66	14102	97.93	521.12	53.01	468.11	2.958	.0263	.3381
+ 60	520.66	15494	107.60	517.93	53.21	464.72	2.704	.0265	.3698
+ 65	525.66	16993	118.03	514.73	53.38	461.35	2.476	.0266	.4039
+ 70	530.66	18605	129.21	511.52	53.57	457.85	2.271	.0268	.4403
+ 75	535.66	20336	141.25	508.29	53.76	454.53	2.087	.0270	.4793
+ 80	540.66	22192	154.11	505.05	53.96	450.70	1.920	.0272	.5208
+ 85	545.66	24178	167.86	501.81	54.15	447.66	1.770	.0273	.5650
+ 90	550.66	26300	182.8	498.11	54.28	443.83	1.632	.0274	.6128
+ 95	555.66	28565	198.37	495.29	54.41	440.88	1.510	.0277	.6623
+ 100	560.66	30980	215.14	492.01	54.54	436.96	1.398	.0279	.7153
+ 105	565.66	33550	232.98	488.72	54.67	434.08	1.296	.0281	.7716
+ 110	570.66	36284	251.97	485.42	54.78	430.64	1.203	.0283	.8312
+ 115	575.66	39188	272.14	482.41	54.91	427.40	1.119	.0285	.8937
+ 120	580.66	42267	293.49	478.79	55.03	423.75	1.045	.0287	9569
+ 125	585.66	45528	316.16	475.45	55.09	420.39	0.970	.0289	1.0309
+ 130	590.66	48978	340.42	472.11	55.16	416.94	0.905	.0291	1.1049
+ 135	595.66	52626	365.16	468.75	55.22	413.53	0.845	.0293	1.1834
+ 140	600.66	56483	392.22	465.39	55.29	410.09	0.791	.0295	1.2642
+ 145	605.66	60550	420.49	462.01	55.34	406.67	0.741	.0297	1.3495
+ 150	610.66	64833	450.20	458.62	55.39	402.23	0.695	.0299	1.4388
+ 155	615.66	69341	481.54	455.22	55.43	399.79	0.652	.0302	1.5337
+ 160	620.66	74086	514.40	451.81	55.46	396.35	0.613	.0304	1.6343
+ 165	625.66	79071	549.04	448.39	55.48	392.94	0.577	.0306	1.7333

TABLE VII.

SATURATED VAPOR OF SULPHUR DIOXIDE (SO_2).

Temperature, Degrees Fahrenheit, T.	Temperature, Absolute, τ.	Pressure, lbs. per sq. ft. p.	Pressure, lbs. per sq. in.	Heat of Vaporization. Thermal Units. h_e.	External Heat, Thermal Units. $\frac{pv}{J}$	Internal Heat, Thermal Units. $h_e - \frac{pv}{J}$	Volume of Vapor per lb. cu. ft. v.	Volume of Liquid per lb. cu. ft. v_1.	Weight of a cu. ft. of Vapor lbs. $\frac{1}{v}$
—20°	440.66°	845.526	5.878	175.829	13.487	162.342	12.40572	.01056	.08068
—15°	445.66°	950.986	6.604	174.790	13.580	161.210	11.12000	.01061	.09001
—10°	450.66°	1133.102	7.868	173.683	13.754	159.829	9.45485	.01066	.10590
— 5°	455.66°	1296.399	9.002	172.506	13.877	158.629	8.33867	.01070	.12008
± 0°	460.66°	1483.226	10.300	171.260	13.991	157.269	7.34900	.01078	.13627
+ 5°	465.66°	1690.709	11.741	169.945	14.095	155.850	6.49701	.01084	.15418
+10°	470.66°	1921.648	13.344	168.562	14.199	154.363	5.75960	.01092	.17395
15°	475.66°	2176.586	15.115	167.109	14.281	152.828	5.11554	.01096	.19909
20°	480.66°	2457.702	17.067	165.587	14.360	151.227	4.55676	.01101	.21999
25°	485.66°	2776.812	19.214	163.997	14.430	149.567	4.06859	.01107	.24645
30°	490.66°	3105.809	21.568	162.337	14.490	147.847	3.64071	.01113	.27551
35°	495.66°	3476.634	24.309	160.608	14.540	146.068	3.26692	.01110	.30734
40°	500.66°	3881.261	26.953	158.811	14.580	144.231	2.93377	.01125	.34217

45°	505.66°	4430.718	30.764	156.944	14.609	142.335	2.57652	.01131	.38983
50°	510.66°	4800.128	33.334	155.009	14.627	140.382	2.38216	.01138	.42180
55°	515.66°	5318.562	36.934	153.004	14.634	138.370	2.15218	.01145	.46713
60°	520.66°	5879.188	40.828	150.931	14.630	136.301	1.94756	.01151	.51652
65°	525.66°	6484.184	45.029	148.788	14.614	134.174	1.76507	.01158	.57029
70°	530.66°	7135.782	49.554	146.577	14.587	131.990	1.60197	.01164	.62880
75°	535.66°	7836.195	54.418	144.297	14.546	129.751	1.45590	.01171	.69243
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